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Применение вычислительных машин в инженерно-экономических расчетах

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The Russian Alphabet and Transliteration

А а	a	К к	k	Х х	kh
Б б	b	Л л	l	Ц ц	ts
В в	v	М м	m	Ч ч	ch
Г г	g	Н н	n	Ш ш	sh
Д д	d	О о	o	Щ щ	shch
Е е	e	П п	p	Ъ ъ	"
Ё ё	e	Р р	r	Ы ы	y
Ж ж	zh	С с	s	Ь ь	'
З з	z	Т т	t	Э э	e
И и	i	У у	u	Ю ю	yu
Й й	y	Ф ф	f	Я я	ya

The Greek Alphabet

Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ ϑ θ	Theta	Π π	Pi	Ω ω	Omega

На английском языке

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Preface

The level of advancement in technological culture primarily depends on the development of energy sources for the needs of man. The use of steam and particularly electricity over the last one hundred years brought industrial revolution and gave a tremendous impetus to the development of society.

In the last few decades worldwide production of electric energy has increased a hundredfold. The electric power of generating plants has grown to 2 700 mln kW. If the rates of growth of generated energy remain the same, in 50 years from now the output of energy will reach 0.2% of the total energy the earth receives from the sun.

Electric generators produce almost all the electric energy used, two-thirds of which is fed to electric motors to be converted to mechanical energy. Each year industry turns out tens of millions of electric machines and transformers. In serial production now are turbine-driven generators of 500, 800, and 12 000 MW, hydroelectric generators of 700 MW, and transformers rated at 1 000 MVA. Today, motors and generators are an essential part of the fabric of living, serving diverse purposes in industry, agriculture, and in the home.

Electric machine engineering naturally owes its advances to the development of the theory of electromechanics—a branch of physics dealing with the processes of electromechanical energy conversion. Electric machines include any electromechanical energy converters (ECs) destined for various purposes. Electromechanical converters come in a great variety of designs and can concentrate energy in magnetic, electric, and electromagnetic fields.

The equations of electric machines are written proceeding from the theory of electric circuits, keeping in mind that energy converts in the air gap and the magnetic field is known. The mathematical model for an infinite spectrum of fields and any number of loops on the rotor and stator is the model of a generalized electromechanical converter—an electric machine with m and n windings on the stator and rotor.

The equations for the generalized converter offer the possibility of working out a mathematical model for practically any problem encountered in modern electric machine engineering.

The present book deals with a mathematical theory of electric machines that uses differential equations as its base. It covers the

mathematical models of electric machines having a circular field and an infinite spectrum of fields in the air gap. Analysis is given of the equations involving nonsinusoidal asymmetric supply voltages and nonlinear parameters and also to multiwinding machines and machines with several degrees of freedom. An attempt is made to adapt the achievements in the area of magnetic-field converters for use in the analysis of electric-field and electromagnetic-field converters.

The book covers topics devoted to the application of electronic computers to the solution of problems in electromechanics. It is expected that the reader is already familiar with computers, programming, and algorithmic languages. The author's objective is to teach the student how to formulate equations for most of the problems in the analysis of the energy conversion processes in electric machines and reduce them to a convenient form for their solution by computers. Much consideration is given to analysis of the obtained solutions. Three chapters are devoted to the synthesis of electric machines and the computer-aided design system; the latter being the highest achievements in electromechanics.

Primary attention is focused on differential equations of electromechanical energy conversion, which form the most general and rigorous mathematical model for describing both transient and steady-state modes of operation. Polynomial models are also given due treatment.

The textbook had its origin in a series of lectures, "Application of Computers for Engineering and Economic Calculations", and in a special course, "The Mathematical Theory of Electric Machines", taught by the author at the Moscow Power Engineering Institute. In organizing the book, the author has also used the results of research conducted at the Electric Machinery Chair and in the Laboratory for the analysis of problems in electric drive, electric machines and apparatus at the same institute.

The present book is designed for students and postgraduates studying electric machines and also for electromechanical and power engineers engaged in the design and service of electric machinery.

I. P. Kopylov

Chapter 1

Introduction to Electromechanics

1.1. Historical Development

The date that marks the beginning of the age of electric machines is considered to be the year 1821 when M. Faraday constructed a motor in which a conductor *2* revolved about a permanent magnet *4* (Fig. 1.1). Mercury *3* and upper support *1* performed the function of a sliding contact. Faraday's motor fed with a dc voltage U to provide field excitation was the first magnetic-field electromechanical energy converter[1].

In 1824 P. Barlow described a motor consisting of two copper gear wheels fastened on one shaft and located between the poles of permanent magnets. Barlow's wheel was in contact with mercury and rotated fast with the passage of current.

In 1831 M. Faraday discovered the law of electromagnetic induction—one of the most important phenomena of electromechanics—which made possible the development of new types of electric machines. The essence of the phenomenon disclosed by Faraday consists in the following. If a magnetic flux linking a conducting loop is made to vary, electromotive forces appear in the loop and an electric current starts circulating over the closed loop.

In 1832 Pixxi suggested an ac generator with a revolving horseshoe permanent magnet *1* and stationary coils *2* wound on steel cores (Fig. 1.2).

In 1834 B. S. Yakobi developed a motor working on the principle of attraction and repulsion of permanent magnets and electromagnets (Fig. 1.3). Switching on and off the electromagnets provided continuous circular motion. In 1838 Russian engineers installed forty motors combined into units on a boat which could run upstream the Neva river with twelve passengers on board. That was the first attempt to harness electric motors for practical purposes.

In 1860 A. Pacinotti and, later, in 1870 Z. Gramme suggested a ring armature (Fig. 1.4). The Gramme ring armature consisted

[1] Throughout this book we will use, where necessary, the general term magnetic-field energy converter to denote the class of machines that covers induction (asynchronous), synchronous, and direct-current types. Also, we will assign the respective names electric-field converter and electromagnetic-field converter to electrostatic converters and converters in which the working field that concentrates energy is an electromagnetic field.—*Translator's note.*

of a ring-type magnetic core *4* (made up from steel wire in early machines) carrying an armature winding *2* in the form of a continuous spiral. In early machines, brushes *3* directly slided over the continuous winding and offered commutation by closing the turns.

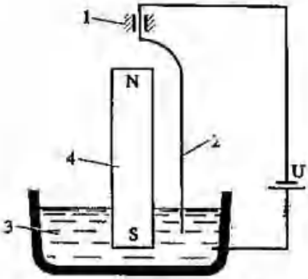

Fig. 1.1. Faraday's motor

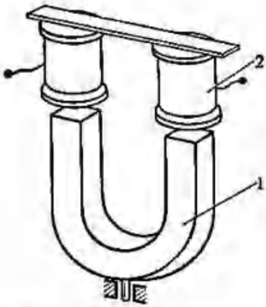

Fig. 1.2. Pixxi's generator

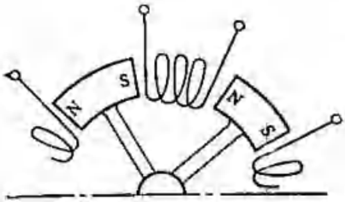

Fig. 1.3. Yakobi's machine

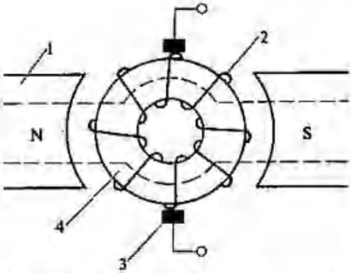

Fig. 1.4. Pacinotti-Gramme's machine

The magnetic field was produced by magnets *1* or electromagnets. While Yakobi's machine had its winding open, in the latter machine the winding was continuous (closed). The Gramme machine made a start for the evolution of commercial electric machines. It had all the basic elements of modern electric machines.

In 1873 F. Hefner-Alteneck and W. Siemens replaced the ring armature by the armature of the drum type. Since 1878 manufacturers have began to produce drum armatures with slots, and, since 1880, armatures from laminations following the suggestion put by Thomas A. Edison.

In 1885 the Hungarian electrical engineers proposed a single-phase, shell-type and core-type, transformer with a closed magnetic circuit.

In 1889 M. O. Dolivo-Dobrovolsky developed a three-phase asynchronous motor and a three-phase transformer. From 1890 the

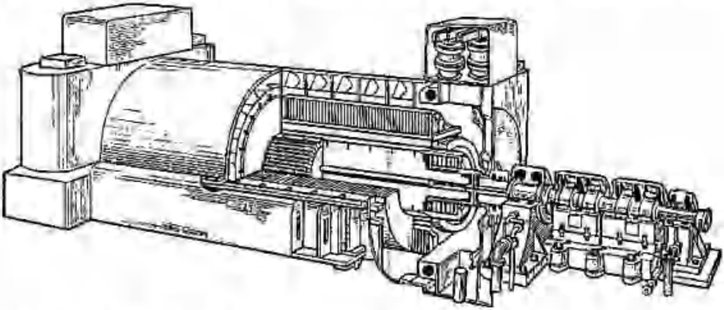

Fig. 1.5. A 1.2 mln kW, 24-kV turbogenerator at rated speed of 3 000 rpm

three-phase system received general recognition and marked the beginning of wide application of alternating current.

At the end of the last century, first district and city power plants came into being. In 1913 the total power of electric stations in Russia amounted to 1 mln kW. In 1975 the total output of electric energy produced in the Soviet Union exceeded 1 000 billion kW h.

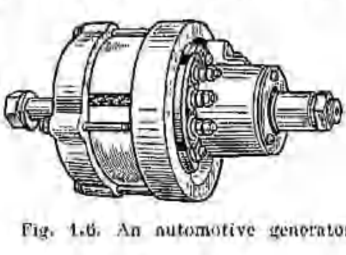

Fig. 1.6. An automotive generator

Electric machines of an efficiency of 99% are now built with the amount of active materials spent on their construction not exceeding 0.5 kg per kW. Fig. 1.5 illustrates a 1.2 mln kW turbine generator operating at the Kostroma power plant.

Electric machine industry produces ac and dc machines to serve a great variety of functions in all branches of industry. Electric machines do jobs in space, under water and under the ground. Special machines are available for work in atomic reactors and spacecraft. Millions of electric machines find use in household appliances, automobiles, tractors, and other kinds of transport. Fig. 1.6 shows an automotive generator. The Soviet industry turns out some dozens of automotive generators of this and other types every minute.

Electric machines enjoy wide application in timing devices, navigation systems, and as different transducers. The power of electric machines varies from fractions of a watt to a million kilowatts, the voltage from fractions of a volt to a million volts, the rotational speed from a few revolutions a day to 500 000 rpm, and the voltage frequency from zero to 10^{12}-10^{14} Hz.

Despite the great progress made in electromechanics, very many problems still remain to be solved. Electromechanical energy conversion occurs in fusion reactors and in biological species too. Further advances in the creation of new types of electric machines will in the main be determined by the level of development of the theory of electromechanical energy conversion.

Although the development of electromechanics is considered to begin with Faraday's discovery, electric machines existed long before his discovery. Much effort was spent on evolving electric-field energy converters, i.e. electrostatic (frictional) machines with electric-field energy storage. In 1650 Otto von Guericke described the first electric machine that represented a rotating ball from sulfur.

At the beginning of the 18th century Francis Hawksbee replaced the sulfur ball by a hollow glass ball fitted on the shaft. In 1743 frictional machines with an isolated metal electrode appeared. The electrode collected electric charges and so the machine could continuously feed power to the external circuit. The subsequent years of the 18th century saw further gradual improvements in the design and performance of frictional machines. Among Russian scientists, M. V. Lomonosov, G. V. Rikhman, A. T. Bolotov, and others were engaged in the work on these machines. By the end of the 18th century one more type of frictional machine came into being in which the rotor was made from glass disks up to two meters in diameter. The machine could produce sparks over a meter long.

In the 19th century work on the improvement of frictional machines continued and resulted in the evolution of unique electric-field energy converters. In 1936 the Van de Graaff generator (electrostatic accelerator) was built which delivered a power of 6 kW at a voltage of 6 mln V. Such setups are designed to test electrical equipment.

Electric-field energy converters which appeared much earlier than magnetic-field energy converters did not find wide practical applications. The emergence of magnetic-field type ECs in the 19th century was a new stage in the history of electric machine engineering and brought about a scientific and technical revolution in this field.

The history of development of the theory of electric machines may conditionally be divided into three stages. The first stage includes the period of creation of early machines and development of the classical theory of electromechanical energy conversion. The second

stage embraces the period of elaboration and introduction of the theory of steady-state processes, complex equations, equivalent circuits, and phasor diagrams. The third stage began in the late 1920s with the formulation of differential equations and development of the theory of transient processes in electric machines. The theory of electric machines was given consideration in the works of A. M. Ampere, G. Ohm, J. P. Joule, Heinrich F. E. Lenz, Herman L. F. Helmholtz, M. V. Lomonosov and other prominent physicists of the 18th and the 19th century. The works of James C. Maxwell who generalized the achievements of electrical engineering in his *Treatise of Electricity and Magnetism*, 1873, hold a particular place. Maxwell introduced the new electromagnetic theory and postulated equations which came to form the theoretical base of electromechanics.

Of much importance are also the works of N. A. Umov (1874) and John H. Poynting (1884) on the transfer and conversion of energy. The first theoretical work concerning electric machines may be considered the work of E. Arnold on the theory and design of windings of electric machines, issued in 1891.

In the 1890s M. O. Dolivo-Dobrovolsky, Gisbert Kapp, and other scientists set forth the fundamentals of the theory and design of transformers. In 1894 A. Heyland theoretically substantiated the circle diagram of an induction machine, and in 1907 K. A. Krug offered an accurate proof of the circle diagram. In the 1920s B. Fortescue suggested the method of symmetric components.

In the 1930s E. Arnold, R. Richter, A. Blondel, L. Dreyfus, M. Vidmar, Charles P. Steinmetz, K. A. Krug, K. I. Shenfer, V. A. Tolvinsky, and M. P. Kostenko considerably extended and advanced the theory of steady-state operation of electric machines.

R. Rudenberg's work was one of the first contributions to the theory of transient processes. This theory, whose origin dates from the beginning of this century, made a tremendous step forward in the 1950s and 1970s owing to the wide application of computers.

The first papers concerned with the mathematical theory of electric machines appeared in the middle of the 1920s, in the 1930s and 1940s. Among the authors, mention should be made of R. Park, A. A. Gorev, G. Kron, and G. N. Petrov. The fundamental works of G. Kron greatly contributed to the development of the mathematical theory. He suggested the model of and derived equations for the generalized (primitive) electric machine.

In the last years the mathematical theory of electric machines (magnetic-field energy converters) has developed to a noticeable extent owing to the efforts of many authors, first of all, B. Adkins, L. N. Gruzov, A. G. Iosifyan, E. Ya. Kazovsky, K. Kovach, V. V. Khrushchev, I. Racz, S. V. Strakhov, D. White, and H. Woodson. The use of electronic computers has enabled researchers to

analyze steady-state processes as a particular case of transients and approach the problem of developing computer-aided design systems.

The theory of electrostatic machines, however, slipped behind despite the pooling of efforts of such prominent scientists as A. F. Ioffe, N. D. Papaleksi, L. I. Mandelshtam, A. E. Kaplyansky, A. A. Vorobyev, and others, largely because their investigations failed to create the production prototypes of these converters.

At present one of the important tasks of the mathematical theory of electric machines is to develop the general theory of all the three classes of energy converters.

The chapters below consider energy converter equations, their transformation and use for most of the basic problems dealt with in the analysis and synthesis of electric machines; present equations and their solutions by computers for machines exhibiting a circular field and an infinite spectrum of fields in the air gap; examine converters involving nonsinusoidal asymmetric voltages, changes in the frequency and amplitude of supply voltage, machines with nonlinear parameters, asymmetric machines; etc. The coverage also includes converters with a few degrees of freedom, linear machines, electric-field electromechanical energy converters, and other electric machines. The theory of energy converters is set forth on the basis of differential equations which describe the dynamic behavior and, as a particular case, the steady-state behavior. The course in the mathematical theory of electric machines gives the base for the mathematical description of the processes of energy conversion taking into account nonlinear, nonsinusoidal, asymmetric aspects and manufacturing factors. Such an analysis is impossible to do employing steady-state equations, equivalent circuits, and phasor diagrams. The electromechanical energy conversion theory presented in this book enables the engineer to use the equations for the generalized electromechanical energy converter as the base from which he can set up equations for solving any problem met with in the practice of electric machine engineering.

1.2. The Laws of Electromechanical Energy Conversion

Although the theory and practice of electromechanical energy conversion have a long history and achieved great successes, the basic energy conversion laws have been stated only quite recently. Let us formulate these laws.

First law. *The efficiency of electromechanical energy conversion cannot equal 100%.*

All energy converters can be divided into simple and complex ones. In simple converters, the energy of one form is converted to the energy of another form. An example is the conversion of electric

energy to heat in an electric heater. In complex converters, which constitute the majority of machines, the energy of one form is converted to the energy of two forms (and, rarer, to three or more forms). These are converters of energy from luminous to electrical form, chemical to mechanical form, nuclear to electrical form, etc. In complex converters there commonly occurs an attendant conversion of energy to heat.

Electromechanical energy converters belong to the group of complex converters because the processes of energy conversion here always go with the conversion of electric energy P_e or mechanical energy P_m to thermal energy P_{th}. ECs exhibit the flows of electromagnetic, mechanical, and thermal energies (Fig. 1.7).

The objective pursued in evolving an EC is to reduce the loss—thermal energy flows—and thus to decrease the overall dimensions

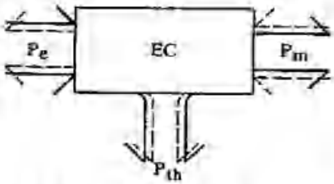

Fig. 1.7. The energy flow distribution in an electric machine

The directions of energy flows in motoring and generating operation are shown by solid and dash lines respectively

Fig. 1.8. The electric machine as a twoport

and cost of the machine. The efficiency of some converters available today reaches 98%, and that of transformers runs as high as 99.8%, which is indicative of exceptional technical achievements.

It is to be borne in mind that high efficiencies are achievable in high-power converters. In low-power ECs the efficiency reaches merely a few percent since the major amount of mechanical or electric energy evolves as heat.

It is impossible to produce an electric machine in which conversion of energy to heat would be nonexistent; otherwise it must be furnished with superconducting windings. As will be shown below, electromechanical energy conversion equations have no solutions at zero resistances.

We can visualize a lossless machine (without iron and having superconducting windings), but to enable such a machine to convert energy, we need to insert a resistance into the current network external to the machine. In this arrangement, it is the electromechanical system beyond the machine that develops losses. An electric

machine can be treated without regard to the external electromechanical system only under definite conditions, when, for example, the line resistance is equal to zero, i.e. the machine operates from or into the bus of infinite power.

The processes of electromechanical energy conversion must be studied with due regard for all electrical and mechanical loops.

An EC that does not develop losses becomes a storage or tank of energy rather than the energy converter. Energy storage devices are electrical engineering arrangements resembling in design electric machines.

Energy storage devices can be built as both static devices and rotating machines, for example, as a gyro with superconducting windings. This is an electric machine that can rotate permanently since there is no loss in it. But an antitorque moment applied to its shaft will bring the machine to a stop. This machine cannot act as an energy converter.

An electromechanical converter can be represented as a twoport (Fig. 1.8) accepting, for example, stimuli (voltage U and electrical frequency f) at a pair of electrical terminals (an electrical port) and producing responses (a torque M on the shaft and rotational frequency n) at a pair of mechanical terminals (a mechanical port). The twoport representation of an electric machine applies to solving problems in electromechanics where the processes of energy conversion inside the machine do not have a dominant significance.

Second law. *All electromechanical converters are reversible, i.e. they can act as motors and as generators.*

The reversibility is an important advantage of ECs over other energy converters such as steam turbines, diesel engines, jet engines, etc. The energy-conversion mode of operation of an electric machine depends on the moment of resistance (torque or antitorque) on its shaft, M_r. If the electric energy is drawn from the power line, the EC operates in the motoring mode. If the flow of mechanical energy delivered to the EC shaft transforms to the flow of electromagnetic energy, the machine operates in the generating mode.

The active power reverses its direction with a change of the operational function from generation to motoring, but the flow of thermal energy does not generally change its direction. Losses in common ECs are irreversible.

There is a great variety of ECs including electric machines which convert heat to electric or mechanical energy. The discussion of such ECs is given in Ch. 9.

To provide linkage between windings (loops) and currents it is necessary to produce an electromagnetic field. The rotating field in electric machines is set up by alternating or direct currents. The reactive power may flow in an EC operating in the steady state from either the stator or rotor, or from both simultaneously.

One of the corollaries of the first and the second law is that an EC also represents an energy concentrator. The electromagnetic energy, being distributed at infinity along an electric power line, is stored in magnetic-field energy converters within the air gap between the stator and rotor. In transformers, the energy is stored in the magnetic core and in the space between the primary and secondary, where leakage fluxes close on themselves, failing to be common to both windings.

The air gap of a comparatively small volume can concentrate huge powers. It is of importance to note that in turbine generators of maximum powers and in induction machines of the single series, the power density (W/mm^3) in the air gap is equal to approximately 0.5. In view of this fact, designing of electric machines can be begun with the estimation of the gap volume and then proceeded with the calculation of windings and geometrical parameters of the magnetic system. Active and reactive flows of energy can be coincident or opposite in direction irrespective of whether the EC runs as a generator or motor. This means that the active power may come from the stator and the reactive power from the rotor, and vice versa.

ECs also operate in the no-load condition at which they convert electric or mechanical power into heat. Synchronous machines connected in parallel with the line and run at no load are called *synchronous capacitors.*

During its operation, an electric machine releases thermal energy. It is possible to produce an electric machine furnished with a thermopile in order to absorb heat inside the machine at the cold junctions as a result of the Peltier effect (thereby preventing it from heating) and to evolve thermal energy at the hot junctions outside the machine. However, the available semiconductor couples offer cooling at low current densities, so the gain resulting from the improved cooling can only be brought about at the cost of an increase in the overall dimensions of the machine and a worsening of its energy characteristics. This attests that the thermal energy fluxes as well as the mechanical energy and electric energy fluxes in an EC must be regarded as closed energy loops.

The condition of resonance exists in electric machines just as it does in most energy converters. Electrical and mechanical phenomena that occur in ECs are resonant. Electric machines exhibit electromechanical resonance at which the rotational speed of the field, f_1, is related to the mechanical rotational speed of the rotor, n, measured in revolutions per second, by the expression

$$f_1 = pn \tag{1.1}$$

where p is the number of pole pairs.

In a two-pole machine, the power line frequency and the synchronous speed of the rotor are the same. Electric machines are built

in such a manner that the wave of a magnetizing force in the air gap distributes itself integrally among the poles, so the processes of energy conversion in two-pole and multipolar machines are essentially identical, the only difference being that in the latter machines the synchronous speed of the field and the mechanical speed of the rotor are a factor of p lower.

Third law. *Electromechanical energy conversion is due to the fields that are stationary with respect to each other.*

The rotor and stator fields in the air gap of a machine, which are stationary with respect to each other, produce a resultant field and electromagnetic torque:

$$M_e = \omega_s^{-1} P_{em} \tag{1.2}$$

where ω_s is the angular velocity (speed) of the field; and P_{em} is the electromagnetic power.

The fields displacing in the air gap with respect to each other produce a flux of thermal energy, thus indirectly affecting the distribution of the fluxes of mechanical and electric energies.

The windings of electric machines must carry polyphase currents and show a proper arrangement to produce a rotating field in the air gap. A rotating field can be set up by a two-phase current system, with the windings displaced 90° in space from one another and the currents shifted in time by 90°; by a three-phase current system, with the windings 120° apart in space and 120° in time; and, in the general case, by an m-phase current system, with the windings displaced $360°/m$ in space and currents shifted $360°/m$ in time. Direct current can also produce a rotating field, in which case the dc winding must rotate. The winding carrying alternating currents to produce a rotating field are usually stationary.

In a synchronous machine, the rotating field is largely set up by the currents in the windings disposed on the stator. The field rotates at a speed ω_s. The rotor runs at the same speed, $\omega_r = \omega_s$, therefore the frequency of the rotor current is $f_2 = 0$, i.e. direct current flows through the rotor winding.

In a dc machine, the field (excitation) winding is on the stator, and the excitation field is stationary. Rotating the armature, which is the rotor here, produces the rotating armature field, which revolves at the same speed as the rotor but in the opposite direction.

In induction machines, the frequency of current in the rotor is

$$f_2 = f_1 s \tag{1.3}$$

where the slip (speed differential that is a fraction of synchronous speed)

$$s = (\omega_s \pm \omega_r)/\omega_s \tag{1.3a}$$

Therefore, the speed (angular velocity) of the rotor ω_r plus the speed with which the rotor field travels with respect to the rotor structure is always equal to the speed of the field ω_s. If the rotor turns at a speed higher than ω_s in the same direction as the field excited by stator currents, the rotor field travels in the opposite direction to the rotor, so the stator and rotor fields are again stationary with respect to each other.

In transformers the windings are stationary, and thus the frequencies in the primary and secondary are the same. It can then be assumed that the fields of the primary and the secondary travel at the same speed. The concept of stationarity of fields in transformers is of little consequence for the analysis of the processes of energy transformation.

The third law facilitates the analysis of energy conversion processes in electric machines and forms the basis for the representation of energy conversion equations.

For electric-field and electromagnetic-field energy converters the field stationarity concept does not have such a great significance as it does for magnetic-field energy converters. These converters are most vividly represented as energy concentrators exhibiting electromechanical resonance.

Since electromechanics is part of physics, all basic physical laws are applicable to electric machines. To these belong first of all the law of energy conservation, Ampere's law (circuital law), Ohm's law, etc. At the root of the equations describing energy conversion in electric machines are Maxwell's equations and Kirchhoff's laws.

1.3. Application of Field Equations to the Solution of Problems in Electromechanics

Electromechanical energy conversion in magnetic-field type machines occurs in the space where the machine concentrates the energy of a magnetic field. Knowing the field, we can estimate voltages, currents, mechanical torques, losses, electrical parameters, and other quantities of interest under the steady-state and transient conditions. The calculation of the electromagnetic field in any energy converter, be it a simple or an intricate type, presents a complicated problem, and its solution involves difficulties even with the use of modern means and most advanced methods available. The electromagnetic field analysis is one of the main aspects that always attracts attention of researchers. The requirements for the accuracy of electromagnetic field calculations become increasingly stringent because of the growth of the specific and total powers of energy converters and more severe temperature conditions in which they have to operate at high efficiency and improved reliability.

Over the past few decades a large number of both special and universal methods have appeared for the analysis and calculation of electromagnetic fields.

Maxwell's equations are the base from which one starts with the calculation of an electromagnetic field. They are usually given in differential form. One of the equations establishes the relation between the vector of magnetic field strength $\overline{H}$ and the vector of current density $\overline{j}$

$$\operatorname{curl} \overline{H} = \overline{j} \tag{1.4}$$

Integrating both sides of the equation over the area S_l for, say, the simple two-dimensional case of a magnetic field

$$\int_{S_l} (\operatorname{curl} \overline{H})_k \, d\overline{S} = \int_{S} (\overline{j})_k \, d\overline{S} \tag{1.5}$$

and applying Stokes theorem

$$\int_{S_l} (\operatorname{curl} \overline{H})_k \, dS = \oint H \, dl$$

we arrive at the well-known circuital law (Ampere's law)

$$\oint \overline{H}_l \, d\overline{l} = \int_{S_l} (\overline{j})_k \, d\overline{S} \tag{1.6}$$

where the area of the surface under consideration is S, inside which there flows the current I of density $\overline{j}$ in the direction of vector $\overline{k}$, the current being confined within the closed loop l. For loops l completely encircling the current-carrying cross-section S, the right-hand side of Eq. (1.6) represents the total current

$$\int_{S_l} (\overline{j})_k \, d\overline{S} = I_k \tag{1.7}$$

The magnetic field vector $\overline{B}$, also referred to as the magnetic induction, or the magnetic flux density, is defined in terms of the permeability μ of a medium and the magnetic field strength H produced in the medium:

$$\overline{B} = \mu \overline{H} \tag{1.8}$$

where

$$\operatorname{div} \overline{B} = 0 \tag{1.9}$$

The divergence of the field is thus zero. This means that there is no "current" flowing in and out of a magnetic field (magnetic lines never end but close on themselves), i.e. free magnetic charges (mono-

poles) do not exist in nature. The magnetic field components $\bar{B}$ and $\bar{H}$ can be found if we solve the field equations for various parts of a converter of definite configuration by observing the boundary conditions of continuity for the normal components of the B field vectors at the interface between two media 1 and 2 (which differ in permeability)

$$B_{1n} = B_{2n} \tag{1.10}$$

and for the tangential components of the field strength

$$H_{1t} = H_{2t} \tag{1.11}$$

providing that current sheets on the boundary surfaces do not exist. As shown by experiment, Eqs. (1.7) through (1.11) permit defining the magnetic field analytically only for a rather limited range of problems with the simplest boundary conditions.

In considering the real parts of electric machines with rather complicated shapes of magnetic cores and current-carrying elements, a number of assumptions have to be made to obtain even an approximate solution. Simplifying assumptions may apply to surface shapes, current distributions, the properties of media, and laws of their motion. In cases where the field sources lie fairly far away from the field region under consideration (i.e. $\bar{J} = 0$), it is sometimes advantageous to introduce the notion of a magnetic scalar potential φ_m. Because of the curl-free character of a scalar field (curl $\bar{H} = 0$), the magnetic field strength $\bar{H}$ can be expressed as

$$\bar{H} = -\text{grad}\ \varphi_m \tag{1.12}$$

For a scalar field, Laplace's equation holds:

$$\nabla^2\varphi_m = \partial^2\varphi_m/\partial x^2 + \partial^2\varphi_m/\partial y^2 + \partial^2\varphi_m/\partial z^2 = 0 \tag{1.13}$$

The field lines here prove discontinuous. The sources and sinks of the field will be the surfaces having different magnetic potentials. The distribution of potentials depends on the distribution of currents in the windings of a converter and is defined up to a constant in any local region.

Most boundary conditions for a scalar magnetic field in electric machines are Dirichlet conditions. This is commonly found to be a favorable factor for the solution of a problem particularly when using approximate methods. The field calculations aim at defining the components of the magnetic field strength along the three axes

$$H_x = -\partial\varphi_m/\partial x, \quad H_y = -\partial\varphi_m/\partial y, \quad H_z = -\partial\varphi_m/\partial z \tag{1.14}$$

Knowing these components and using (1.8), we can find the B field vector components and then magnetic fluxes and flux linkages. The unit of measure of a magnetic potential is the ampere, therefore this quantity corresponds to the magnetomotive force (mmf) as

regards its meaning. The function of the flux in a potential field φ_m proves to correspond to the magnetic flux.

The calculation practice of rotational electromagnetic fields widely uses the notion of a magnetic vector potential $\bar{A}$ defined by the relation

$$\bar{B} = \operatorname{curl} \bar{A} \tag{1.15}$$

Solving simultaneously (1.6), (1.8), and (1.9), and then (1.15) gives Poisson's equation

$$\nabla^2 \bar{A} = -\mu \bar{j} \tag{1.16}$$

in which the magnetic vector potential calculated up to a constant acquires a definite physical meaning. The circulation of the vector potential over the loop is found to be equal to the magnetic flux through the surface bounded by this loop. What is important is that the shape of the surface is of no consequence and thus can be arbitrary. In the three-dimensional case, Eq. (1.16) is written for each of the three components given as the projections on to the corresponding coordinate axes. It is often permissible to consider the field of an electric machine as a flat, two-dimensional, pattern with one current component, for example, along the z axis:

$$\partial^2 A_z/\partial x^2 + \partial A_z/\partial y^2 = -\mu \bar{j}_z \tag{1.17}$$

In this case the magnetic vector potential takes on the meaning of the magnetic flux per unit length in the z direction. The B field vectors along the x and y axes are given by

$$B_x = \partial A_z/\partial y, \quad B_y = -\partial A_z/\partial x \tag{1.18}$$

The solution to the problem involving the determination of the magnetic field in electric machines is most commonly sought under the boundary conditions of the second kind (Neumann conditions). The function of the flux in the vector field $\bar{A}$ corresponds to the magnetomotive force, i.e. the function of the potential is proportional to the magnetic flux.

For defining the magnetic field, it is usual to employ similitude methods and the methods of physical and mathematical modeling. Experience attests that the notions of scalar and vector magnetic potentials equally well hold in modeling of magnetic fields, although the realization of boundary conditions when using either of these two notions is substantially different.

Where there is a need to solve the problem with consideration for induced currents, the notion of the magnetic vector potential is the only one acceptable, in which case Poisson's equation must be replaced by the so-called heat-conduction equation.

Most diverse methods apply to solve the obtained equations for a magnetic field under the conditions (1.10) and (1.11) at the boundaries between different media. Historically the methods of direct solution have developed most intensively, which commonly give an accurate or approximate analytical result. Among those, we should note the method of images and the method of separation of variables. Conformal transformations of the regions of interest, by which complex boundary conditions undergo substantial changes and become appreciably simpler, play a noticeable part in the development of the methods for the solution to magnetic field problems. The solution to Laplace's equation is worked out for relatively simple areas and then applied to the initial region. The invariants, i.e. quantities invariable in transformations, are magnetic potentials, magnetic fluxes, and the moduli of the magnetic flux density vectors and field strength vectors. The solution itself in the transformed plane is found accurately, whenever possible, or approximately using an analytical or numerical method. The methods of conformal transformations mainly apply to irrotational fields. The methods of integral equations are suitable for the solution of a number of rotational field problems. The last few decades have seen an exceptionally rapid development of the approximate numerical techniques based on the methods of finite differences and finite elements.

The progress in computer engineering and the creation of fast computers with a large memory capacity have enabled the effective introduction of these approximate methods. They permit obtaining the solution of a desired function (potential) in the field region for each particular case. A substantial disadvantage of these methods is that they do not allow for deriving the general expression for the solution, so it becomes necessary to obtain a new solution with any change of the parameters affecting the field. However, the potentialities of computer engineering greatly offset this inconvenience.

Electromechanical energy conversion is the result of interaction of electromagnetic forces appearing in an energy converter. The determination of these forces is the most important stage in designing a converter. There are a few approaches to attacking this problem. A mechanical interaction of currents, or what is sometimes called pondermotive interaction, obeys Ampere's law. For a conductor carrying current i and placed in an external magnetic field $\overline{B}$, the emf f is given by the vector product:

$$\overline{f} = [\overline{l}\overline{B}]i \tag{1.19}$$

where l is the unit vector along the wire carrying current i.

Where the magnetic field is known from the solution of Maxwell's equations, it is convenient to express emfs in terms of the current

called the tensor of tension, the expression for which can be reduced to the form

$$T_n = \mu_0 H_n \overline{H} - \mu_0 (\overline{n} H^2/2) \tag{1.20}$$

where H_n is the vector component of the magnetic field strength $\overline{H}$ in the direction of the unit vector $\overline{n}$ normal to the surface region under study. Upon integrating the tension tensor over the entire surface where the magnetic field is substantially high in magnitude, we can then go to the computation of emfs and electromagnetic torques.

It is sometimes expedient to determine electromagnetic forces and torques from the expression of mutual specific energy $\partial W/\partial V$ referred to unit volume, which is equal to the scalar product of the current density and the vector potential of an external magnetic field:

$$\partial W/\partial V = \overline{A}\overline{j} = -\overline{B}\overline{H} \tag{1.21}$$

The next Maxwell's equation, which is of much importance, relates the vector of electric field strength $\overline{E}$ to the magnetic flux density:

$$\text{curl } \overline{E} = -d\overline{B}/dt \tag{1.22}$$

In its integral form, the expression allows us to pass to the expression for the emf E of a loop (disregarding the gradient of a scalar electric potential):

$$E = \int\limits_{S_l} -(d\overline{B}/dt)\, dS + \oint [\overline{V} \times \overline{B}]\, dl \tag{1.23}$$

The vectors of $\overline{B}$ and $\overline{H}$ give us ample information on the magnetic field and hence on all integral quantities such as currents, emfs, voltages, forces, and torques.

The classical theory of electric machines relies on the equations of circuit theory which defines the parameters in integral notation.

The most important parameter of an energy converter is its inductance L defined as the ratio of the instantaneous values of flux linkage Ψ produced by the current i to this current:

$$L = \Psi/i \tag{1.24}$$

If the flux due to current in a winding or conductor links only this winding, we can talk about self-inductance; where the flux links one winding due to current in the other, we can talk about mutual inductance. To define the flux linkage for the field describable by Laplace's equation, it is necessary to apply Eqs. (1.14) and (1.18) in order to go to the expression for the magnetic flux density and then integrate the magnetic fluxes for a conductor over its entire cross-section S. The flux linkage, when expressed in terms of the

magnetic vector potential, is defined with respect to A_0 taken as the reference for counting off the running values of vector potential A_i existing in the cross-section S:

$$\Psi = \int_S (A_i - A_0)\, dS_i/S \qquad (1.25)$$

The problem of determining the flux linkage practically reduces to simple arithmetic operations if the conductor is broken down into a finite number of elementary areas each of which has a definite value of A_i found from the calculation of the field.

For the case when the flux for all points in the cross-section of the conductor of a winding (with a certain number of turns) is constant, the flux linkage can be expressed as

$$L = \Psi/i = \omega\Phi/i \qquad (1.26)$$

Introduce the notion of permeance Λ

$$\Lambda = \Phi/F \qquad (1.27)$$

where F is the magnetomotive force (mmf) of a conductor (winding). The inductance now becomes independent of the current and flux and is only a function of permeance:

$$L = (\omega F\Lambda)/i = (\omega i \omega \Lambda)/i = \omega^2\Lambda \qquad (1.28)$$

In a particular case when air gaps are taken into consideration

$$L = \omega^2\Lambda = \omega^2\mu_0\lambda \qquad (1.29)$$

where $\lambda = \Lambda/\mu_0$ is the coefficient of permeance for fluxes produced by the mmf. The resultant relation (1.29) masks somewhat the nature of origin of inductance and makes this parameter apparently dependent only on the geometrical dimensions and types of material. However, we should recall the initial relation (1.24) from which it unambiguously follows that such a parameter as inductance is not at all intrinsic in any conductor or winding but is indicative of the conditions of existence of a magnetic field in an energy converter. Inductances do not remain constant but vary quite appreciably when (a) fluxes change slowly, (b) short-circuited contours lie on the paths of magnetic fluxes varying in time and amplitude, (c) hysteresis makes itself felt, and (d) the portions of converter magnetic circuits display nonlinear characteristics of magnetization. In the theory of electric machines, for example, this fact is taken into consideration in a number of ways, but a sufficiently consistent approach does not exist. The reason is that the task of quantitatively considering all the influences is extremely complex.

It is easy to calculate the emf (at i = constant) in terms of the self- and mutual inductances proceeding from the changes in the

intrinsic energy of the field in motion

$$f = -\partial W/\partial x = -(i^2/2)(\partial L/\partial x) \tag{1.30}$$

This formula shows that a change in inductance is the requisite condition for the electromechanical conversion of energy.

Despite a relatively simple form of field equations (Laplace's and Poisson's equations) and a simple character of boundary conditions, the solution for the field of an energy converter having various boundaries, a large number of spatially arranged coils, let alone the nonlinearity phenomena and hysteresis that must be taken into account, has been found only in the last years by use of the numerical methods proceeding from yet rather numerous assumptions. With the use of analytical and semigraphical methods of calculation the number of assumptions grows still more. In particular, we can enumerate the following assumptions.

1. The main field which determines energy conversion in electric machines and gives rise to the main self- and mutual inductances of windings is plane-parallel.
2. Various leakage inductive reactances are independent of each other and of the main field. It is common to isolate permeances corresponding to slot, end, and differential leakage fluxes of ac windings.
3. The surfaces of stator and rotor cores of electric machines are smooth; the actual saliency is given due consideration by introducing air-gap coefficients.
4. The permeability of ferromagnetic section is taken infinite at the preliminary calculation stage.
5. The use of the superposition principle is permissible.
6. The processes of energy conversion are dependent on the fundamental harmonics of currents and magnetic fluxes.
7. The effect of eddy currents induced in magnetic circuits is negligible. We have cited but a few instances of all of the possible constraints.

In the list of approximate methods, the numerical methods used for the solution of field equations occupy distinct positions and have great significance. The finite difference method (FDM) is particularly popular. This method was in extensive use well before the introduction of digital computers to the calculation practice. The development of high-speed large-capacity memory computers with extensive generalized program libraries and the introduction of efficient algorithmic languages has made popular the methods of calculation of electromagnetic fields on the basis of finite difference approximations of continuity equations of the most diverse forms.

The main idea underlying the application of the FDM in electromagnetic calculations comes to the replacement of the continuous distribution of a scalar or vector magnetic potential by a discrete

distribution of the same function in a limited number of points within the region being studied. The points at which the values of the function have to be found are distributed over the region of interest; in other words, a coordinate grid is drawn on the region. In the FDM, this grid shows a regular pattern. In most extensive use are the rectangular (or quadratic, in a particular case) system and the polar system of coordinates.

Figure 1.9 shows how the rectangular grid (network) divides a salient-pole synchronous machine region one pole pitch in length into a few meshes. The coordinate system and the form of meshes of the grid are so chosen as to approximate most accurately the boundaries of the region and to introduce the minimum possible errors into the configurations of neighboring regions. The grid plotting at this stage is often done by the trial and error method, and depends on the experience and skill of the investigator; the procedure lends itself to automation only for local zones.

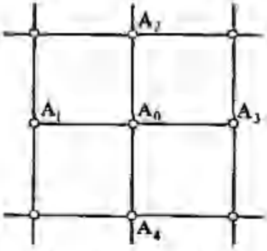

Fig. 1.9. Superimposing the square-mesh grid on the area under analysis

In accordance with the FDM, the field equations written as partial derivatives are rearranged to the finite difference form using the expressions for a Taylor series. In the case of a quadratic grid with a pitch h, the Laplacian assumes a simple form

$$\nabla^2 A \approx (1/h^2) \sum_{i=1}^{4} (A_i - A_0) \ldots \tag{1.31}$$

for a point representing a value of the function A_0 and surrounded by points $A_1 = A_4$ (see Fig. 1.9). The error of digitization here depends on the fourth-order derivatives in the sought-for function and can be reduced by decreasing the grid pitch h. The solution to Laplace's equation in the finite-difference form amounts to performing elementary arithmetic operations. The number of the nodes of the solution may in practice be very high and usually ranges into a few thousands. Therefore the solution to the obtained system of high-order equations requires the use of iterative or statistical methods. The direct solution to the system of equations using, for example, Gauss' method proves impossible. With the iterative method of calculation, the values of the function sought are preset at the first stages either arbitrarily or on the basis of certain physical considerations which subsequently improve the convergence of the

solution. By performing the multiple sequential tracing of all nodes of the grid and solving the finite-difference relation, it becomes possible to decrease the remainder of the field equation to the maximum permissible value. The number of iterations (repetitive tracings) can run into a few tens, hundreds, and even thousands. One cannot always be confident that the solution tends to the ideal value, thereby ensuring the convergence. The iterative method is rather routine, is easy to formalize for solving problems on digital computers, and is safe from calculation errors since possible errors are recoverable at subsequent steps. The effective versions of the FDM are available at present, which give good convergence at a high accuracy of the results.

A form of grid marked out on the region of interest affects the accuracy of the solution. This circumstance has recently stimulated the search for the best forms of layout of regions. It is possible to optimize sequentially the grid structure by calculating the derivatives of higher order at a definite stage of the solution with a view to raise the mesh density of the grid at the next stage in the region of higher values of these derivatives.

The method of finite elements (FEM) developed in the last years displays exceptional flexibility in breaking down the space of an electromagnetic field destined for calculation. Worked out first for the needs of structural mechanics, this method has turned out to be rather convenient for the calculation of electromagnetic fields in electric machines which have boundaries complex in configuration and exhibit nonlinearities and induced currents. The region for the function sought is broken down into a finite number of elements mostly in the form of triangles with straight or curvilinear sides. The dimensions of elements may differ substantially depending on the expected intensity of changes of the field. The desired function inside the elements is assumed to obey a certain law. In a simple case, first-power spline functions are applicable. Thus in the two-dimensional case, the function $A(x, y)$ for a triangular element with coordinates at the vertices, x_l and y_l, x_m and y_m, x_n and y_n, can be written as

$$A(x, y) = N_l A_l + N_m A_m + N_n A_n \tag{1.32}$$

where

$$N_l = (1/2\Delta)\,[a_l + b_l x + c_l y]$$

$$\begin{cases} a_l = x_m y_n - x_n y_m \\ b_l = y_m - y_n \\ c_l = x_m - x_n \end{cases} \qquad \Delta = 1/2 \begin{vmatrix} 1 & x_l & y_l \\ 1 & x_m & y_m \\ 1 & x_n & y_n \end{vmatrix}$$

A similar approach applies to determine the values of N_m and N_n.

Thus each element is describable by its own polynomial, which is so chosen as to preserve the continuity of the function along the

element boundaries. The values at grid nodes are found by using the variational principles, and in this respect the FEM is often stated in the context of the Ritz and Galerkin methods. With the variational formulation, the solution to the problem involving a two-dimensional magnetic field defined by Poisson's equation (1.16) is equivalent to the condition of minimization of a certain energy functional

$$F=\int_R\int\Big[\int_0^B(1/\mu)\,B\,dB\Big]\,dx\,dy-\int_R\int jA\,dx\,dy \qquad (1.33)$$

inside the region of integration R. The functional displays such a property that any function which minimizes it satisfies both differential equations and boundary conditions. In the case of nonlinear dependences the process of minimization involves the solution of the system of nonlinear algebraic equations, commonly using the Newton-Raphson method which gives good convergence.

The calculation of magnetic fields in electric machines with the aid of the finite-difference and finite-element methods enables a more accurate evaluation of the characteristics and parameters of electric machines. However, the FDM and FEM call for retaining a number of assumptions the legitimacy of which are not always unquestionable. One of these assumptions made in evaluating magnetic fields is that the toothed stator and rotor cores are in fixed mutual position. The position itself is most often chosen arbitrarily without sufficient reason, and the results of field calculations are taken valid for other possible mutual positions. Whenever the attempts are made to calculate magnetic fields with the toothed cores in motion, the computer-aided calculations prove so time-consuming that they become impracticable. It takes an especially long time and large memory size to calculate the air-gap band noted for the most intensive magnetic field. On the other hand, the inhomogeneity of media in this band shows a rather regular character, for which reason a large share of repeated calculations can be done away with.

Owing to the efforts of a number of scientists it has become possible to evolve the calculation method based on the representation of the fields of real windings as a set of fields of the simplest loops disposed on core teeth (Fig. 1.10). Every loop encircles one tooth. The planes of the cross-section of loop wires coincide with the planes of the cross-section of real winding wires placed in the slots. The loop may also enclose a few teeth or extend along the entire gap. Also, loops may have different sides located in slots of various shapes and sizes.

The essential point of this method called the method of permeances is that the loop field must be defined not for real but for specific

boundary conditions which can be obtained only artificially. Beyonp the confines of the loop, the permeance of the air gap between the rotor and stator is assumed to be infinite. Under such boundary conditions the field traversing the gap extends only in one direction and gets concentrated in the area that differs insignificantly from the area bounded by the loop itself. The loop mmf here corresponds to the mmf in the gap. In going away from the loop in opposite directions, the loop field decays fast. The loop field under artificial boundary conditions exhibits an interesting feature. The magnetic

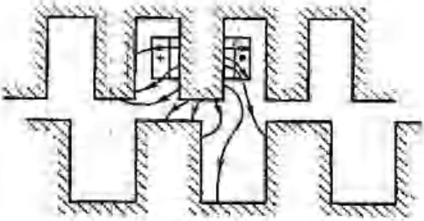

Fig. 1.10. The field of a simplest loop

flux through the gap due to the loop current is the same as the unipolar flux linking the loop when the difference of scalar magnetic potentials between the cores is equal to the loop current. Also, the permeance for the loop flux through the surface of an unexcited core corresponds to the permeance for the loop flux linkage in unipolar magnetization. This is the case for any form of the two-sided saliency and for any arrangement of loop conductors in the slots or in the grap. This fundamental property of fluxes and flux linkages of tooth-induced loops opens the way of evolving a new method to enable the development of mathematical models for describing the fields in electric machines with due regard for the two-sided saliency of cores.

This mathematical model though resembling to a definite extent the model applied in the FEM contains a special feature. A portion of model elements representing the air gap permeance $R_{\delta i}$ are not permanent and calculated beforehand either with the aid of rather circumstantial networks employed in the FDM and FEM or analytically by use of the methods of conformal transformations. The data of this calculation are entered into the computer memory in the form of approximation curves or tables. The teeth and yokes of cores are broken down into a number of elements whose dimensions can be taken appreciably larger (without the introduction of noticeable errors) than is the case with the FDM or FEM. The nonlinear characteristics of these elements are defined starting from the BH-

curves for corresponding materials. As is done in the frameworks of other methods, here too the permeability inside an individual element is considered constant. The magnetic state of core elements is first set roughly and then specified more accurately after solving the system of the nonlinear equations by the Newton-Raphson iterative method.

The mathematical model based on the permeance method uses a relatively large-size mesh pattern and gives a high accuracy of field reproduction, especially in the gap band. This opens up possibilities for the calculation of fields in the transient operation of electric machines with consideration for the effects of saliency, discreteness of the winding structure, saturation, and induced currents. The equations for all loops in the permeance method representation do not necessitate additional coordinate transformations.

Although the progress in the development of electric machine models on the basis of field equations is appreciable, the most material advancements are made by use of the equations written in the notation adopted in electric circuit theory. Therefore in the further presentation of the text we will basically employ the equations of the generalized electromechanical energy converter.

1.4. The Primitive Four-Winding Machine

All electric machines are identical in the sense that they convert energy from electrical to mechanical form or from mechanical to electrical form. But electric machines even of the same series differ from one another in performance.

The basic types of electric machines can be reduced to a generalized, or primitive, model representing a set of two pairs of windings moving with respect to each other. In Fig. 1.11 is shown the idealized model of a symmetric machine having a smooth air-gap structure and sinusoidal windings, with the permeance equal to zero. A sinusoidally varying voltage applied to the winding produces a circular field in the air gap. With the windings being symmetric, a sinusoidal symmetric voltage sets up a sinusoidal field in the gap.

The term 'primitive machine' stands for *an idealized two-pole two-phase symmetric (balanced) machine having one pair of windings on the rotor and the other pair on the stator* as shown in Fig. 1.11. Here w_α^s, w_β^s are the stator windings along the α and β axes; w_α^r, w_β^r are the rotor windings along the α and β axes; u_α^s, u_β^s, u_α^r, u_β^r are voltages along the α and β axes on the stator and rotor respectively; and ω_r is the angular speed of the rotor.

The analysis of the two-pole machine as a model enables us to extend the results and describe the processes occurring in a real multipolar machine. The two-phase machine has four windings

and is describable by four voltage equations (a minimum number of equations in comparison with those used for describing single-phase, three-phase, and m-phase machines). Consider an idealized uniform-air-gap machine whose windings are taken to be in the form of current sheets where the mmf distribution is sinusoidal. Our idealized machine has no saturation, nor nonlinear resistances, and therefore exhibits a sinusoidal field in the air gap when the windings are fed with sinusoidal voltage.

The idealized machine model is the analog of an induction machine when the stator windings w_α^s and w_β^s accept sinusoidal voltages

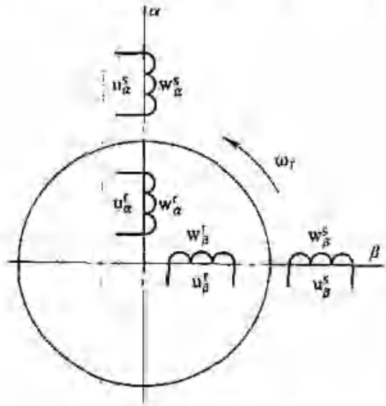

Fig. 1.11. A primitive machine

at frequency f_1, 90° apart in time. The rotor windings carry currents of frequency $f_2 = f_1 s$, either produced by the voltage applied to the rotor or induced by the currents in the stator windings. In an induction machine, the rotor angular speed is $\omega_r \neq \omega_s$ (ω_s is the synchronous speed of the field), and the rotor and stator fields are stationary with respect to each other since the mechanical rotor speed ω_s plus/minus the rotor field speed relative to ω_r is equal to ω_s.

The idealized machine model represents a synchronous machine if an ac voltage is put across the stator windings and a dc voltage across the rotor windings, and vice versa. Here $\omega_r = \omega_s$, i.e. the stator and rotor fields are stationary with respect to each other. If a dc voltage drives current through the stator windings, the rotor field travels in the direction opposite to that of the rotor, so the stator and rotor fields are stationary relative to the stationary reference frame. With dc supply to all the windings, it is enough to

have one field winding in which the resultant magnetizing force is equal to the geometric sum of the magnetizing forces of each winding.

In dc machines, the armature winding carries a multiphase alternating current rectified mechanically by means of a commutator — a frequency converter (FC). By reducing a polyphase system to a two-phase one, we obtain the model of a dc machine (Fig. 1.12). As in a synchronous machine, the armature field of the dc machine rotates

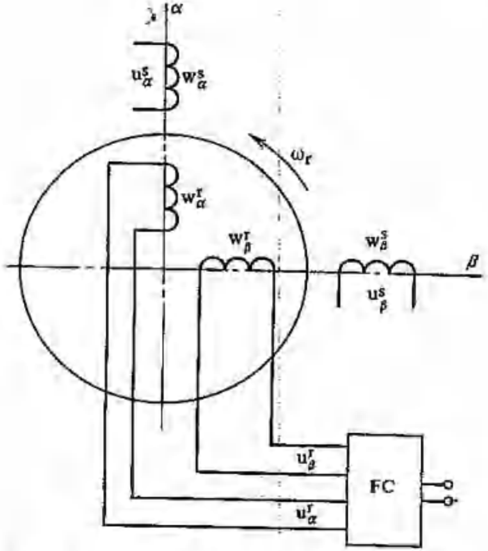

Fig. 1.12. The model of a dc machine and an ac commutator machine

in the opposite sense with respect to the armature. When $\omega_r = \omega_s$ the armature field is stationary relative to the field winding and to the stationary reference frame. It should be noted that the slip in synchronous machines and dc machines equals zero. A commutator can be replaced by a semiconductor frequency converter, reed relay converter, etc. The processes of energy conversion in the air gap do not change with the replacement of one type of FC by the other. However, a conventional commutator holds a fixed tie between the frequency and the rotor speed ω_r, while a semiconductor FC may afford the possibility of securing controllable feedback to regulate f_2 according to ω_r. As regards its power supply, a semiconductor-commutator machine is a dc machine. Historically, this type of dc

machine received several names—rectifier-type machine, semiconductor-commutator machine, contactless machine, etc.

In an ac commutator machine, alternating currents exist in the stator and rotor windings, and the frequency converter transforms the alternating current at the bus frequency into that of slip frequency (see Fig. 1.12). As in other electric machines, here the stator field is stationary relative to the rotor field. These machines can be of the single-phase, three-phase, or multiphase types; the stator and rotor windings can be connected in series or parallel, or can have magnetic coupling.

The primitive machine with a rotor speed $\omega_r = 0$ can represent an electromagnetic converter—a transformer. In this case it is sufficient to consider separately the pair of windings on the stator and rotor along the α axis or β axis because with the rotor at standstill there is no coupling between the windings shifted 90° apart in space. Although transformers perform electromagnetic conversion of energy, they belong to electric machines because of the generality of equations and for historical reasons.

The classification of electric machines into individual types is largely conventional. One and the same machine can operate as a synchronous and as an asynchronous machine. In electric machines there occurs electromechanical and electromagnetic energy conversion simultaneously.

The processes of electromechanical energy conversion in the primitive machine are described by voltage equations (1.34) and equation of motion (1.35)

$$\begin{Vmatrix} u_\alpha^s \\ u_\alpha^r \\ u_\beta^r \\ u_\beta^s \end{Vmatrix} = \begin{Vmatrix} r_\alpha^s + (d/dt)\,L_\alpha^s & (d/dt)\,M & 0 & 0 \\ (d/dt)\,M & r_\alpha^r + (d/dt)\,L_\alpha^r & L_\beta^r\omega_r & M\omega_r \\ -M\omega_r & -L_\alpha^r\omega_r & r_\beta^r + (d/dt)\,L_\beta^r & (d/dt)\,M \\ 0 & 0 & (d/dt)\,M & r_\beta^s + (d/dt)\,L_\beta^s \end{Vmatrix} \times \begin{Vmatrix} i_\alpha^s \\ i_\alpha^r \\ i_\beta^r \\ i_\beta^s \end{Vmatrix} \tag{1.34}$$

$$(1/p)\,J\,d\omega_r/dt \pm M_r = M_e \tag{1.35}$$

Eqs. (1.34) and (1.35) together with the equation for an electromagnetic torque form the fundamental system of equations of electromechanical energy conversion.

In Eqs. (1.34), u_α^s, u_β^s, u_α^r, u_β^r, i_α^s, i_β^s, i_α^r, i_β^r are the voltages and currents in the stator and rotor windings on the α and β axes respectively; r_α^s, r_β^s, r_α^r, r_β^r are the resistances of stator and rotor windings respectively; M is mutual inductance; and L_α^s, L_β^s, L_α^r, L_β^r are total inductances of the stator and rotor windings along the α and β axes respectively.

Winding inductances are defined by the known relations

$$L_\alpha^s = M + l_\alpha^s, \quad L_\alpha^r = M + l_\alpha^r$$
$$L_\beta^s = M + l_\beta^s, \quad L_\beta^r = M + l_\beta^r \tag{1.36}$$

where l_α^s, l_β^s, l_α^r, l_β^r are leakage inductances of the stator and rotor windings along the α and β axes respectively.

The mutual inductance and leakage inductances are found by the known methods involving the calculations or experimental analysis, i.e. using equivalent circuits and design formulas. The assumption is that there is a working flux which links the stator and rotor windings and also leakage fluxes linking only one winding.

Equations (1.34) describe a hypothetical machine having the same number of turns on the stator and on the rotor, with the windings being pseudostationary. To preserve the power invariance in an actual machine and in the machine with stationary windings, the equations have to contain the emfs of rotation, expressed as $L_\beta^r\omega_r i_\beta^r + M\omega_r i_\beta^s$ for the rotor winding along the α axis and as $-L_\alpha^r\omega_r i_\alpha^r - M\omega_r i_\alpha^s$ for the β-axis winding.

Kirchhoff's equations (1.34) include voltages, voltage drops across resistances, emfs of rotation that exist only in rotating windings, and transformer emfs:

$$L_\alpha^s\,(d/dt)\,i_\alpha^s + M\,(d/dt)\,i_\alpha^r, \qquad M\,(d/dt)\,i_\alpha^s + L_\alpha^r\,(d/dt)\,i_\alpha^r$$

The transformer emfs for the β-axis windings are written in a similar form.

In the equation of motion (1.35), p stands for the number of pole pairs, and J for the moment of inertia. If the analysis is made of an electric machine together with its drive mechanism, the quantity J must represent the rotor moment of inertia and the normalized moment of inertia of the mechanism.

In the analysis of electric machines, the moment of resistance M_r (torque) is usually taken constant. In the analysis of electromechanical systems, M_r can be a function of ω_r or time.

The electromagnetic torque M_e—the torque produced by a converter—is given by the products of currents flowing in the windings:

$$M_e = (m/2)\,M\,(i_\beta^s i_\alpha^r - i_\alpha^s i_\beta^r) \tag{1.37}$$

where m is the number of phases.

The electromechanical energy conversion equations suggested by Gabriel Kron in the 1930s comprise the system of five equations (1.34) and (1.35) involving five independent variables (voltages and M_r) and five dependent variables (current and angular speed). The coefficients ahead of the dependent variables, namely, resistances, inductances, mutual inductances, and the moment of inertia, are the parameters of an energy converter.

In the mathematical theory, the coefficients at variables may vary with the form of equations used, therefore it is of importance to have a clear idea of the parameters and mathematical description of the processes of energy conversion. The parameters of a machine can be constant, periodic, and nonlinear.

The analytical solution of the equations for electromechanical conversion does not exist because the equations contain product terms. The equations are solvable with the aid of computing devices, the solutions being approximate. This approach also permits handling equations with nonlinear coefficients.

The accuracy of solution of equations depends on the class of computers used. Computers can solve a simple problem even to a higher accuracy than is necessary for the engineering purpose. On the other hand, many factors which affect the processes of energy conversion in a real machine cannot be taken into account. Even the energy conversion equations with constant coefficients are nonlinear since the torque equation contains the products of variables. The addition of nonlinear terms only makes the problem more difficult.

Independent and dependent variables in (1.34) and (1.35) may vary in value, and then they describe what is called the current drive.

The system of equations (1.34) consisting of four voltage equations and the equation of motion (1.35) describes transient and steady-state modes of operation. To obtain the steady-state equations, we should replace the differential operator d/dt by $j\omega$ and work with complex equations. In the steady conditions the voltage equations can be dealt with independently of the equation of motion. The courses in electric machinery commonly cover voltage equations, and the course in electric drive mainly considers the equation of motion.

The electromagnetic torque M_e is equal to the product of currents in all of the four windings. The torque (1.37) is set up by the currents in the stator and rotor windings disposed on different axes, with the stator current shifted in phase with respect to the rotor current. If the rotor and stator windings of the primitive machine carry only active ac components, the torque M_e is zero since the coupling between the windings due to reactive currents that produce the magnetic field is absent.

The solutions to the equations of electromechanical energy conversion do not exist if any of the parameters entering into the equations is zero or goes to infinity. If the resistances and inductive reactances are at infinity the currents are equal to zero and the machine does not develop the torque M_e. At $J = \infty$, the energy converter picks up speed infinitely long. At $J = 0$, the machine cannot come up to its steady-state velocity because the rotor res-

ponses to all changes in the products of currents continuously. If the mutual inductance is zero, the magnetic linkage between the windings is nonexistent and $M_e = 0$ (1.5). If there is no resistance in the loops through which the currents complete their paths, the device will act as a storage of energy. The time constants are at infinity, the shift between currents is zero, and $M_e = 0$.

It is possible to obtain optimum relations between the parameters at which an electric machine may have a maximum efficiency, higher $\cos \varphi$, a minimum mass or a desirable form of output characteristics. It should be noted, however, that Eqs. (1.34) and (1.35) are unsuitable for use in optimization studies because the minimum values of currents (dependent variables in these equations) are not yet indicative of an optimum machine.

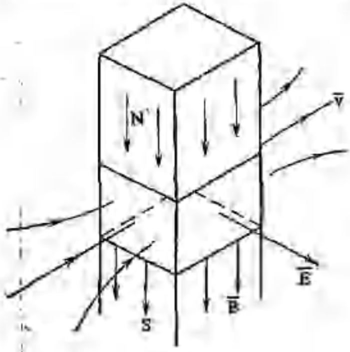

Fig. 1.13. The principle of an MHD generator

$\overline{B}$, $\overline{v}$, and $\overline{E}$ are magnetic flux density vector, conducting gas or liquid velocity, and emf induced on channel walls respectively

Considering voltage equations (1.34), we should point out that the terms defining the transformer emf include the inductances and currents under the derivative sign. In most electric machines currents are varying quantities, but the conversion of energy from electrical to mechanical form is possible if currents are constant and inductances undergo variations in a sinusoidal manner. The machines performing energy conversion in this manner comprise the class of parametric devices, among which inductor machines are most popular. In the general case, both inductances and currents in electric machines vary sinusoidally.

Given the mathematical description of the processes in electric machines, let us inquire into the nature of energy conversion in the machines. The general conclusion that can be drawn from the consideration of the laws and equations of energy conversion comes to the following: *electromechanical energy conversion is possible if any of the quantities entering the energy conversion equations undergoes variations.*

Most of electric machines are said to operate in one mode or another if their windings carry alternating currents. In these machines the parameters may vary too. Energy conversion is possible at constant voltages and currents but at varying parameters. Energy conversion can occur when inductive reactances and resistances

entering the equations undergo variations. With a change in the moment of inertia, a machine stores kinetic energy and gives it up to the line.

Faraday's machine (see Fig. 1.1) and the magnetohydrodynamic (MHD) generator (Fig. 1.13) are the most complicated case for the explanation of electromechanical energy conversion. In Faraday's machine with a permanent magnet, the dc circuit changes state. It has a portion that is stationary and a portion that moves about the magnet. The sliding contact is obligatory. If the loop is made uniform without the sliding contact, the motor will not run even when the current source is made to rotate together with the loop.

In the MHD generator, the varying parameter is the velocity of plasma in the nozzle and outside it. In Faraday's motor, transition from the rotating part of the current-carrying loop to the stationary part occurs in a jumplike manner within the sliding contact region, while in the MHD generator the velocity of the active medium changes smoothly.

1.5. Application of Computers to the Solution of Problems in Electromechanics

Since the equations of electromechanical energy conversion are nonlinear, the analytical solution exists only under certain assumptions, when $\omega_r = 0$ or the speed varies linearly, in which case voltage equations (1.34) are solvable independent of the equation of motion (1.35). The analysis of transient processes with a varying rotational speed is possible only with the aid of computing devices because the equations contain the products of variables.

Electronic computers can be classified under three main headings: *analog computers*, *digital computers*, and *hybrid computers*.

An analog computer represents all variables by continuously varying physical quantities (currents and voltages) whose change gives the solution to the problem being investigated. Any dynamic characteristic is reproduced by a recorder, for example, on the screen of a cathode-ray oscillograph. In solving problems on an analog computer, it is well to state the problem first in an incompletely definite form and then refine the statement in the process of the analysis of the problem. A disadvantage of analog computers is that they have low accuracy and limited versatility. But the accuracy up to a few percent is often quite sufficient for many engineering studies because the specified accuracy of initial data is yet lower.

What makes an analog computer insufficiently universal is that transition from the solution of one problem to that of the other requires changing the flow diagram of the machine. Analog computers available today handle problems involving the integration of

ordinary differential equations, algebraic and transcendental equations, and partial differential equations. They are convenient for use in the analysis of dynamic operation of energy converters.

Digital computers find use where it is necessary to solve mathematical problems to a high accuracy. The input and output information here is in the discrete form, so these machines realize the numerical methods for the solution of problems. The calculation accuracy attainable on a digital computer depends on the quantity of bits, the limits being set by the size of computer facilities employed. Modern digital computers can automatically perform a complete computation with a speed 100 000 times as fast as a human being does and thus offer the greatest possibilities for carrying out calculations. As distinguished from analog computers, digital ones handle problems with a definitely stated solution algorithm, for which the instruction (program) is written and given to the machine. Digital computers of today are capable of solving a wide range of problems. In going from the solution of one problem to that of the other, it is only necessary to change the program without modifying the computer flow diagram.

Large digital computers are costly, sophisticated, and highly universal installations mainly set up at computing centers whose personnel service the machines, prepare the problems to be solved and program them, i.e. write the problems in a machine language, or code in a suitable form required for the automated solution.

A digital computer operates with discrete quantities—numbers represented in a definite notation. The main advantages of a digital computer are a high accuracy of computation, up to 20 decimal digits and over, and inherent versatility which allows for the solution of a wide class of problems. A disadvantage of this type of computer is that programming, debugging, and decoding of the results obtained in discrete form consume a great deal of time.

The first digital computers were built around electromechanical relays and then, later, around vacuum tubes. The on-line memory of the machines relied on tube triggers, mercury delay lines, cathode-ray tubes, and, later, ferrite cores. Vacuum tube-based machines with a speed in the order of 10-20 thousand operations in a second belong to the first generation of computers. They appeared in 1946 and were built up to the early 1960s.

The second-generation digital computers that began to appear in 1960s are the machines based on semiconductor discrete elements using magnetic-core memories. The machines of the second generation occupy a hundredth the space of the first-generation computers, consume a hundredth the amount of energy, and can perform about a million operations per second.

The third-generation computers use semiconductor small-scale integrated circuits (on the average 10 gates in a chip), magnetic-core

memories and, partially, magnetic-disk memories. The computing system of the third generation displays three characteristic features as follows: employs integrated circuits; has input-output channels and the developed network of peripheral units; and is made complete with software which forms an integral part of the computing system.

The cost of software systems grows steadily with each passing year. While at the beginning of the 1960s the cost of programs was 30% and that of equipment 70%, at present the cost of software reaches one half the total cost of hardware.

Examples of the third-generation computers include the IBM360 and the Soviet-made EC machine that closely resembles the former in parameters (EC is the abbreviation of Russian words meaning the unified system).

The IBM360 system represents a family of the third-generation machines developed by the world's largest American computer-building corporation.

The IBM360 displays a number of distinguishing features, of which the most important are the following: the program compatibility of various types of computers entering into the family, which provides for the applicability of programs in going from one model of the machine to another; the possibility of connection of a large number of input-output devices and standard integration of input-output devices with input-output channels; the capability to operate in real time in control systems; and the possibility of combining small computing-power machines into a single system.

The IBM360 computer is a universal system designed for serving economic (business, commerce) and scientific purposes and also for solving the problems of data transfer and control. The standard system of programs offers the basic computation capabilities of the machine. This command system may include means for processing data in decimal notation. The addition of floating-point features gives a scientific command system, and the addition of security facilities to the economic and scientific command systems provides a universal command system. A few types of the IBM360 machine can be combined with the aid of central processors to form a computing complex. The IBM360 system uses solid integrated circuits noted for high speed and small size, which ensures high reliability of the computers.

The machines of the EC type employ the standard network of interconnection of peripheral units, the so-called input-output interface based on the program control of these units.

The hardware of the EC machine can be divided into four groups (Fig. 1.14). Group I includes processors together with the register-based working storage, arithmetic and logic elements, and control devices. The devices of Groups II and III link processors to peripheral units which form Group IV. To group II belong selector and mul-

tiplex channels. The selector channel operates in the burst mode to provide for a high-speed data input to and readout from only one peripheral unit for a certain length of time (nearly a second and over). The multiplex channel permits a simultaneous data transfer for a large number of input-output units. The link between the units of Groups II and III forms what is called the input-output interface of the standard design, which is a detachable 30-wire connection ensuring the transfer of control signals and data.

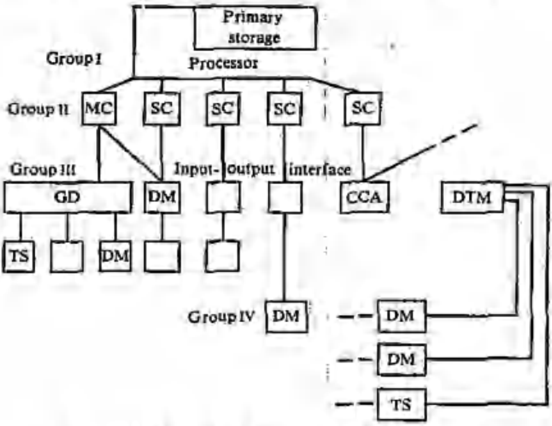

Fig. 1.14. The hardware of an EC computer

MC — multiplex channels; *SC* — selector channels; *GD* — group devices; *DM* — demodulators; *TS* — tape storage units; *CCA* — channel-to-channel adapters; *DTM* — data transfer multiplexers

The devices of group III intended to link the interface to various peripheral units include individual interface devices, group arrangements servicing a few peripheral units, channel-to-channel adapters providing direct connection between the selector channels of processors, and multiplexers for data transfer over a few channels.

Peripheral units include magnetic-tape, magnetic-drum, or diode storages; input-output devices for punch cards and tapes; printers; data terminals and consoles; the means (modems) for data teleprocessing and communication with control units.

The mathematical program package (software) for the EC computer includes the programs of three categories: operating systems ensuring the link between the operator and user and distributing the jobs and system resources; maintenance programs or test routines (debugging, checking, and diagnostic routines); and the packs of application

programs, which are functionally complete sets arranged for the solution of a definite class of problems.

The EC unitary system represents a family of program-compatible computing setups of the following types: EC-1010, EC-1020, EC-1021, EC-1030, EC-1040, EC-1050, EC-1060. The working storage capacity ranges from 8 to 10^4 kilobytes.

The basic features of the EC computer are its universality, adaptability for various applications, and the possibility of a gradual buildup of the computing power over a wide range. The versatility is due to the instruction set involving fixed-point and floating-point computations, logic and decimal operations, operations with variable-length words, and also due to various data formats, multiprogramming possibilities, and the advanced system of software.

The adaptability for uses stems from the changeable structure of the EC system (replaceability of memories, channels, peripheral equipment). A gradual increase in the computing power can be achieved by several methods, namely, by increasing the number of peripheral units and the working storage capacity, producing multimachine computing complexes, replacing the processor by a faster-speed type, etc. The program compatibility of the EC computer comes from the unified logical structure (standardization of the instruction set, data representation form, and address system).

During the last 25 years the computer speed, storage capacity, and reliability have increased many times. The overall dimensions, the energy consumed, and the specific cost of computers decrease very fast concurrent with the improvement of their parameters and characteristics.

At the start of the 1970s *the* first *fourth-generation computers* appeared, which began to use medium-scale ICs (about 100 gates on a chip) and large-scale ICs (thousands of gates on a chip). What distinguishes the fourth-generation computers is that they widely employ semiconductor storages, enlarged instruction sets, microprogramming, built-in subroutines, automated program debugging, peripheral units and channels of diversed types and improved quality, interfaces, specialized processors. These computers exhibit enhanced reliability and form the basis for the construction of multimachine and multiprocessor computing complexes.

The emergence of an automatic universal digital computer that performs arithmetic and logical operations with a high speed opens up new qualitative possibilities for conducting the theoretical investigations in conjunction with check experiments.

Hybrid computers which comprise digital machines and analog devices hold much promise for the efficient combination of the elements of a hybrid installation to enable the most rational solution of problems.

The digital computer in a hybrid complex is a control machine

which simultaneously gives the source information for the further solution of problems on analog devices. This complex, when connected to the system of appropriate transducers, can control an experiment and keep the link from the moment of data analysis to the moment of obtaining the result. This also offers new possibilities of the search for an optimal mode of operation on the principle of self-instruction of the system.

At present the ways are sought for the construction of high-performance computing systems by using a cascaded setup composed of a large number of identical universal digital computers program-organized for the realization of a specified algorithm. The development of such structures involves the refinement of the requirements for versatility, performance, computation accuracy, and directly depends on the class of problems to be solved.

By their structure, the fourth-generation machines are multiprocessor setups devoted to the common memory block and the common extent of peripheral devices. An aggregate of computing facilities forms a center connected to numerous subscribers by communication lines. Such a network offers the possibility for the communal use of computers by an individual or a group of researchers who may contact the center by telegraph or telephone from any region of the country, relay the message for the solution of a problem and receive the answer on the given date.

Both analog and digital computers find use for the solution of problems in electromechanics. If some ten equations describing the transient processes in electric machines are enough and the parameters entering into the equations are constant, it is well to solve the problems on an analog computer. Where the number of equations is larger, the parameters are nonlinear and there is a need for solving problems for the optimization of an energy converter, it is necessary to choose a digital computer.

In the analysis of electric machines, it is expedient to employ both analytical methods and analog and digital computers. The experience in choosing the combination of methods of analysis determines the degree of accuracy and profoundness of the solution of the problem.

In solving a problem in electromechanics, the researcher should primarily formulate the equations for the processes under study to a sufficient degree of accuracy and then choose a computer to form a mathematical model. Next he should refine the model with the aim to estimate the time it would take to solve the problem and the expected accuracy of the solution. The final step involves drawing the plan of the experiments to be run.

Despite their great opportunities, computer facilities can solve a rather limited range of problems in electromechanics. Taking into account even two or three harmonics in the air gap and two or three loops on the stator and rotor necessitates solving a few tens of

equations. Consequently, the researcher should thoroughly choose the mathematical model, keeping in mind the power of computer facilities, and estimate the time required for the solution of the problem and the possible solution accuracy.

Chapter 2
Electromechanical Energy Conversion Involving a Circular Field

2.1. The Equations of the Generalized Electric Machine

Consider a two-phase two-pole electric machine (Fig. 2.1). It has two orthogonal systems of stator and rotor windings w_a^s, w_b^s

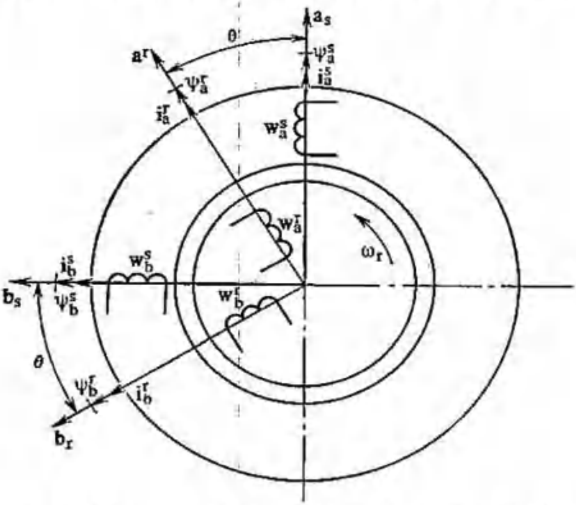

Fig. 2.1. The machine model arranged in a nontransformed coordinate system

and w_a^r, w_b^r, respectively, lying on the stator and rotor axes a_s, b_s and a_r, b_r. The rectangular coordinate frames of the stator and

rotor move with respect to each other, and the angle θ between the axes determines the relative rotational velocity. With the stator being stationary,

$$\omega_r = d\theta/dt \tag{2.1}$$

The differential equations of voltages in natural or phase (non-transformed) coordinates have the form

$$\begin{aligned} u_a^s &= i_a^s r_a^s + d\Psi_a^s/dt \\ u_b^s &= i_b^s r_b^s + d\Psi_b^s/dt \\ -u_a^r &= i_a^r r_a^r + d\Psi_a^r/dt \\ -u_b^r &= i_b^r r_b^r + d\Psi_b^r/dt \end{aligned} \tag{2.2}$$

In Eqs. (2.2), the frequencies of currents in the stator and rotor are different, and the 'minus' signs before the rotor voltages denote that the active power flows from the stator to the shaft (motoring action).

The flux linkages of the windings are

$$\begin{aligned} \Psi_a^s &= L_a^s i_a^s + M\cos\theta i_a^r + M\sin\theta i_b^r \\ \Psi_b^s &= L_b^s i_b^s + M\cos\theta i_b^r - M\sin\theta i_a^r \\ \Psi_a^r &= L_a^r i_a^r + M\cos\theta i_a^s - M\sin\theta i_b^s \\ \Psi_b^r &= L_b^r i_b^r + M\cos\theta i_b^s + M\sin\theta i_a^s \end{aligned} \tag{2.3}$$

Here the coefficients ahead of the currents vary with the same rate as the currents.

If we substitute expressions (2.3) into (2.2), the resultant equations will be too awkward and contain periodic coefficients. To simplify the solution of equations, it is necessary to have the same frequencies in the stator and rotor windings and ensure the invariance of power, i.e. to enable the power coming to the shaft, the losses, and the energy consumed to be the same as they are in a real machine.

Look into the processes of energy conversion in a machine within the air gap (the spacing between the rotor and stator) which concentrates the energy of a magnetic field. A rotating field is set up in the air gap of a real machine owing to a definite distribution of the windings in space and to the time shift between currents and voltages. With the sinusoidally varying voltages impressed on the terminals of an ideal machine, a circular field appears in the air gap. By the third law of electromechanics, there is a rigid link between the frequencies of currents in the stator and rotor, the stator and rotor fields being stationary with respect to each other. Account must also be taken here of the mechanical speeds of the rotor and stator. For a stationary stator, $\omega_s = \omega_r \pm \omega_{fr}$ (where ω_{fr}

is the speed of the rotor field relative to the rotor). It is convenient to represent the circular field in the air gap by the resultant magnetic flux density vector

$$\overline{B}^s = B^s_\alpha + jB^s_\beta, \quad \overline{B}^r = B^r_\alpha + jB^r_\beta \tag{2.4}$$

and by the resultant flux linkage vector

$$\overline{\Psi}^s = \Psi^s_\alpha + j\Psi^s_\beta, \quad \overline{\Psi}^r = \Psi^r_\alpha + j\Psi^r_\beta \tag{2.5}$$

The stator and rotor voltages and currents can be represented as unit resultant vectors $\overline{U}^s$, $\overline{U}^r$ and $\overline{I}^s$, $\overline{I}^r$, respectively.

Since the windings in the primitive machine lie 90° apart in space, the voltages

$$U^s_a = U_m \sin \omega t, \quad U^s_b = U_m \cos \omega t \tag{2.6}$$

impressed across the windings produce the resultant field $\overline{B}^s$ and Ψ^s in the air gap.

For voltages (2.2) given in terms of the resultant vectors, the equations assume the form

$$\begin{aligned} \overline{U}^s &= \overline{I}^s R^s + d\overline{\Psi}^s/dt \\ -\overline{U}^r &= \overline{I}^r R^r + d\overline{\Psi}^r/dt \end{aligned} \tag{2.7}$$

Here $R^s = r^s_\alpha = r^s_\beta$, $R^r = r^r_\alpha = r^r_\beta$.

Examine the processes of energy conversion as viewed from the coordinate reference frame rotating at an arbitrary speed ω_c (the observer's speed). For this coordinate system the power invariance depends on ω_c and frequency f_c. Illustrate a change in frequency with the rotation of the coordinate system by an example of the commutator machine shown in Fig. 2.2.

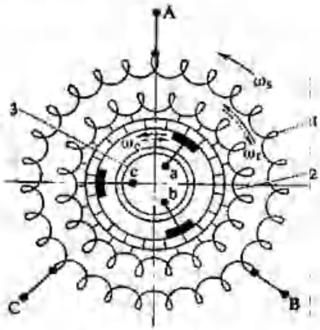

Fig. 2.2. A commutator machine with revolving brushes

In this machine, the stator carries a three-phase winding *1*, each phase winding *A*, *B*, and *C* being fed with ac voltage which produces in the gap a field revolving at a synchronous speed ω_s. The rotor *2* runs at a speed ω_r and the frequency in the rotor winding is $f_2 = f_1 s$. The brushes rigidly connected to the coordinate axes slide over the rotor winding and rotate together with the brush ring *3* at a speed ω_c. The number of brushes is equal to the number of phases. As seen from the figure, the ma-

chine has three brushes, and therefore the system is of the three-phase type (a, b, c).

Since the brushes are connected to the coordinate axes, the power taken from the three-phase system and frequency f_c depend on the speed ω_c.

Because $\cos\theta + j\sin\theta = \exp(+j\theta)$, vector equations (2.7) for the coordinate axes rotating at an arbitrary speed

$$\omega_c = d\theta_c/dt \tag{2.8}$$

have the form

$$\bar{U}^s \exp j\theta_c = R^s\bar{I}^s \exp j\theta_c + (d/dt)\,(\bar{\Psi}^s \exp j\theta_c) \tag{2.9}$$

$$\bar{U}^r \exp j\,(\theta_c - \theta) = R^r\bar{I}^r \exp j\,(\theta_c - \theta) + (d/dt)\,(\bar{\Psi}^r \exp j\,(\theta_c - \theta))$$

Taking the derivatives in (2.9) yields

$$\begin{aligned} \bar{U}^s &= R^s\bar{I}^s + d\bar{\Psi}^s/dt + j\omega_c\bar{\Psi}^s \\ \bar{U}^r &= R^r\bar{I}^r + d\bar{\Psi}^r/dt + j\,(\omega_c - \omega_r)\,\bar{\Psi}^r \end{aligned} \tag{2.10}$$

The obtained voltage equations for the resultant vectors are defined with respect to the coordinate reference frame rotating at an arbitrary speed with the rotor. These are the simplest and most general Kirchhoff's equations for the primitive machine. The equations expressed in the variable-speed rotating reference frame are in rare use. Of most interest are the equations written in coordinates α and β when $\omega_c = 0$ and the equations in the system of coordinates d and q when $\omega_c = \omega_r$. The latter system is the rotating reference frame, which is most popular for the analysis of synchronous machines when $\omega_c = \omega_r = \omega_s$. The rotor and stator fields appear stationary for the observer when he views them from the rotor. Here the modeling of energy conversion processes involves direct currents.

For the stationary reference frame ($\omega_c = 0$), with the reference axes α and β being rigidly connected to the stator, Eqs. (2.10) take on the form

$$\begin{aligned} \bar{U}^s &= R^s\bar{I}^s + d\bar{\Psi}^s/dt \\ \bar{U}^r &= R^r\bar{I}^r + d\bar{\Psi}^r/dt - j\omega_r\bar{\Psi}^r \end{aligned} \tag{2.11}$$

Resolving the resultant vectors along the axes α and β gives the equations of voltages for the primitive machine in terms of flux linkages:

$$\begin{aligned} u_\alpha^s &= i_\alpha^s r_\alpha^s + d\Psi_\alpha^s/dt \\ u_\beta^s &= i_\beta^s r_\beta^s + d\Psi_\beta^s/dt \\ u_\alpha^r &= i_\alpha^r r_\alpha^r + d\Psi_\alpha^r/dt + \omega_r\Psi_\beta^r \\ u_\beta^r &= i_\beta^r r_\beta^r + d\Psi_\beta^r/dt - \omega_r\Psi_\alpha^r \end{aligned} \tag{2.12}$$

By substituting the flux linkages

$$\begin{aligned}
\Psi_\alpha^s &= L_\alpha^s i_\alpha^s + M i_\alpha^r \\
\Psi_\beta^s &= L_\beta^s i_\beta^s + M i_\beta^r \\
\Psi_\alpha^r &= L_\alpha^r i_\alpha^r + M i_\alpha^s \\
\Psi_\beta^r &= L_\beta^r i_\beta^r + M i_\beta^s
\end{aligned} \tag{2.13}$$

into (2.12) we arrive at the energy conversion equations in the coordinate system α, β, expressed in terms of currents.

In going from the nontransformed coordinate system to the system α, β, we should refer to Fig. 2.1 and determine the projections of rotor voltages and currents on the stator axis from the relations

$$\begin{aligned}
u_\alpha^r &= u_a^r \cos\theta + u_b^r \sin\theta \\
u_\beta^r &= -u_a^r \sin\theta - u_b^r \cos\theta
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
i_\alpha^r &= i_a^r \cos\theta + i_b^r \sin\theta \\
i_\beta^r &= -i_a^r \sin\theta + i_b^r \cos\theta
\end{aligned} \tag{2.15}$$

The transformation matrix here is

$$G = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \tag{2.16}$$

The inverse transformation matrix is

$$G^{-1} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \tag{2.17}$$

As mentioned above, the reference frame with the reference axes d and q rigidly connected to the rotor structure is very popular. In this system, $\omega_c = \omega_r$. From (2.10) it follows that

$$\begin{aligned}
\overline{U}^s &= R^s\overline{I}^s + d\overline{\Psi}^s/dt + j\omega_r\overline{\Psi}^s \\
\overline{U}^r &= R^r\overline{I}^r + d\overline{\Psi}^r/dt
\end{aligned} \tag{2.18}$$

Resolving the resultant vectors along the d and q axes yields the equations for the primitive machine in the coordinate system d, q:

$$\begin{aligned}
u_d^s &= i_d^s r_d^s + d\Psi_d^s/dt - \omega_r \Psi_q^s \\
u_q^s &= i_q^s r_q^s + d\Psi_q^s/dt + \omega_r \Psi_d^s \\
u_d^r &= i_d^r r_d^r + d\Psi_d^r/dt \\
u_q^r &= i_q^r r_q^r + d\Psi_q^r/dt
\end{aligned} \tag{2.19}$$

Replacing the flux linkages by currents, inductances, and mutual inductances, we derive the direct- and quadrature-axis equations expressed in terms of currents, in the same way as we did in the α-β coordinate system:

$$\begin{vmatrix} u_d^s \\ u_d^r \\ u_q^r \\ u_q^s \end{vmatrix} = \begin{vmatrix} r_d^s + (d/dt)\,L_d^s & (d/dt)\,M & -L_q^s\omega_r & -M\omega_r \\ (d/dt)\,M & r_d^r + (d/dt)\,L_d^r & 0 & 0 \\ 0 & 0 & r_q^r + (d/dt)\,L_q^r & (d/dt)\,M \\ M\omega_r & L_d^s\omega_r & (d/dt)\,M & r_q^s + (d/dt)\,L_q^s \end{vmatrix} \times \begin{vmatrix} i_d^s \\ i_d^r \\ i_q^r \\ i_q^s \end{vmatrix} \tag{2.20}$$

$$(1/p)\,J\,(d\omega_r/dt) \pm M_s = pM\,(i_q^s i_d^r - i_d^s i_q^r) \tag{2.21}$$

The energy conversion equations in the system of coordinates u and v rotating at an arbitrary speed ω_c have the form

$$\begin{vmatrix} u_u^s \\ u_u^r \\ u_v^r \\ u_v^s \end{vmatrix} = \begin{vmatrix} r_u^s + (d/dt)\,L_u^s & (d/dt)\,M & M\omega_c & L_v^s\omega_c \\ (d/dt)\,M & r_u^r + (d/dt)\,L_u^r & L_v^r(\omega_c - \omega_r) & M\,(\omega_c - \omega_r) \\ -M\,(\omega_c - \omega_r) & -L_u^r(\omega_c - \omega_r) & r_v^r + (d/dt)\,L_v^r & (d/dt)\,M \\ -L_u^s\omega_c & -M\omega_c & (d/dt)\,M & r_v^s + (d/dt)\,L_v^s \end{vmatrix} \times \begin{vmatrix} i_u^s \\ i_u^r \\ i_v^r \\ i_v^s \end{vmatrix} \tag{2.22}$$

$$(1/p)\,J\,(d\omega_r/dt) \pm M_r = pM\,(i_v^s i_u^r - i_u^s i_v^r) \tag{2.23}$$

Equations (2.22) and (2.23) are most general. Substituting $\omega_c = 0$ in Eqs. (2.22) and (2.23), we can obtain the α-β equations (1.34) and (1.35). The d-q equations (2.20) and (2.21) follow from (2.22) and (2.23) if $\omega_c = \omega_r$.

The coordinate reference frames α, β; d, q; and u, v are in most extensive use. They permit applying equations to practically all problems dealt with in electromechanics. *The right choice of the reference frame simplifies equations, enables deriving equations with constant coefficients but does not reduce the number of unknowns.*

The system α, β proves most suitable for the analysis of induction machines. The system d, q applies to the description of energy conversion processes in synchronous machines. This system is especially convenient for use in the analysis of salient-pole machines, where the coordinates extend along the direct and quadrature axes d and q of the machine. The system rotating at an arbitrary speed finds use for the analysis of machines where both the rotor and stator are rotating members.

In some cases use is also made of the physical reference frame (for which the equations have periodic coefficients), for example, when analyzing an induction motor whose rotor and stator windings

receive power from a thyristor frequency converter. Here it is advisable to use the system of six voltage equations rather than to reduce the machine under study to the two-phase type. This approach permits applying the actual-wave form voltages of the frequency converter to the machine windings.

The equations expressed in one reference frame or another are brought to a suitable form in accordance with the rules of mathematics. One of the important ways of transformation comes to the replacement of variables:

$$i_{0\alpha} = i_\alpha^s + i_\alpha^r, \quad i_{0\beta} = i_\beta^s + i_\beta^r$$

where $i_{0\alpha}$ and $i_{0\beta}$ are the instantaneous current components at no load along the α and β axes.

Using Eqs. (2.13) for magnetic flux linkages, we determine currents:

$$i_\alpha^s = (L^r\Psi_\alpha^s - M\Psi_\alpha^r)/(L^sL^r - M^2), \; i_\beta^s = (L^r\Psi_\beta^s - M\Psi_\beta^r)/(L^sL^r - M^2)$$
$$i_\alpha^r = (L^s\Psi_\alpha^r - M\Psi_\alpha^s)/(L^sL^r - M^2), \; i_\beta^r = (L^s\Psi_\beta^r - M\Psi_\beta^s)/(L^sL^r - M^2)$$

Substituting the above current equations into the torque equation gives the expression for the torque in terms of the flux linkages

$$M_e = (m/2)\,[pM/(L^sL^r - M^2)]\,(\Psi_\beta^s\Psi_\alpha^r - \Psi_\alpha^s\Psi_\beta^r) \tag{2.24}$$

Voltage equations (2.12) and torque equation (2.24) give a most stable model of energy conversion processes simulated on an analog computer.

The electromagnetic torque can be defined in terms of the stator flux linkages and currents

$$M_e = (m/2)\,(\Psi_\alpha^s i_\beta^s - \Psi_\beta^s i_\alpha^s) \tag{2.25}$$

That equation (2.25) is valid is easy to confirm if we substitute in (2.25) the expressions

$$\Psi_\alpha^s = L_\alpha^s i_\alpha^s + Mi_\alpha^r, \quad \Psi_\beta^s = L_\beta^s i_\beta^s + Mi_\beta^r$$

then perform simple transformations, putting $L_\alpha^s = L_\beta^s$, and obtain the value of torque expressed in terms of currents.

The torque is also definable in terms of the rotor flux linkages and currents

$$M_e = (m/2)\,(\Psi_\alpha^r i_\beta^r - \Psi_\beta^r i_\alpha^r) \tag{2.26}$$

The validity of (2.26) is borne out by substituting in (2.26) the expressions

$$\Psi_\alpha^r = L_\alpha^r i_\alpha^r + Mi_\alpha^s, \quad \Psi_\beta^r = L_\beta^r i_\beta^r + Mi_\beta^s$$

and equating L_α^r to L_β^r.

The electromagnetic torque can also be defined in terms of the field energy in the air gap or found from the expression for the Poynting vector.

The described approaches to the modification of equations for the primitive machine do not at all make up an exhaustive list. We have discussed only the basic ways and many other courses are open to make transformations.

2.2. Steady-State Equations

The equations for an energy converter in the steady-state conditions of operation are derived from differential equations by replacing in the electromechanical equations the differential operator d/dt by $j\omega$.

The steady-state equations for the primitive machine in the coordinate system α, β can be obtained from (1.34) in the following form

$$\begin{vmatrix} \dot{U}_\alpha^s \\ \dot{U}_\alpha^r \\ -\dot{U}_\beta^r \\ -\dot{U}_\beta^s \end{vmatrix} = \begin{vmatrix} r_1 + jx_1 & jx_m & 0 & 0 \\ jx_m & r_2 + jx_2 & \nu x_2 & \nu x_m \\ -\nu x_m & -\nu x_2 & r_2 + jx_2 & jx_m \\ 0 & 0 & jx_m & r_1 + jx_1 \end{vmatrix} \times \begin{vmatrix} \dot{I}_\alpha^s \\ \dot{I}_\alpha^r \\ \dot{I}_\beta^r \\ \dot{I}_\beta^s \end{vmatrix} \tag{2.27}$$

Here, $\nu = \omega_r/\omega_s$ is the relative speed; r_1 and r_2 are the resistances of stator and rotor windings respectively; $x_1 = \omega L_1$ is the inductive reactance of the stator winding; $x_2 = \omega L_2$ is the inductive reactance of the rotor winding; and $x_m = \omega M$ is the reactance of mutual induction.

The equation of motion in the steady state ($d/dt = 0$) is

$$M_s = pM_e \tag{2.28}$$

The electromagnetic torque equation is given by

$$M_e = (mp/2)\, M\, (I_{\alpha a}^r I_{\beta a}^s + I_{\alpha r}^r I_{\beta r}^s - I_{\beta a}^r I_{\alpha a}^s - I_{\beta r}^r I_{\alpha r}^s) \tag{2.29}$$

where $I_{\alpha a}^s$, $I_{\beta a}^s$ and $I_{\alpha a}^r$, $I_{\beta a}^r$ are the active current components along the α and β axes of the stator and rotor respectively; and $I_{\alpha r}^s$, $I_{\beta r}^s$ and $I_{\alpha r}^r$, $I_{\beta r}^r$ are the reactive current components along the α and β axes of the stator and rotor respectively.

By use of steady-state equations (2.27), (2.28), and (2.29) for the primitive machine we can formulate equations for asynchronous and synchronous machines and also for transformers.

Derive the equations for a two-winding transformer and an induction machine. It should be kept in mind that the equations of the primitive machine neglect the presence of a few current networks in actual machines, asymmetry, saturation, and other factors.

Therefore, the procedure of deriving the steady-state equations, which appeared earlier than the differential equations in the theory of electric machines, involves certain difficulties, whatever the type of machine these equations have to be set up for. It appears logical to conduct all new studies in the theory of electromechanical converters in such a manner that the steady-state conditions would be a partial case of transient conditions and the static equations would be a partial case of dynamic equations.

The steady-state equations can be written in any coordinates, just like the differential equations.

The equations for a single-phase two-winding transformer, derived from Eq. (1.34) for the primitive machine on condition that $\omega_r = 0$ (coupling between the windings along different axes is nonexistent), have the form

$$\begin{vmatrix} u_1 \\ -u_2 \end{vmatrix} = \begin{vmatrix} r_1 + (d/dt)\,L_1 & (d/dt)\,M \\ (d/dt)\,M & r_2 + (d/dt)\,L_2 \end{vmatrix} \times \begin{vmatrix} i_1 \\ i_2 \end{vmatrix} \tag{2.30}$$

In (2.30), the subscripts 1 and 2 identify the primary and the secondary respectively, and the 'minus' sign before u_2 means that power flows to the primary. Replacing d/dt by $j\omega$, from (2.30) we obtain the steady-state equations for the transformer

$$\begin{aligned} \dot{U}_1 &= \dot{I}_1 r_1 + j\omega L_1 \dot{I}_1 + j\omega M \dot{I}_2 \\ -\dot{U}_2 &= j\omega M \dot{I}_1 + \dot{I}_2 r_2 + j\omega L_2 \dot{I}_2 \end{aligned} \tag{2.31}$$

Here

$$\begin{aligned} j\omega L_1 \dot{I}_1 &= j\omega (M + l_{\sigma_1})\,\dot{I}_1 = j\omega M \dot{I}_1 + j\omega l_{\sigma_1} \dot{I}_1 \\ j\omega L_2 \dot{I}_2 &= j\omega (M + l_{\sigma_2})\,\dot{I}_2 = j\omega M \dot{I}_2 + j\omega l_{\sigma_2} \dot{I}_2 \end{aligned} \tag{2.32}$$

Replacing variables $\dot{I}_0 = \dot{I}_1 + \dot{I}_2$ in (2.32) gives

$$\begin{aligned} \dot{U}_1 &= \dot{I}_1 r_1 + j\omega M \dot{I}_1 + j\omega l_{\sigma_1} \dot{I}_1 + j\omega M \dot{I}_2 \\ &= \dot{I}_1 r_1 + j\omega M (\dot{I}_1 + \dot{I}_2) + j\omega l_{\sigma_1} \dot{I}_1 = \dot{I}_1 r_1 + j\omega M \dot{I}_0 + j\dot{I}_1 x_1 \\ &= \dot{I}_1 r_1 - \dot{E}_1 + j\dot{I}_1 x_1 = -\dot{E}_1 + \dot{I}_1 z_1 \end{aligned} \tag{2.33}$$

where $\dot{E}_1 = -j\omega M \dot{I}_0$, $x_1 = \omega l_{\sigma_1}$, $z_1 = r_1 + jx_1$.

Performing the same transformations for the secondary-winding equation given in (2.31), we get

$$-\dot{U}_2 = \dot{I}_2 r_2 + j\omega M (\dot{I}_1 + \dot{I}_2) + j\omega l_{\sigma_2} \dot{I}_2 = \dot{I}_2 r_2 - \dot{E}_2 + j\dot{I}_2 x_2 \tag{2.34}$$

Transforming (2.34) yields the equation for the secondary winding

$$\dot{U}_2 = \dot{E}_2 - \dot{I}_2 z_2 \tag{2.35}$$

where $\dot{E}_2 = \dot{E}_1 = -j\omega M\dot{I}_0$, $z_2 = r_2 + jx_2$.

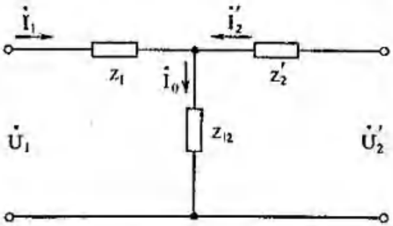

Fig. 2.3. The equivalent circuit of a transformer

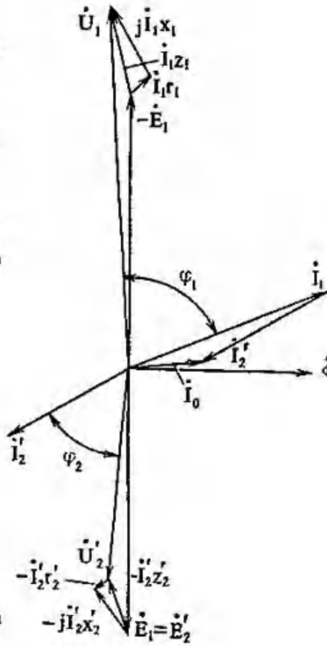

Fig. 2.4. The phasor diagram of a transformer

We have thus arrived at the known system of equations describing the processes of electromagnetic energy conversion in a transformer:

$$\dot{U}_1 = -\dot{E}_1 + \dot{I}_1 z_1, \quad \dot{U}_2 = \dot{E}_2 - \dot{I}_2 z_2, \quad \dot{I}_1 + \dot{I}_2 = \dot{I}_0 \tag{2.36}$$

Equations (2.36) follow from the equivalent circuit (Fig. 2.3) and phasor diagram (Fig. 2.4) of a transformer, which are dealt with in the general course in electric machines.

For induction machines, the steady-state voltage equations found from (1.34) have the form

$$
\begin{aligned}
\dot{U}^s_\alpha &= R^s\dot{I}^s_\alpha + j\omega L^s\dot{I}^s_\alpha + j\omega M\dot{I}^r_\alpha \\
\dot{U}^s_\beta &= R^s\dot{I}^s_\beta + j\omega L^s\dot{I}^s_\beta + j\omega M\dot{I}^r_\beta \\
-\dot{U}^r_\alpha &= R^r\dot{I}^r_\alpha + j\omega L^r\dot{I}^r_\alpha + j\omega M\dot{I}^s_\alpha + M\dot{I}^s_\beta\omega_r + L^r\dot{I}^r_\beta\omega_r \\
-\dot{U}^r_\beta &= R^r\dot{I}^r_\beta + j\omega L^r\dot{I}^r_\beta + j\omega M\dot{I}^s_\beta - M\dot{I}^s_\alpha\omega_r - L^r\dot{I}^r_\alpha\omega_r
\end{aligned}
\tag{2.37}
$$

For induction machines with a short-circuited rotor, $\dot{U}^r_\alpha = 0$ and $\dot{U}^r_\beta = 0$. Taking into account

$$
j\omega L^s = j\omega M + j\omega l^s_\sigma, \quad j\omega L^r = j\omega M + j\omega l^r_\sigma
$$

$$
x_0 = \omega M, \quad x^s = \omega l^s_\sigma, \quad x^r = \omega l^r_\sigma
$$

and also the relative speed $\nu = \omega_r/\omega_s$, the equations for an induction machine assume the form

$$
\begin{aligned}
\dot{U}^s_\alpha &= R^s\dot{I}^s_\alpha + jx^s\dot{I}^s_\alpha + jx_0\dot{I}^s_\alpha + jx_0\dot{I}^r_\alpha \\
\dot{U}^s_\beta &= R^s\dot{I}^s_\beta + jx^s\dot{I}^s_\beta + jx_0\dot{I}^s_\beta + jx_0\dot{I}^r_\beta \\
0 &= -R^r\dot{I}^r_\alpha - jx^r\dot{I}^r_\alpha - jx_0\dot{I}^r_\alpha - jx_0\dot{I}^s_\alpha - x_0\dot{I}^r_\beta\nu - (x^r + x_0)\dot{I}^r_\beta\nu \\
0 &= -R^r\dot{I}^r_\beta - jx^r\dot{I}^r_\beta - jx_0\dot{I}^r_\beta - jx_0\dot{I}^s_\beta + x_0\dot{I}^s_\alpha\nu + (x^r + x_0)\dot{I}^r_\alpha\nu
\end{aligned}
\tag{2.38}
$$

Considering that $\dot{I}^s_\beta = j\dot{I}^s_\alpha$, $\dot{I}^r_\beta = j\dot{I}^r_\alpha$ and omitting the intermediate operations, for the stator and rotor windings lying along the same axis we get

$$
\begin{aligned}
&\dot{U}_s = R_s\dot{I}_s + jx_s\dot{I}_s + jx_0\dot{I}_0 \\
&0 = -R_r\dot{I}_r - jx_r(1-\nu)\dot{I}_r - jx_0(1-\nu)\dot{I}_r - jx_0(1-\nu)\dot{I}_s \\
&\dot{I}_0 = \dot{I}_s + \dot{I}_r
\end{aligned}
\tag{2.39}
$$

We introduce the slip

$$
s = (\omega_s \pm \omega_r)/\omega_s = 1 \pm \nu \tag{2.40}
$$

and transform the rotor winding to the stator winding; then Eqs. (2.39) will take on the form

$$
\begin{aligned}
\dot{U}_s &= R_s\dot{I}_s + jx_s\dot{I}_s + jx_0\dot{I}_0 \\
0 &= -R'_r\dot{I}'_r - jx'_r\dot{I}'_r s - jx_0\dot{I}_0 s \\
\dot{I}_0 &= \dot{I}_s + \dot{I}'_r
\end{aligned}
\tag{2.41}
$$

Because $\dot{E}_0 = -j\dot{I}_0 z_0$ and $z_0 = r_0 + jx_0$, we assume that $\dot{E}_s = \dot{E}_r = \dot{E}_0$. The impedance z_0 of the excitation current-carrying

branch includes the resistance r_0 equivalent to the iron loss. After transformations, Eqs. (2.41) are written as

$$\begin{aligned}\dot{U}_s &= R_s\dot{I}_s + jx_s\dot{I}_s - \dot{E}_s \\ 0 &= \dot{E}'_r - jx'_r\dot{I}'_r - R'_r\dot{I}'_r/s \\ \dot{I}_0 &= \dot{I}_s + \dot{I}'_r\end{aligned} \tag{2.42}$$

Here $R'_r/s = R'_r + R'_r(1-s)/s$.

Introducing the stator and rotor impedances

$$z_s = R_s + jx_s, \quad z'_r = R'_r + jx'_r$$

yields the equations for an induction machine

$$\begin{aligned}\dot{U}_s &= -\dot{E}_0 + \dot{I}_s z_s \\ 0 &= \dot{E}_0 - \dot{I}'_r z'_r - \dot{I}'_r R'_r(1-s)/s \\ \dot{I}_0 &= \dot{I}_s + \dot{I}'_r\end{aligned} \tag{2.43}$$

The equivalent circuit and the phasor diagram for the above equations appear in Fig. 2.5 and Fig. 2.6 respectively. A circle

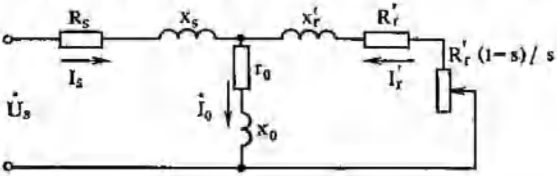

Fig. 2.5. The equivalent circuit of an induction machine

diagram exists for a transformed equivalent circuit. The complex equations (2.43), equivalent circuit, and circle diagram are the basic elements of the theory of steady-state operation of induction machines.

The steady-state equations for synchronous machines without a damper winding result from Eqs. (1.34) for the primitive machine. The equations for a synchronous machine that do not take into account the damper winding are too approximate (they are given below following the discussion of multiwinding machines). It should be borne in mind that the phasor diagrams of synchronous machines are drawn for fairly simplified equations; these diagrams have a qualitative meaning for most synchronous machines.

The phasor diagram for a nonsalient-pole machine (Fig. 2.7) corresponds to the equation

$$\dot{E} = \dot{U} + \dot{I}_a r_a + j\dot{I}_a x_s \qquad (2.44)$$

where $\dot{I}_a$ is the armature current; $\dot{U}$ is voltage; $\dot{E}$ is the open-circuit voltage or emf; r_a is the armature resistance; $x_s = x_\sigma + x_{ad}$ is the

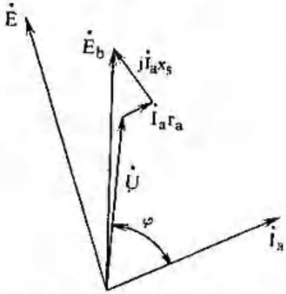

Fig. 2.7. The phasor diagram of a nonsalient-pole synchronous machine

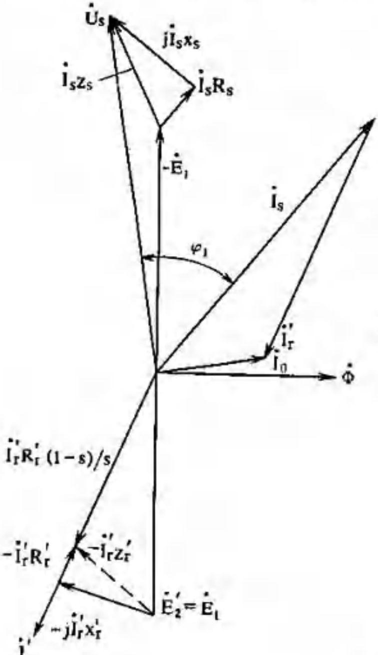

Fig. 2.6. The phasor diagram of an induction machine

synchronous machine reactance; x_σ is the leakage inductive reactance of the armature; and x_{ad} is the inductive reactance of the armature (opposition to the armature mmf).

For a salient-pole synchronous machine, the phasor diagram (Fig. 2.8) is plotted by use of the equation

$$\dot{E} = \dot{U} + \dot{I}_a r_a + j\dot{I}_d x_d + j\dot{I}_q x_q \qquad (2.45)$$

where $\dot{I}_d$ and $\dot{I}_q$ are currents along the direct and quadrature axes of the machine respectively; x_d and x_q are the inductive reactances along the d and q axes respectively.

For dc machines the classical theory uses a yet simpler equation

$$U = E \pm I_a r_{in} \tag{2.46}$$

where $E = f(I_{in}, F_{qd})$; I_{in} is the excitation current; F_{qd} is the

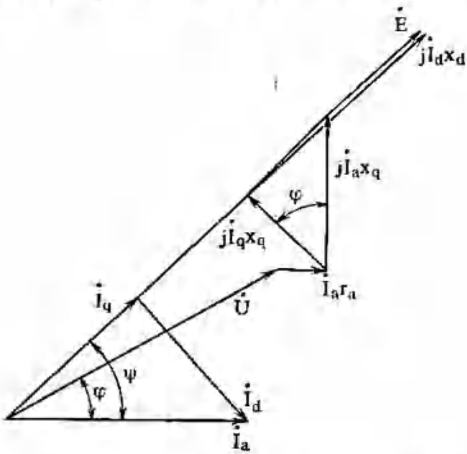

Fig. 2.8. The phasor diagram of a salient-pole synchronous machine

direct-axis component of the armature cross mmf; and r_{in} is the internal resistance of the machine.

The equations for synchronous machines and dc machines can be set up by representing a machine as a twoport (see Fig. 1.8) involving r_a and x_s for a nonsalient-pole machine; r_a, x_d, and x_q for a salient-pole machine; and r_{in} for a dc machine. Complicated nonlinear couplings are taken into account by introducing nonlinear dependences of the parameters on currents.

The solution of most problems in electromechanics requires extensive studies into steady-state processes, therefore theory largely deals with static conditions. Differential equations describing the processes of electromechanical energy conversion enable investigating transient processes and, as a particular case, steady-state processes. Computing facilities offer a fairly fast estimation of steady-state processes by solving differential equations. However, in handling optimization problems or investigating complicated

connection diagrams of electric machines, such an approach comes out impracticable because it takes a great deal of machine time.

The proper establishment of mathematical models of electric machines that would enable a ready simulation of both steady-state and transient performance on computers is one of the main tasks to be handled in the mathematical theory, for one needs first to learn how to formulate energy conversion equations before placing the problems on a computer.

2.3. Application of Analog Computers to the Analysis of Electric Machines

Analog computers find wide use for the analysis of transients in electric machines. In analog simulation, consideration should be given to the features peculiar to a simulation model (analog), and the ways should be sought to combine the convenient notation of equations with the favorable conditions for the solution of these equations on the analog computer.

Preliminary to simulation, Eqs. (1.34) expressed in terms of currents need to be transformed to bring them to forms suitable for setting up an analog:

$$\begin{aligned}
di_\alpha^s/dt &= (1/L^s)\,u_\alpha^s - (R^s/L^s)\,i_\alpha^s - (M/L^s)\,(di_\alpha^r/dt)\\
di_\beta^s/dt &= (1/L^s)\,u_\beta^s - (R^s/L^s)\,i_\beta^s - (M/L^s)\,(di_\beta^r/dt)\\
di_\alpha^r/dt &= (R^r/L^r)\,i_\alpha^r - (M/L^r)\,(di_\alpha^s/dt) - \omega_r\,[i_\beta^r + (M/L^r)\,i_\beta^s]\\
di_\beta^r/dt &= (R^r/L^r)\,i_\beta^r - (M/L^r)\,(di_\beta^s/dt) + \omega_r\,[i_\alpha^r + (M/L^r)\,i_\alpha^s]
\end{aligned} \tag{2.47}$$

The block diagram of a computer analog for the solution of Eqs. (2.47) is shown in Fig. 2.9. The solution of (2.47) comes to the integration of currents. The positive feedback loops interconnecting summing amplifiers *1*, *7*, *4*, and *10* of Fig. 2.9 cause self-excitation of the model since here the amplifier gains k_2, k_9, k_3, and k_{13} are higher than unity.

To effect stability, we integrate (2.47), omit the derivatives, and obtain the equations convenient for simulation (see Fig. A1 in Appendix I). These equations have a final form ($\dot{p} = d/dt$):

$$\begin{aligned}
i_\alpha^s &= (1/\dot{p})\,(a_1 u_\alpha^s - a_2 i_\alpha^s) - a_3 i_\alpha^r\\
i_\beta^s &= (1/\dot{p})\,(a_4 u_\beta^s - a_5 i_\beta^s) - a_6 i_\beta^r\\
i_\alpha^r &= (1/\dot{p})\,[-a_7 i_\alpha^r - \omega_r\,(i_\beta^r + a_8 i_\beta^s)] - a_9 i_\alpha^s\\
i_\beta^r &= (1/\dot{p})\,[-a_{10} i_\beta^r + \omega_r\,(i_\alpha^r + a_{11} i_\alpha^s)] - a_{12} i_\beta^s\\
M_e &= a_{13}\,(i_\alpha^r i_\beta^s - i_\beta^r i_\alpha^s), \quad \dot{p}\omega_r = a_{14}\,(M_e - M_r)
\end{aligned} \tag{2.48}$$

where $a_1 = a_4 = 1/L^s$; $a_2 = a_5 = R^s/L^s$, $a_3 = a_6 = M/L^s$; $a_7 = a_{10} = R^r/L^r$; $a_8 = a_9 = a_{11} = a_{12} = M/L^r$; $a_{13} = (m/2)\,pM$; $a_{14} = p/J$.

For the convenience of designation of quantities on the diagram, we reduce the expressions between the brackets in (2.48) to the form

$$a_1 u_\alpha^s - a_2 i_\alpha^s = A_{s\alpha},\quad a_4 u_\beta^s - a_5 i_\beta^s = A_{s\beta},\quad -a_7 i_\beta^r - \omega_r C_1 = B_{r\alpha}$$

$$-a_{10} i_\beta^r + \omega_r C = B_{r\beta},\quad i_\beta^r + a_8 i_\beta^s = C_1,\quad i_\alpha^r + a_{11} i_\alpha^s = C_2$$

In the model of Fig. A1 the gain factors of the summers (*2* and *8*, *5* and *11*), shown by dash lines, are close to unity and the model is

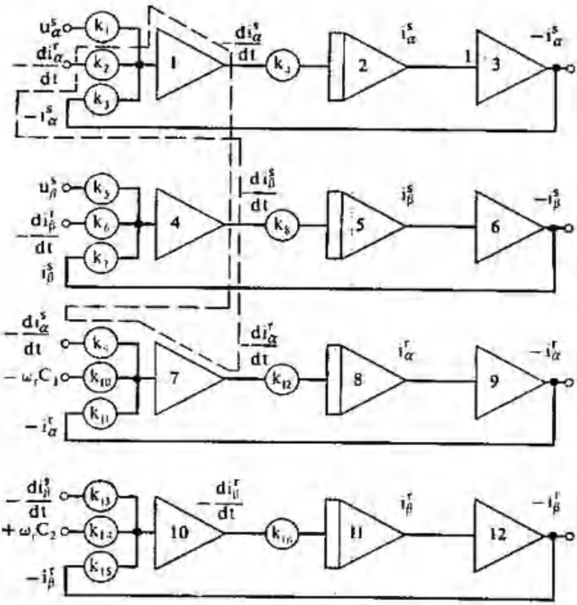

Fig. 2.9. The computer analog for the system of Eqs. (2.47)

more stable. Negative feedback paths of amplifiers with gains k_3, k_6, k_9, k_{13} make the model more stable in performance.

The notation of Eqs. (2.12) through (2.24) for flux linkages is brought to the form convenient for simulation

$$
\begin{aligned}
d\Psi^s_\alpha/dt &= a_1 u^s_\alpha - a_2\Psi^s_\alpha + a_3\Psi^r_\alpha \\
d\Psi^s_\beta/dt &= a_4 u^s_\beta - a_5\Psi^s_\beta + a_6\Psi^r_\beta \\
d\Psi^r_\alpha/dt &= -a_7\Psi^r_\alpha + a_8\Psi^s_\alpha - \omega_r\Psi^r_\beta \\
d\Psi^r_\beta/dt &= -a_9\Psi^r_\beta + a_{10}\Psi^s_\beta + \omega_r\Psi^r_\alpha \\
M_e &= a_{11}(\Psi^s_\beta\Psi^r_\alpha - \Psi^s_\alpha\Psi^r_\beta) \\
d\omega_r/dt &= a_{12}(M_e - M_r)
\end{aligned}
\qquad (2.49)
$$

where $a_1 = a_4 = u_{n.ph}\sqrt{2}$; $a_2 = a_5 = R^sL^r/(L^sL^r - M^2)$; $a_3 = a_6 = R^sM/(L^sL^r - M^2)$; $a_7 = a_9 = R^rL^s/(L^sL^r - M^2)$; $a_8 = a_{10} = R^rM/(L^sL^r - M^2)$; $a_{11} = (mp/2)\,M/(L^sL^r - M^2)$; $a_{12} = p/J$.
Current equations are

$$
i^s_\alpha = a_{13}\Psi^s_\alpha - a_{14}\Psi^r_\alpha, \quad i^s_\beta = a_{15}\Psi^s_\beta - a_{16}\Psi^r_\beta
$$
$$
i^r_\alpha = a_{17}\Psi^r_\alpha - a_{18}\Psi^s_\alpha, \quad i^r_\beta = a_{19}\Psi^r_\beta - a_{20}\Psi^s_\beta
$$

where $a_{13} = a_{15} = L^r/(L^sL^r - M^2)$; $a_{17} = a_{19} = L^s/(L^sL^r - M^2)$; $a_{14} = a_{16} = a_{18} = a_{20} = M/(L^sL^r - M^2)$.

The block diagram for Eqs. (2.49) appears in Appendix 1. The setup of Fig. A2 is stable, the negative feedback signal exceeds the positive feedback signal, and hf generation is absent.

The notation of the equations for currents and flux linkages is chosen such as to be convenient for the simulation procedure:

$$
\begin{aligned}
&d\Psi^s_\alpha/dt = a_1u^s_\alpha - a_2 i^s_\alpha, && d\Psi^s_\beta/dt = a_3u^s_\beta - a_4 i^s_\beta \\
&d\Psi^r_\alpha/dt = -\omega_r\Psi^r_\beta - a_5 i^r_\alpha, && d\Psi^r_\beta/dt = \omega_r\Psi^r_\alpha - a_6 i^r_\beta \\
&i^s_\alpha = a_7\Psi^s_\alpha - a_8\Psi^r_\alpha, && i^s_\beta = a_9\Psi^s_\beta - a_{10}\Psi^s_\alpha \\
&i^r_\alpha = a_{11}\Psi^r_\alpha - a_{12}\Psi^s_\alpha, && i^r_\beta = a_{12}\Psi^r_\beta - a_{14}\Psi^s_\beta
\end{aligned}
\qquad (2.50)
$$
$$
M_e = a_{15}(i^s_\beta\Psi^r_\alpha - i^s_\alpha\Psi^r_\beta), \quad d\omega_r/dt = a_{16}(M_e - M_s)
$$

where $a_1 = a_3 = U_{n.ph}\sqrt{2}$; $a_2 = a_4 = R^s$; $a_5 = a_6 = R^r$, $a_7 = a_9 = L^r/(L^sL^r - M^2)$; $a_8 = a_{10} = a_{12} = a_{14} = M/(L^sL^r - M^2)$; $a_{11} = a_{13} = M/(L^sL^r - M^2)$; $a_{15} = (mp/2)\,(M/L^r)$; $a_{16} = p/J$.

This model is stable over the wide range of parameters and is applicable to the solution of complicated problems (see Fig. A3).

The choice of a simulation model depends on the gains of computing blocks, permissible time of integration and the stability in operation. An important stage in preparing a problem for simulation is the choice of scales of variables and the calculation of gains of the model's computing blocks. In the simulation on an analog computer, each physical quantity of a real object has its analog as a voltage. Scale factors are chosen to establish the relation between the quan-

tities and voltages. The maximum possible values of variables should be taken so as not to fall outside the limits of the computer working range. The voltage is given by

$$u = M_x x \tag{2.51}$$

where u is the model voltage; x is a dependent variable; and M_x is the scale of the dependent variable. The time of transients in the model is

$$t_m = M_t t_r \tag{2.52}$$

where M_t is the time scale; and t_r is the time of transients in a real object.

In choosing the scales, it is convenient to take the following quantities as decisive factors: the nominal-phase current $I_{n.ph}\sqrt{2}$; nominal-phase voltage $U_{n.ph}\sqrt{2}$; nominal moment M_n; $\Psi_n = U_{n.ph}\sqrt{2}/\omega_0$; $\omega_r = 2\pi np/60$. Here $\omega_0 = 2\pi f$, and ω_r is the angular velocity in radians per second.

Then, $M_u = 100/U_{n.ph}\sqrt{2}$ is the voltage scale; $M_I = 100/I_{n.ph} \times \sqrt{2}\, k_{r_I}$ is the current scale; $M_{M_e} = 100/M_n k_{rM_e}$ is the electromagnetic torque scale; $M_\Psi = 100/\Psi_n k_{r\Psi}$ is the flux linkage scale, and $M_{\omega_r} = 100/\omega_r$ is the velocity scale. The time scale is chosen proceeding from the deceleration or acceleration of the process of solution, as defined by (2.52). In choosing the scales, it is necessary to keep within the working range of an analog computer ($u = \pm 100$ V).

The gain factors of computing blocks are defined as $k_\Sigma = M_{out} a_i / M_{in}$ for a summing amplifier; $k_\int = M_{out} a_i / M_{in} M_t$ for an integrating amplifier; $k_\Sigma = M_{out} a_i / 0.01 M_{1in} M_{2in}$ for a summer receiving the product of variables; $k_\int = M_{out} a_i / 0.01 M_{1in} M_{2in} M_t$ for an integrator receiving the product of variables. Here M_{out} is the scale for representation of an output variable of an amplifier; M_{in} is the scale for representation of an input variable of an amplifier; and a_i is the coefficient in the initial equation, which stands before the variable arriving at the amplifier input.

The gains of amplifiers in most analog computers must be chosen between 0.01 and 10.

The successful implementation of a simulation model with the aid of an analog computer depends on the ratio between the electromagnetic and electromechanical time constants. If these constants differ by a factor of 10^3 to 10^5, the simulation on an analog computer becomes difficult (this is the case for the simulation models of gyro motors, hf machines, etc.).

If the simulation in real quantities meets with difficulties, it is advisable to convert to relative units. The right choice of the basic

quantities enables the gains of the model to be brought close to unity. This decreases the dynamic errors of the model's computing blocks and raises the computer capability.

The convenient basic quantities are $i_b = I_{n.ph}\sqrt{2}$; $u_b = U_{n.ph}\sqrt{2}$; and $\omega_b = \omega_0 = 2\pi f_0$, and also their derivatives such as the base power $P_b = (m/2)\, i_b u_b$, base moment $M_b = P_b/\omega_b$, flux linkage $\Psi_b = u_b/\omega_b$, time $t_b = 1/\omega_b$, impedance $z_b = u_b/i_b$, inductance $L_b = x_b/\omega_b$, and moment of inertia $J_b = M_b/\omega_b$.

The mathematical model of an energy converter, where the differential equations for flux linkages are written in quantities expressed in dimensionless units, has the form

$$
\begin{gathered}
d\dot{\Psi}^s_\alpha/d\tau = \dot{a}_1\dot{u}^s_\alpha - \dot{a}_2\dot{\Psi}^s_\alpha + \dot{a}_3\dot{\Psi}^r_\alpha, \quad d\dot{\Psi}^s_\beta/d\tau = \dot{a}_4\dot{u}^s_\beta - \dot{a}_5\dot{\Psi}^s_\beta + \dot{a}_6\dot{\Psi}^r_\beta \\
d\dot{\Psi}^r_\alpha/d\tau = -\dot{a}_7\dot{\Psi}^r_\alpha + \dot{a}_8\dot{\Psi}^s_\beta - \dot{\omega}_r\dot{\Psi}^r_\beta \\
d\dot{\Psi}^r_\beta/d\tau = -\dot{a}_9\dot{\Psi}^r_\beta + \dot{a}_{10}\dot{\Psi}^s_\beta + \dot{\omega}_r\dot{\Psi}^r_\alpha, \quad M_e = \dot{a}_{11}(\dot{\Psi}^s_\beta\dot{\Psi}^r_\alpha - \dot{\Psi}^s_\alpha\dot{\Psi}^r_\beta) \\
d\omega_r/dt = \dot{a}_{12}(M_e - M_s)
\end{gathered}
\tag{2.53}
$$

where $\dot{a}_1 = \dot{a}_4 = 1$; $\dot{a}_2 = \dot{a}_5 = \frac{R^s L^r}{L^s L^r - M^2} t_b$; $\dot{a}_3 = \dot{a}_6 = \frac{R^s M}{L^s L^r - M^2} t_b$; $\dot{a}_7 = \dot{a}_9 = \frac{R^r L^s}{L^s L^r - M^2} t_b$; $\dot{a}_8 = \dot{a}_{10} = \frac{R^r M}{L^s L^r - M^2} t_b$; $\dot{a}_{11} = (mP_n/2) \times$ $\times\, [M/(L^s L^r - M^2)]\, (\Psi_b^2/M_b)$; $\dot{a}_{12} = P_n M_b / J\omega_b^2$.

The block diagram for (2.53) to be solved on a computer remains the same as before.

The solution of equations in relative units enables handling the problems which otherwise (when using real units) lead to unstable models. The model of an energy converter simulated on an analog computer represents a control system containing positive and negative feedback loops. The degree to which one feedback path or another affects the stability depends on the relations between the parameters of the machine being modeled. A simulation model is chosen proceeding from the type of energy converter, its characteristics and transient behavior. In Appendix I are given the equations and the block diagrams of the main types of energy converters.

2.4. Transient Processes in Electric Machines

Transients in electric machines arise from changes in the voltages and frequencies at machine terminals, the load on the shaft, machine parameters, during connection of a machine to or its disconnection

from the bus, etc. In real conditions, transient processes can naturally occur during a simultaneous variation of a few factors. The combinations of the factors affecting the dynamics can be manifold, so the researcher must have enough experience and knowledge to choose the prevailing set of factors, thereby simplifying the problem. There is a great variety of transients which are much more complex than steady-state processes, the latter being a particular case of the former. By their importance and the influence they have on the operation of machines, the transients can be divided into the processes brought about during starting, braking, reversing, restarting, and load variation. These processes can appear at symmetric and

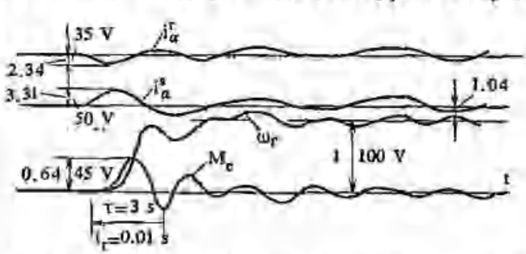

Fig. 2.10. The oscillogram of starting the УАД-22 type 4-W motor

asymmetric voltages in symmetric and asymmetric machines. The dynamics of asynchronous and synchronous machines has its own features. A commutator or any other frequency converter introduces its own specific features into the dynamic behavior of the machine. The transients in transformers and other electromagnetic energy converters differ from the transients in rotating electric machines.

Transients often determine the choice of the installed power of equipment, the mass of electric machines and electromagnetic loads they have to carry. This is particularly the case for impact-load heavy-duty drives, reversing quick-acting drives, etc. To analyze transient processes, we should formulate the mathematical model of the transients, convert the equations to the forms convenient for the simulation of the processes on a computer and solve these equations.

At present a large amount of material is available on the investigation of dynamic processes by means of computers; many problems have become classical and form part of the laboratory work assigned to students in most colleges.

Figures 2.10 through 2.13 illustrate the start oscillograms for motors ranging in power from 4 W to 500 kW. As seen from the oscillograms, the starting procedure pattern differs with the relations between the parameters. The УАД-22 motor comes up to its steady-state velocity

for two or three periods, but the rotor yet oscillates about its steady angular velocity for some time. The 500-kW motor gains speed very slowly, but it does not overspeed after approaching the steady-state velocity. The starting conditions for motors of the 4A series, for example, 75-kW motors, are most typical (Fig. 2.12).

The process of reversal differs from the process of starting by the effect of parameters on the impact torque M_{tm}, impact current

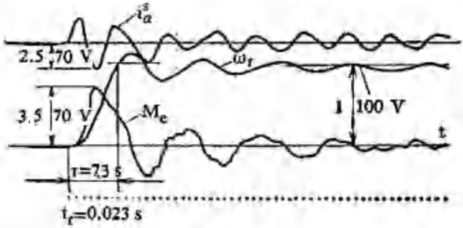

Fig. 2.11. The oscillogram of starting the 4AA-63A4 type 250-W induction motor

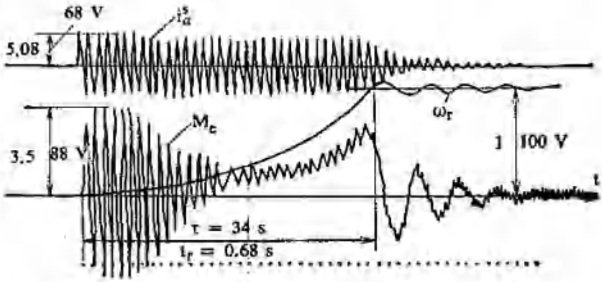

Fig. 2.12. The oscillogram of starting the 4A-250S4 type 75-kW induction motor

I_{im}, and the starting (speedup) time t_{st} (Fig. 2.14). For its reversal, a motor is cut off and then connected to the line with the reverse phase sequence. The transient here depends on the commutation time and on whether or not the field in the air gap has died out. Where the switching is instantaneous, the processes of field decay and field buildup go on concurrently, so the impact currents and torques grow. The schematic of a program unit for reversing is shown in Fig. A7 and Fig. A8.

In restarting a motor, the inrushes of current and the thrust produced are the highest. The schematic diagram for restart and the

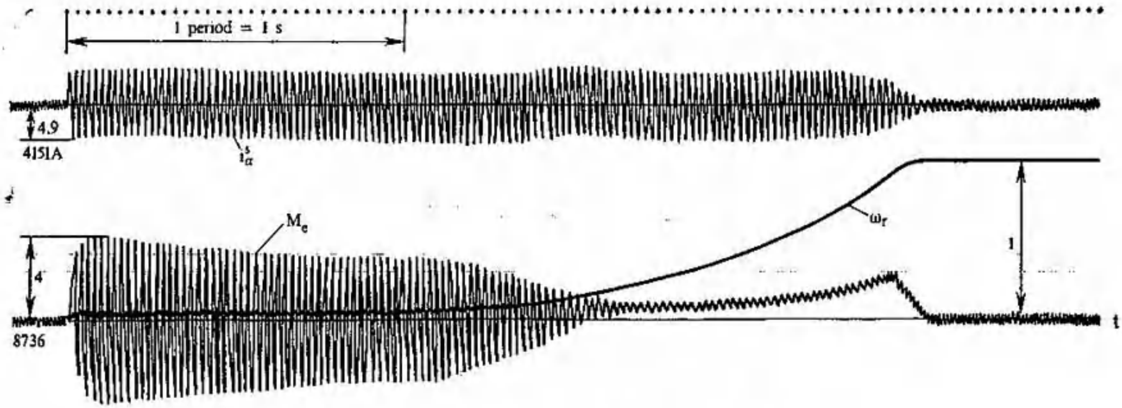

Fig. 2.13. The oscillogram of starting the A2-122-12 motor
$P_2 = 500$ kW, $U_{n.ph} = 380$ V, $f = 50$ Hz

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system of equations for this condition are the same as at starting with the difference that the initial velocity ω_r is other than zero (Figs. 2.15, 2.16). The analysis of restart processes involving an

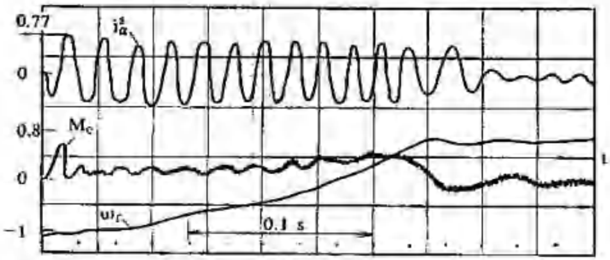


Fig. 2.14. Reversing the АД-31/4 induction motor
$P_2 = 2.2$ kW, $U = 220$ V, $n_n = 1\,485$ rpm, $I_l = 8.3$ A, $M_{st} = 14.5$ N m, $J = 0.012$ kg m^2

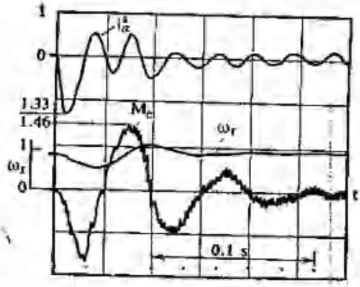


Fig. 2.15. Restarting the АД-31/4 motor at the initial speed of $0.9n_n$ with the field decayed

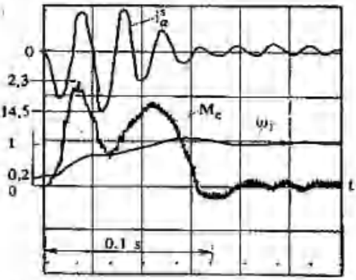


Fig. 2.16. Restarting a motor at the initial speed of $0.2n_n$ with the field decayed

undamped field requires the computation of initial conditions for magnetic flux linkages. The transients at restarting in the undamped field conditions are most complex.

From the qualitative analysis of the processes of starting, reversing, and restarting it follows that these processes differ from each other by the character of variation of currents, torques, and angular velocities. The effect of parameters on the course of various transients is different. The analysis of transients is made by solving the system of transient dynamic equations.

A diversity of transients in electric machines stems from the combined effect of parameters, their nonlinear relationships, effect of elements connected to the stator and rotor, feedback paths, asymmetric and non-sinusoidal pattern of voltages, running conditions, and design versions of energy converters.

In Appendix II are given the data on motors of the 4A series, their parameters, base quantities, coefficients in equations written in flux linkage notation, scales of variables, and gain factors of the amplifiers in a simulation model. Table 2.1 lists the values of basic

Table 2.1

Basic Quantities in Transients

Quantity	Motor type							
	4A-80AA4	4A-112M4	4A-132M4	4A-180M4	4A-250LA	4A-250S4	4A-250M4	4AH-250M4
I_{st}	11.0	19.5	33.3	88.1	113.2	213.8	232.7	270.4
M_{tm}	4.3	4.5	4.25	3.25	3.16	3.5	3.42	2.37
I^{s}_{tm}	3.55	5.12	5.26	5.21	4.86	5.08	4.65	4.35
I^{r}_{tm}	3.5	5.12	5.21	4.92	4.8	4.85	4.5	3.64

quantities (in relative units) describing transients in motors. The dynamic processes are of importance in the analysis of a number of drives, therefore it is expedient that the guides to electric machines should include the characteristics of transients along with steady-state characteristics.

The theory of synchronous machines widely uses the notions of steady-state, subtransient, and transient conditions and the parameters describing these conditions. Transients in induction machines were given insufficient treatment. Because of a narrow gap in these machines, there has been no need, until recently, to introduce the parameters of steady-state and transient conditions.

The development of the general theory of electric machines and the theory of transients in induction machines has necessitated the introduction of transient parameters for induction machines. Inductive reactances greatly vary until a motor attains its operating speed. In Appendix II are given the transient parameters for some machines of the 4A series. The analytical evaluation of inductive parameters was made for the steady-state conditions. Transient parameters can be defined by using computers for the solution of energy conversion equations.

2.5. The Effect of Parameters on the Dynamic Characteristics of Induction Machines

As an example of application of computers for the simulation of dynamic processes, consider the transients in an induction machine with a circular field in the air gap. Such a field can exist in an ideal machine supplied from a sinusoidal symmetric voltage source. In real electric machines, the air gap contains a host of the fields of higher harmonics along with the field of the fundamental harmonic, therefore the analysis of the processes in machines that takes into consideration only the fundamental harmonic applies to the ideal machine and gives approximate results.

If the electromechanical equations for an energy converter having a circular field in the air gap are cast in the form convenient for the simulation of processes on computers, it is possible to solve the equations and thus investigate the processes using the obtained results. The solution of equations for a circular field does not present difficulties. Given the simulation model for the solution of electromechanical equations, we can use the results from the processed oscillograms or take the readings on digital measuring devices and thus evaluate the dependent and independent variables and the time from one event to the next (starting, reversing, braking, pulling in synchronism, restarting, etc.).

Computers offer the possibility of analyzing transients, i.e. determining currents, impact torques, and the duration of a transient process with a change in one parameter, which is impossible to do in investigating a real object.

Analyzing the start oscillograms of induction motors of various powers, it is easy to reveal that the course of transients (variations in currents, torques, oscillations) varies from motor to motor because these energy converters heavily differ in parameters.

Figure 2.17 displays the plots of M_{im}, i_{im}, t_{st} versus R^r at starting. Figs. 2.18, 2.19 and 2.20 illustrate how M_{im}, i_{im} and t_{st} vary with mutual inductance (the size of an air gap), the moment of inertia and rotor leakage inductance respectively. The plots are drawn for an A2-102-8 induction motor. Here $P_2 = 100$ kW, $2p = 8$, $U = 220$ V, $R^s = 0.03\ \Omega$, $R^r = 0.024\ \Omega$, $M = 106 \times 10^{-4}$ H, $L^s = L^r = 151.1 \times 10^{-4}$ H, and $J = 6$ kg m^2.

The curves of Figs. 2.17 through 2.20 permit us to estimate the effect of parameters on the dynamic characteristics of a motor at starting. However, these plots are unsuitable for estimating the dynamically optimum parameters at which, for example, the starting time and the impact torque would be at a minimum.

The curves of Figs. 2.17 through 2.20 are seen to display extrema. Let us note that *a decrease or increase of only one of the parameters*

cannot lead to optimum results. An analog computer solves the system of five equations. There are optimum relations between the equation coefficients at which the requisite quantities exhibit extrema.

Figure 2.21 illustrates the plots of M_{im}, i_{im}, and t_{st} versus R^r for the reversal process in the A2-102-8 motor; Figs. 2.22, 2.23,

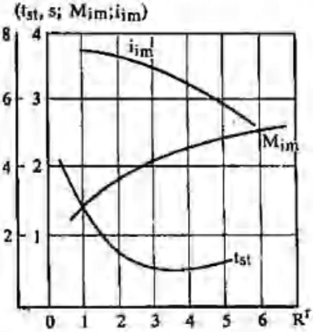


Fig. 2.17. Starting time t_{st}, current i_{im}, and torque M_{im} versus rotor resistance

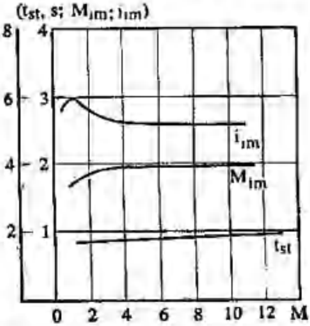


Fig. 2.18. The curves of t_{st}, M_{im}, and i_{im} versus mutual inductance

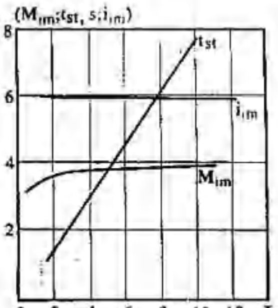


Fig. 2.19. The curves of t_{st}, M_{im}, and i_{im} versus moment of inertia

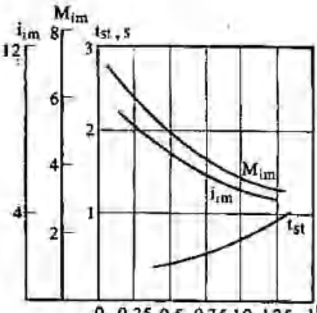


Fig. 2.20. The curves of t_{st}, M_{im}, and i_{im} versus rotor leakage inductance

and 2.24 display the plots of the same quantities as functions of mutual inductance M, moment of inertia J, and rotor leakage inductance l_σ^r respectively. The transients in reversing are more complex than those at starting. It has to be noted that the effect of parameters on the processes occurring in reversing differs from the

effect they have on the processes at starting. *Optimum parameters in starting, reversing or other dynamic conditions differ from one another.*

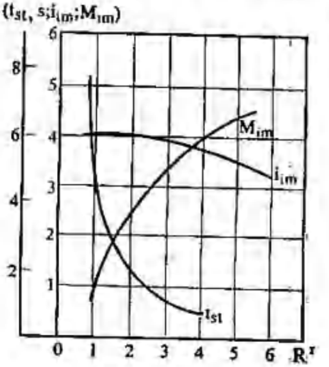


Fig. 2.21. The curves of t_{st}, M_{im}, and i_{im} versus rotor resistance in reversing

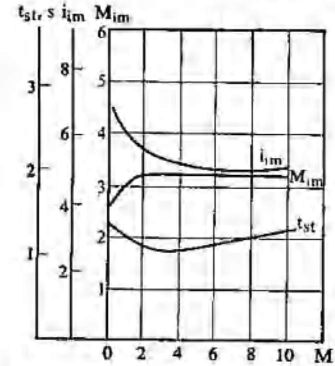


Fig. 2.22. The curves of t_{st}, M_{im} and i_{im} versus mutual inductance in reversing

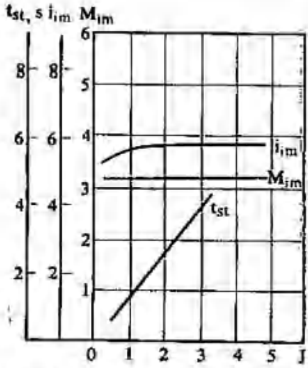


Fig. 2.23. The curves of t_{st}, M_{im} and i_{im} versus moment of inertia in reversing

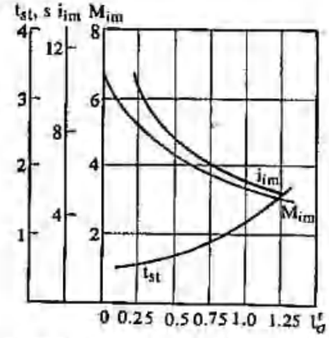


Fig. 2.24. The curves of t_{st}, M_{im} and i_{im} versus leakage inductance in reversing

It is of interest to look at the process of reversing with the field still undamped. With a rather fast reversal of phases at machine terminals, the field in the air gap has no time to give up its stored energy. In other words, the field has not yet decayed at the instant

of connection of the motor to the bus. The analysis of the process of reversal thus requires estimating the initial values of the undamped field by defining the flux linkage between the stator and rotor windings. Reversing contactors operate in the severe conditions (an ac contactor closes in 0.03 to 0.05 s). The most effective technique of studying transients in these devices is to use an analog computer set up for programmable work.

One of the interesting transient modes is the mode at restarting. The system of equations and the simulation model for its solution will be the same as at starting with the exception that the angular velocity ω_r will be other than zero. Investigations reveal that the restarting process at ω_r close to the nominal (with short-time supply interruptions) is accompanied by heavy surges of currents and torques. These surges exceed the maximum values of currents and torques in starting and reversing. The restarting process with the field undamped lends itself to the analysis after estimation of the initial conditions for flux linkages.

Since the rotor has a store of kinetic energy and the field has not decayed, restarting must appear at first glance to be an easy mode for an electric machine. However since restarting involves the disconnection of a machine from and its connection to the supply line, the two attendant transient processes with complicated changes in currents and torques superpose one on the other.

Given the mathematical model of an induction machine, we can analyze the operation of the machine both in the braking and in the generating mode. In investigating the generator action, it suffices to change only the sign of the torque in the electromechanical equations. If the line voltage is constant, the induction generator operates in parallel with and into the infinite power line. Certain difficulties arise in the analysis of an autonomous induction generator when it draws the reactive power from capacitors and the voltage and frequency undergo changes.

Chapter 3

Generalized *m-n* Winding Converter

3.1. The Infinite Arbitrary Spectrum of Fields in the Air Gap

As is known, the circular field in the air gap can be thought to exist only in an idealized machine. *In real machines, the air gap exhibits an infinite spectrum of harmonics differing in amplitude and*

frequency along with the fundamental harmonic. These harmonics revolve both in the forward and in the backward direction with respect to the revolving fundamental harmonic. The angular velocities of the harmonics can be higher and lower than that of the fundamental wave and their amplitudes can vary in rotation. All harmonics may be divided into two types, time and space harmonics.

Time harmonics are the ones which get into the air gap of a machine from the outside. *Space harmonics* appear in the air gap on account of the specifics of the converter's internal structure. It should be kept in mind that this classification of harmonics is rather conditional; the names 'time and space harmonics' arose for historical reasons in the course of development of the theory of electric machines.

Considering the EC as a twoport (see Fig. 1.8), we should note that the converter has two inputs, one on the side of electrical terminals and the other on the side of mechanical terminals. Time harmonics arise from nonsinusoidal, asymmetric voltages and nonlinear changes in the amplitude and frequency of voltages. They also result from nonlinear changes in the torque and speed. In the general case, time harmonics appear from the simultaneous action of nonlinear factors at two input terminals. These harmonics may also get into the air gap of an electric machine from thermal terminals (see Fig. 1.1). Heat shocks (sharp temperature variations of the machine frame) cause upper harmonics in the air gap as a result of changes in the machine parameters.

Nonsinusoidal voltages which give rise to time harmonics may result from nonlinear elements such as saturable reactors and semiconductor elements disposed ahead of the motor, a nonsinusoidal waveform of the generator voltage or distortion of the waveform of the supply voltage, etc. If the supply voltage contains a constant component, a harmonic spectrum emerges, which includes an infinite range of even harmonics along with odd harmonics.

In the absence of time harmonics in the air gap of a machine, space harmonics originate from the nonsinusoidal distribution of turns and magnetizing forces, air gap non-uniformity due to the presence of teeth and slots in the rotor and stator, gap ellipticity and conicity, and nonlinearity of the parameters entering into electromechanical equations.

Consider in more detail space harmonics, in particular the harmonics of magnetizing forces. The windings of electric machines are current loops producing magnetizing forces. The simplest winding (loop) is a turn (or a coil consisting of several turns) whose pitch y is equal to the pole pitch τ (Fig. 3.1*a*). Such a loop produces a rectangular magnetomotive force (mmf). At $y < \tau$ (Fig. 3.1*b*) the mmf takes the form of a trapezoid. Where two or more coils are involved, the mmf assumes the form of a steplike curve (Fig. 3.1*c*).

Developing the mmf as a harmonic progression, we should note the fact that where the mmf distribution is rectangular in shape, upper harmonics have maximum amplitudes; the amplitudes become lower with a shortened winding pitch and lower still further where the

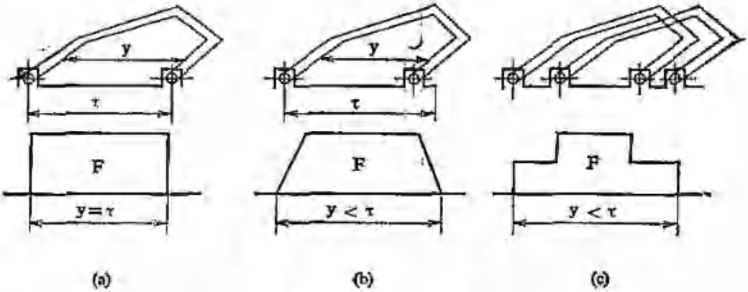


Fig. 3.1. Current loops

winding consists of a few coils. Only in the case of the sinusoidal distribution of turns over a smooth cylindrical surface of the gap upper harmonics of the mmf are nonexistent.

For a coil winding or for one turn, as shown in Fig. 3.1*a*, the amplitudes of harmonics are given by

$$\begin{aligned} F_1 &= (4/\pi)\, F_c \\ F_3 &= (1/3)\,(4/\pi)\, F_c \\ F_5 &= (1/5)\,(4/\pi)\, F_c \\ &\cdots\cdots\cdots \\ F_\nu &= (1/\nu)\,(4/\pi)\, F_c \end{aligned} \tag{3.1}$$

where $F_c = Iw/2$; I is the current flowing in a turn; and w is the number of turns in the coil.

In a winding comprising q coils, the mmf approximates a sinusoid and the amplitudes of harmonics become lower.

In three-phase symmetric windings there appear harmonics of the order

$$\nu = 6c \pm 1$$

where $c = 0, 1, 2, \ldots$. Harmonics of the order $6c + 1$ (7, 13, 19 . . .) revolve in the forward direction with respect to the rotating first harmonic at a speed which is a factor of 7, 13, 19, . . . lower than the speed of the first harmonic. Harmonics of the order $6c - 1$ (5, 11, 17, . . .) revolve at a speed of 1/5, 1/11, 1/17 . . . the speed of the first harmonic in the direction opposite to that of the first harmonic.

For two-phase symmetric windings, $\nu = 4c \pm 1$. Harmonics of the order $4c + 1$ revolve in the same direction as the first harmonic, and harmonics of the order $4c - 1$ revolve in the opposite direction to the first harmonic.

Because the phases of windings are asymmetric, each upper harmonic of the mmf may have a forward and a backward wave. Thus, only a nonsinusoidal and asymmetric distribution of the mmf may give rise to two sets of harmonics in the air gap.

A set of saliency-induced harmonics has a heavy effect on the characteristics of an energy converter. The windings of electric machines are distributed in slots. Since the permeance of the air gap is nonuniform, the field pattern in the gap depends both on the mmf distribution and on the gap permeance:

$$B(x) = \lambda_\delta(x) F(x)$$

The magnetic flux density as a function of the coordinate counted off around the circumference is proportional to the gap's specific permeance $\lambda_\delta(x)$ expressed in terms of the permeance per unit area of the gap. In electric machines the tooth pitch is usually the same around the entire circumference and the gap permeance is represented in the form of a periodic curve which can be expanded into a Fourie series. For a machine exhibiting saliency on both sides of the gap, the analytical estimation of the per-unit-area permeance even with the rotor at standstill, let alone in motion, presents a difficult problem. The quantity λ_δ can be defined as a product

$$\lambda_\delta = \lambda_{\delta_1} \lambda_{\delta_2}$$

where λ_{δ_1} and λ_{δ_2} are the per-unit-area permeances of the stator and rotor respectively, calculated separately on the assumption that saliency results from the slots only on one side of the gap.

The equivalent opening of a slot varies as a result of saturation, so the amplitudes of saliency-induced harmonics may change with load. Skewing of slots on the stator and rotor for one tooth pitch can reduce the amplitudes of these harmonics. With the slots skewed, the emfs due to saliency-induced harmonics over the active length of the machine are offset and the currents caused by the emfs are close to zero.

The magnitude of saliency-induced harmonics depends on the proportion of the number of slots in the stator to that of slots in the rotor. Some proportions are unfavourable because they are responsible for appreciable vibrations and noise. For three-phase induction machines, the unacceptable proportions are the following:

$$\begin{gathered} Z_1 - Z_2 = 0,\ 1,\ 2,\ 3,\ 4;\ \ Z_1 - Z_2 = p,\ p \pm 1 \\ Z_1 - Z_2 = 2p;\ \ 2p \pm 1;\ \ 2p \pm 2;\ \ 2p \pm 3 \\ 2p \pm 4;\ \ Z_1 - Z_2 = 3p \end{gathered} \tag{3.2}$$

Making the right choice of the proportion of the number of slots in the stator to that in the rotor, properly shortening the winding pitch, and deciding on slot opening and skewing, it becomes possible to reduce the amplitudes of space harmonics or make one of the harmonics prevalent over the others in the spectrum. In this case the fundamental, or first, harmonic is an upper space harmonic since its amplitude is the highest over the others.

Space harmonics generally revolve at a lower velocity than the first harmonic. The angular velocity of a space harmonic B_ν is a function of voltage u_1 and its frequency f_1, and the pole pitch is a function of the number of slots and teeth. Therefore the velocity of B_ν is a factor of ν lower than that of the first harmonic. This explains why motors operating on upper space harmonics are of the slow-speed type, and thus can perform the functions of *multipolar* machines or gearmotors.

Slow-speed motors exhibit harmonics whose frequencies are lower than that of the first harmonic. These are *subharmonics* which are also present in ordinary machines. In multipolar machines, subharmonics arise due to the difference between permeances under the poles and as a result of modulation of the first harmonic.

Space harmonics also arise from the nonlinearity of machine parameters. What exerts the highest influence on the spectrum of harmonics caused by the nonlinearity of parameters is saturation, i.e. the nonlinear dependence of mutual inductance on current or time since current is a function of time. All coefficients entering into the electromechanical equations may be nonlinear. In an actual machine, resistances vary on account of current displacement (current density variation), and inductances depend on saturation; in some drives, the moment of inertia undergoes changes. Nonlinear variations in the parameters tend to develop certain spectra of harmonics in the air gap. The effect of nonlinear parameters on the machine characteristics will be dealt with in Ch. 7.

The appearance of space harmonics is attributable to manufacturing factors, to which belong the air gap nonuniformity due to eccentricity of the stator with respect to the rotor, rotor conicity, axial misalignment between rotor and stator, and other factors of this type treated below in the text.

Machine asymmetry too is responsible for the emergence of harmonics in the air gap. A field of the backward (negative) sequence and a field of the zero sequence appear in an asymmetric machine. A pulsating zero-sequence field may be thought of as consisting of a forward and a backward field.

Mention should also be made of heterodyne-frequency harmonics because electromechanical ECs are nonlinear systems; even two harmonics present in an energy converter are enough to give rise to an infinite spectra of harmonics at heterodyne frequencies.

If the possibility exists for the flow of currents in the stator and rotor, which enable the stator and rotor fields to be stationary with respect to each other, an electromagnetic torque results from the pair of harmonics. In a short-circuited (squirrel-cage) rotor induction machine, all harmonics present in the air gap may give rise to torques. Upper harmonics show up most vividly when starting a machine. For each harmonic there comes a point where the rotor speed equals the field speed, so the rotor may "get stuck" at this speed under the influence of the synchronous torque due to the space harmonic.

Thus there is an infinite arbitrary spectrum of harmonics making up the air-gap field. This spectrum can be broken down into sets of harmonics according to their origin. Earlier in the text we have mentioned the classification of harmonics into time and space types. In turn, space harmonics can be divided into types associated with the mmf, saliency, manufacturing factors, nonlinearity of parameters, and heterodyne frequencies.

Of the infinite sets of harmonics, only some affect the characteristics of an EC, since a great many of the harmonics have infinitely low amplitudes. Therefore only a small number of harmonics are given consideration in the analysis of energy conversion processes.

To gain an insight into complicated interactions of harmonics, we first need to construct a mathematical model to describe energy conversion processes involving an infinite spectrum of harmonics.

3.2. The Generalized Energy Converter

In magnetic-field energy converters, the working field is a magnetic field with its energy concentrated in the air gap—the space where electromechanical energy conversion takes place. As mentioned earlier, in the air gap of an electric machine there exists an infinite harmonic spectrum dependent on the supply voltage, load on the machine, and machine design. Any processes in a machine, of whatever character, lead to changes in the air-gap field.

In the mathematical model of a machine, each harmonic may be set up by a pair of windings on the stator and on the rotor if currents of respective amplitudes and frequencies, shifted in phase with respect to each other, flow in the windings. Given an infinite set of windings, it is possible to produce a field of any shape.

In Section 1.4 we have described the generalized, or primitive, machine defined as an idealized unsaturated machine with a uniform air-gap structure, which carries two pairs of windings on the stator and rotor. Here we shall deal with the generalized electromechanical energy converter—the mathematical model of a real machine—which may have many loops (phase windings) on the stator and rotor.

Such a generalized convertor enables us to write equations with due regard for the field in the air gap and all current-carrying loops.

Summing up, *the generalized energy converter is an idealized two-pole two-phase electric machine with m-n windings on the stator and rotor, respectively, arranged along the α and β axes as shown in Fig. 3.2.*

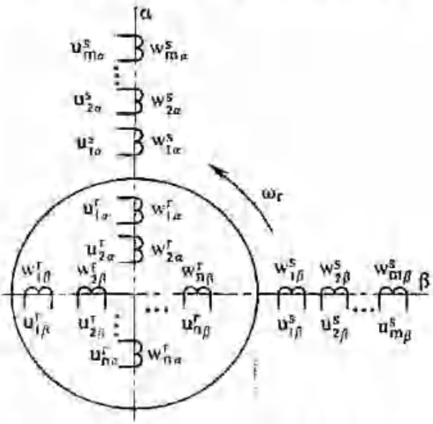


Fig. 3.2. A generalized electromechanical energy converter

Using the model of the generalized converter we can describe symmetric multiphase multipolar machines on the assumption that they are transformable to an equivalent form to match the two-phase two-pole machine.

As seen from Fig. 3.2, each phase winding has its designation: subscripts α and β identify the axes along which windings w lie; 1, 2, . . ., m, n stand for the ordinal number of the windings on the stator and rotor respectively; and superscripts s and r denote the stator and rotor windings, respectively, supplied with voltages u.

Each pair of the windings is fed from an individual supply source or all windings arranged in any kinds of networks draw current from a single source. In the generalized converter, magnetic link between the groups of windings on the same axis may not exist. Each pair of windings (coils) on the stator produces a circular field in the air gap. As is known, the generalized converter is an unsaturated machine, which allows us to use the principle of superposition. The field in the air gap can be set up by applying to the windings the voltages of different amplitudes and frequencies, shifted in

phase with respect to each other. From the viewpoint of mathematical theory, electric machines differ from one another by the form of the field in the air gap, number of windings, and the parameters of windings.

The generalized converter is a useful tool for describing any electric machine. For example, a single-phase single-winding motor with a pulsating field in the air gap can be represented by a mathematical model comprising two pairs of windings on the stator and two

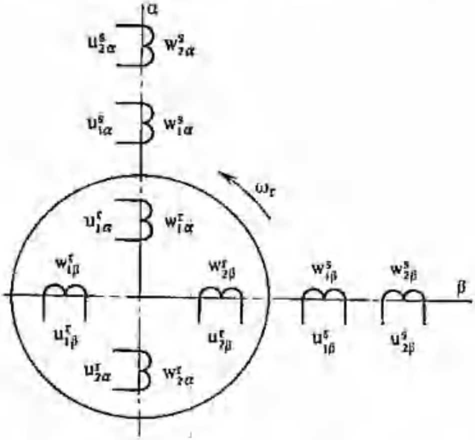


Fig. 3.3. The model of a single-phase motor

pairs on the rotor (Fig. 3.3). Windings $w^s_{1\alpha}$ and $w^s_{1\beta}$ build up a forward (positive-sequence) field ($u^s_{1\alpha} = U_m \sin \omega t$, $u^s_{1\beta} = U_m \cos \omega t$). Windings $w^s_{2\alpha}$ and $w^s_{2\beta}$ are fed with voltages $u^s_{2\alpha} = U_m \cos \omega t$ and $u^s_{2\beta} = U_m \sin \omega t$ which set up a backward (negative-sequence) field. If the rotor is of the squirrel-cage type, $u^r_{1\alpha}$, $u^r_{1\beta}$, $u^r_{2\alpha}$, and $u^r_{2\beta}$ are equal to zero.

By changing the voltages across the windings which produce the forward and backward fields, it is possible to go over from the pulsating field to the elliptic field and then to the circular field if the backward field in the air gap does not exist. If, apart from the forward and the backward field, upper harmonics are present in the gap, it is necessary to add a requisite number of pairs of windings to the model and set up the field by applying to the stator windings the voltages of corresponding amplitudes and frequences, which show a definite phase sequence and phase shift.

Most electric machines have several windings. If eddy current loops are taken into account, all machines may be thought of as multiwinding machines. As mentioned earlier, the generalized converter serves as a powerful tool for the analysis of actual machines with many windings.

Equations describing the behavior of the majority of electric machines can be set up using the equations for the generalized energy converter. For this we need to expand the mmf of the field (whose shape in the gap is known) into a harmonic series, construct the model from a few pairs of windings and apply the voltages of corresponding amplitudes and frequencies to these windings. Although the field pattern in a rotating machine is practically impossible to define, the mathematical model of an energy converter enables us to solve many problems to a sufficient accuracy by specifying voltages on the input terminals of the converter.

The analysis of working processes in electric machines relies on two statements: (a) all static and dynamic characteristics of a machine are governed by the processes occurring in the air gap; (b) an electric machine is represented as a system of linear electric circuits moving with respect to one another.

3.3. The Equations of the Generalized Energy Converter

The voltage equations for the generalized energy converter are written in the form of a complex matrix similar to the Kron matrix for the primitive machine of Fig. 1.11:

$$\begin{vmatrix} u_\alpha^s \\ u_\alpha^r \\ u_\beta^s \\ u_\beta^r \end{vmatrix} = \begin{vmatrix} A_\alpha^s & A_\alpha^{sr} & 0 & 0 \\ A_\alpha^{rs} & A_\alpha^r & D_\beta & B_{\beta\alpha} \\ B_{\alpha\beta} & D_\alpha & A_\beta^r & A_\beta^{rs} \\ 0 & 0 & A_\beta^{sr} & A_\beta^s \end{vmatrix} \times \begin{vmatrix} i_\alpha^s \\ i_\alpha^r \\ i_\beta^r \\ i_\beta^s \end{vmatrix} \tag{3.3}$$

Each element in the above matrix is a submatrix. Here u_α^s, u_α^r, u_β^r, and u_β^s are matrix columns:

$$u_\alpha^s = \begin{vmatrix} u_{1\alpha}^s \\ u_{2\alpha}^s \\ \vdots \\ u_{m\alpha}^s \end{vmatrix}, \quad u_\alpha^r = \begin{vmatrix} u_{1\alpha}^r \\ u_{2\alpha}^r \\ \vdots \\ u_{n\alpha}^r \end{vmatrix}, \quad u_\beta^r = \begin{vmatrix} u_{1\beta}^r \\ u_{2\beta}^r \\ \vdots \\ u_{n\beta}^r \end{vmatrix}; \quad u_\beta^s = \begin{vmatrix} u_{1\beta}^s \\ u_{2\beta}^s \\ \vdots \\ u_{m\beta}^s \end{vmatrix} \tag{3.4}$$

Also, i_α^s, i_α^r, i_β^r, i_β^s are matrix columns:

$$i_\alpha^s = \begin{vmatrix} i_{1\alpha}^s \\ i_{2\alpha}^s \\ \vdots \\ i_{m\alpha}^s \end{vmatrix}, \quad i_\alpha^r = \begin{vmatrix} i_{1\alpha}^r \\ i_{2\alpha}^r \\ \vdots \\ i_{n\alpha}^r \end{vmatrix}, \quad i_\beta^r = \begin{vmatrix} i_{1\beta}^r \\ i_{2\beta}^r \\ \vdots \\ i_{n\beta}^r \end{vmatrix}, \quad i_\beta^s = \begin{vmatrix} i_{1\beta}^s \\ i_{2\beta}^s \\ \vdots \\ i_{m\beta}^s \end{vmatrix} \qquad (3.5)$$

In Eqs. (3.4), $u_{1\alpha}^s$, $u_{2\alpha}^s$, ..., $u_{m\alpha}^s$; $u_{1\alpha}^r$, $u_{2\alpha}^r$, ..., $u_{n\alpha}^r$; $u_{1\beta}^r$, $u_{2\beta}^r$, ..., $u_{n\beta}^r$; $u_{1\beta}^s$, $u_{2\beta}^s$, ..., $u_{m\beta}^s$ are the stator and rotor voltages along the α and β axes. In Eqs. (3.5), $i_{1\alpha}^s$, $i_{2\alpha}^s$, ..., $i_{m\alpha}^s$; $i_{1\alpha}^r$, $i_{2\alpha}^r$, ..., $i_{n\alpha}^r$; $i_{1\beta}^r$, $i_{2\beta}^r$, ..., $i_{n\beta}^r$; $i_{1\beta}^s$, $i_{2\beta}^s$, ..., $i_{m\beta}^s$ are the currents along the α and β axes in the stator and rotor.

Voltage equations (3.3) may be written in a more general form

$$[u] = [z] \times [I] \qquad (3.6)$$

The impedance matrix $[z]$ includes 12 submatrices. Four impedance matrices lie along the diagonal:

$$A_\alpha^s = \begin{vmatrix} r_{1\alpha}^s + (d/dt) L_{1\alpha}^s & (d/dt) M_{12\alpha}^s & \ldots & (d/dt) M_{1m\alpha}^s \\ (d/dt) M_{21\alpha}^s & r_{2\alpha}^s + (d/dt) L_{2\alpha}^s & \ldots & (d/dt) M_{2m\alpha}^s \\ \vdots & & \vdots & \vdots \\ (d/dt) M_{m1\alpha}^s & (d/dt) M_{m2\alpha}^s & \ldots & r_{m\alpha}^s + (d/dt) L_{m\alpha}^s \end{vmatrix} \qquad (3.7)$$

Denote the resistances of stator and rotor windings on the α and β axes as $r_{1\alpha}^s$, $r_{2\alpha}^s$, ..., $r_{m\alpha}^s$; $r_{1\alpha}^r$, $r_{2\alpha}^r$, ..., $r_{n\alpha}^r$; $r_{1\beta}^s$, $r_{2\beta}^s$, ..., $r_{m\beta}^s$; $r_{1\beta}^r$, $r_{2\beta}^r$, ..., $r_{n\beta}^r$. Next, denote the total inductances of stator and rotor windings on the α and β axes as $L_{1\alpha}^s$, $L_{2\alpha}^s$, ..., $L_{m\alpha}^s$; $L_{1\alpha}^r$, $L_{2\alpha}^r$, ..., $L_{n\alpha}^r$; $L_{1\beta}^s$, $L_{2\beta}^s$, ..., $L_{m\beta}^s$; $L_{1\beta}^r$, $L_{2\beta}^r$, ..., $L_{n\beta}^r$. Finally, designate the mutual inductances between stator windings on the α axis as $M_{12\alpha}^s$, ..., $M_{m1\alpha}^s$. Thus $M_{12\alpha}^s$ identifies the mutual inductance between the first and the second stator winding on the α axis.

In a machine with the equal number of turns on the stator and rotor the mutual inductances for all windings along the same axis are identical. The total inductance L is defined as a sum of the mutual inductance and the leakage inductance of the given winding.

The submatrix A_β^s is similar to A_α^{sr} and comes out after the substitution of β for α. Submatrices A_α^r and A_β^s in (3.3) come from A_α^s and A_β^s by substituting r for s. Submatrices A_α^{sr}, A_α^{rs}, A_β^{sr}, and A_β^{rs} are indicative of linkage between stator and rotor windings. The submatrix A_α^{sr} has the form

$$A_\alpha^{sr} = \begin{vmatrix} (d/dt) M_{11\alpha}^{sr} & (d/dt) M_{12\alpha}^{sr} & \ldots & (d/dt) M_{1n\alpha}^{sr} \\ (d/dt) M_{21\alpha}^{sr} & (d/dt) M_{22\alpha}^{sr} & \ldots & (d/dt) M_{2n\alpha}^{sr} \\ \vdots & \vdots & \vdots & \vdots \\ (d/dt) M_{m1\alpha}^{sr} & (d/dt) M_{m2\alpha}^{sr} & \ldots & (d/dt) M_{mn\alpha}^{sr} \end{vmatrix} \qquad (3.8)$$

Here $M^{sr}_{11\alpha}$ is the mutual inductance between the first windings on the stator and rotor along the α axis; $M^{sr}_{mn\alpha}$ is the mutual inductance between the mth stator winding and the nth rotor winding along the α axis. The submatrix A^{sr}_{β} follows from the matrix (3.8) after substituting β for α.

The submatrix A^{rs}_{α} results from A^{sr}_{α} by interchanging the positions of superscripts s and r and also m and n; A^{rs}_{β} results from A^{rs}_{α} by changing α for β. The inductance $M^{sr}_{mn\alpha}$ is generally not equal to $M^{rs}_{nm\alpha}$, though in many cases these inductances may be taken equal to each other. Submatrices D_α and D_β and also $B_{\alpha\beta}$ and $B_{\beta\alpha}$ are related to the rotating emf. The submatrices D_α and D_β corresponding to the total inductive reactances of rotor windings along the α and β axes have the form

$$D_\alpha = \begin{vmatrix} -L^r_{1\alpha}\omega_r & -M^r_{12\alpha}\omega_r & \dots & -M^r_{1n\alpha}\omega_r \\ -M^r_{21\alpha}\omega_r & -L^r_{2\alpha}\omega_r & \dots & -M^r_{2n\alpha}\omega_r \\ \vdots & \vdots & \ddots & \vdots \\ -M^r_{n1\alpha}\omega_r & -M^r_{n2\alpha}\omega_r & \dots & -L^r_{n\alpha}\omega_r \end{vmatrix} \tag{3.9}$$

$$D_\beta = \begin{vmatrix} L^r_{1\beta}\omega_r & M^r_{12\beta}\omega_r & \dots & M^r_{1n\beta}\omega_r \\ M^r_{21\beta}\omega_r & L^r_{2\beta}\omega_r & \dots & M^r_{2n\beta}\omega_r \\ \vdots & \vdots & \ddots & \vdots \\ M^r_{n1\beta}\omega_r & M^r_{n2\beta}\omega_r & \dots & L^r_{n\beta}\omega_r \end{vmatrix} \tag{3.10}$$

In (3.9) and (3.10), $M^r_{12\alpha}$ and $M^r_{12\beta}$ are mutual inductances between the first and the second rotor winding along the α and β axes respectively.

The submatrices $B_{\alpha\beta}$ and $B_{\beta\alpha}$ differ in sign and correspond to mutual inductances between the stator and rotor windings:

$$B_{\alpha\beta} = \begin{vmatrix} -M_{11\alpha}\omega_r & -M_{12\alpha}\omega_r & \dots & -M_{1m\alpha}\omega_r \\ -M_{21\alpha}\omega_r & -M_{22\alpha}\omega_r & \dots & -M_{2m\alpha}\omega_r \\ \vdots & \vdots & \ddots & \vdots \\ -M_{n1\alpha}\omega_r & -M_{n2\alpha}\omega_r & \dots & -M_{nm\alpha}\omega_r \end{vmatrix} \tag{3.11}$$

where $M_{11\alpha}, \dots, M_{1m\alpha}$ are mutual inductances between the first rotor winding along the β axis and the first stator winding along the α axis; $M_{n1\alpha}, \dots, M_{nm\alpha}$ are mutual inductances between the nth rotor winding and the mth stator winding. The submatrix $B_{\alpha\beta}$ relates the rotor windings along the α axis to the stator windings along the β axis. The signs in the matrix are positive. The submatrix $B_{\beta\alpha}$ follows from (3.11) by interchanging the position of α and β.

The electromagnetic torque in the generalized energy converter is defined as the products of all currents flowing in the loops of the machine.

The equations of motion for a motor with m-n windings on the stator and rotor, respectively, along the α and β axes are written in the form

$$\begin{aligned} M_r = J d\omega_r/dt &+ M^{sr}_{11\alpha} i^s_{1\alpha} i^r_{1\beta} - M^{sr}_{11\beta} i^s_{1\beta} i^r_{1\alpha} \\ &+ M^{sr}_{12\alpha} i^s_{1\alpha} i^r_{2\beta} - M^{sr}_{12\beta} i^s_{1\beta} i^r_{2\alpha} + \ldots + M^{sr}_{1n\alpha} i^s_{1\alpha} i^r_{n\beta} - M^{sr}_{1n\beta} i^s_{1\beta} i^r_{n\alpha} \\ &+ M^{sr}_{21\alpha} i^s_{2\alpha} i^r_{1\beta} - M^{sr}_{21\beta} i^s_{2\beta} i^r_{1\alpha} + M^{sr}_{22\alpha} i^s_{2\alpha} i^r_{2\beta} - M^{sr}_{22\beta} i^s_{2\beta} i^r_{2\alpha} \\ &+ \ldots + M^{sr}_{2n\alpha} i^s_{2\alpha} i^r_{n\beta} - M^{sr}_{2n\beta} i^s_{2\beta} i^r_{n\alpha} \\ &+ \ldots + M^{sr}_{mn\alpha} i^s_{m\alpha} i^r_{n\beta} - M^{sr}_{mn\beta} i^s_{m\beta} i^r_{n\alpha} \end{aligned} \quad (3.12)$$

The expression for the electromagnetic torque of a symmetric machine, where the mutual inductances along the α and β axes are the same for all windings and equal to M, is derived from (3.12) and is written as

$$\begin{aligned} M_e = M \, [(i^s_{1\alpha} + i^s_{2\alpha} + i^s_{3\alpha} + \ldots + i^s_{m\alpha})(i^r_{1\beta} + i^r_{2\beta} + i^r_{3\beta} + \ldots + i^r_{m\beta}) \\ - (i^s_{1\beta} + i^s_{2\beta} + i^s_{3\beta} + \ldots + i^s_{m\beta})(i^r_{1\alpha} + i^r_{2\alpha} + i^r_{3\alpha} + \ldots + i^r_{m\alpha})] \end{aligned} \quad (3.13)$$

From (3.13) it follows that the electromagnetic torque can be defined by two products of the four nonsinusoidal currents. The terms in (3.12) and (3.13) can be divided into two groups. To the first group belong the terms associated with the buildup of starting, braking, or generating torques, and to the second belong the terms having to do with pulsating torques. The torques involved with the first group are set up by the stator and rotor fields that are stationary relative to each other. Pulsating torques are due to the fields moving with respect to each other.

The expressions for the former torques take the form

$$\begin{matrix} M^{sr}_{11\alpha} i^s_{1\alpha} i^r_{1\beta} - M^{sr}_{11\beta} i^s_{1\beta} i^r_{1\alpha} \\ \cdots\cdots\cdots\cdots\cdots \\ M^{sr}_{mm\alpha} i^s_{m\alpha} i^r_{m\beta} - M^{sr}_{mm\beta} i^s_{m\beta} i^r_{m\alpha} \end{matrix} \quad (3.14)$$

Pulsating torques are given by

$$\begin{matrix} M^{sr}_{12\alpha} i^s_{1\alpha} i^r_{2\beta} - M^{sr}_{12\beta} i^s_{1\beta} i^r_{2\alpha} \\ \cdots\cdots\cdots\cdots\cdots \\ M^{sr}_{mn\alpha} i^s_{m\alpha} i^r_{n\beta} - M^{sr}_{mn\beta} i^s_{m\beta} i^r_{n\alpha} \end{matrix} \quad (3.15)$$

The latter torques cause vibrations and do not afford the mean torque component.

Expressions (3.3) through (3.12) are the most general electromechanical equations for the case where a stator or rotor is stationary. Where both the stator and rotor are in motion, expression (3.13) contains four more submatrices to allow for stator rotation.

Proceeding from expressions (3.3) through (3.12), it becomes possible to derive equations for multiwinding machines, machines

supplied with nonsinusoidal asymmetric voltages, nonsinusoidal magnetic-field machines, and most of the other electric machines. The equations here are written in the α-β coordinate system, though they can also be set up in other coordinates.

Equations (3.3) through (3.12) contain integral parameters despite the fact that a winding consists of turns. In the analysis of surges, the treatment of the processes in a winding should be given with regard to voltage distribution among the turns. Note also that voltages distribute themselves nonuniformly among the turns at the slot bottom, near the slot wedge, at the top and in the middle of the winding, and also at the end and slot turn portions. It is thus possible to formulate a mathematical model assuming that the energy converter has distributed parameters.

The mathematical model becomes more complex if a machine has two, three, or n degrees of freedom. This is a machine where the stator and rotor are both rotating members, or the rotor has the shape of a sphere with the spherical stator being either stationary or in motion. In the electromechanical equations for electric-field energy converters, capacitances substitute for inductances.

The concept of the generalized converter continues to develop in keeping with the advancement of the theory of electromechanical energy conversion. G. Kron was the first to introduce the notion of a primitive machine in the 1930s. The generalized electromechanical energy converter represents the model for describing energy conversion processes in magnetic-field machines having one degree of freedom, an infinite spectrum of harmonics, and any number of loops on the stator and rotor. The notion of the generalized converter can be extended to cover machines with any number of degrees of freedom, electric-field converters, and electromagnetic field converters.

Chapter 4

Typical Equations of Electric Machines

4.1. Transition from Simple to More Complex Equations

In a particular case, the model of the generalized energy converter enables us to derive electromechanical energy conversion equations for machines with a few harmonics in the air gap, multiwinding

machines, energy converters supplied from nonsinusoidal voltage sources, and for other actual machines showing a complicated interaction between harmonics. Given the equation for the most complex m-n winding machine, we can gradually simplify the mathematical description in the course of analysis to transform the model to the simplest one specific to a circular field in the air gap, the equations of which have been given earlier in the text. But a useful approach is to begin with simple equations and then gradually transform them to a more complex form.

As is known, the system of equations comprising four voltage equations (1.34) and the associated equation of motion (1.35) describes the processes of energy conversion where the field in the air gap is circular in shape. The expression for the electromagnetic torque includes two products of currents.

A circular field in the air gap can be set up by a few windings on the stator and rotor. First consider a machine model having a pair of windings on the stator, which produces a circular field in the air gap and two pairs of windings on the rotor (Fig. 4.1). The voltage equations derived from the equations for an m-n winding machine have the form

$$\begin{vmatrix} u^s_{1\alpha} \\ u^r_{1\alpha} \\ u^r_{2\alpha} \\ u^r_{1\beta} \\ u^r_{2\beta} \\ u^s_{1\beta} \end{vmatrix} = \begin{vmatrix} R^s_{1\alpha} + \frac{d}{dt} L^s_{1\alpha} & \frac{d}{dt} M_{11} & \frac{d}{dt} M_{12} & 0 & 0 & 0 \\ \frac{d}{dt} M_{11} & R^r_{1\alpha} + \frac{d}{dt} L^r_{1\alpha} & \frac{d}{dt} M^r_{12} & L^r_{1\beta}\omega_r & M^r_{12}\omega_r & M_{11}\omega_r \\ \frac{d}{dt} M_{21} & \frac{d}{dt} M^r_{21} & R^r_{2\alpha} + \frac{d}{dt} L^r_{2\alpha} & M^r_{21}\omega_r & L^r_{2\beta}\omega_r & M_{21}\omega_r \\ -M_{11}\omega_r & -L^r_{1\alpha}\omega_r & -M^r_{12}\omega_r & R^r_{1\beta} + \frac{d}{dt} L^r_{1\beta} & \frac{d}{dt} M^r_{12} & \frac{d}{dt} M_{11} \\ -M_{21}\omega_r & -M^r_{21}\omega_r & -L^r_{2\alpha}\omega_r & \frac{d}{dt} M^r_{21} & R^r_{2\beta} + \frac{d}{dt} L^r_{2\beta} & \frac{d}{dt} M_{21} \\ 0 & 0 & 0 & \frac{d}{dt} M_{11} & \frac{d}{dt} M_{12} & R^s_{1\beta} + \frac{d}{dt} L^s_{1\beta} \end{vmatrix} \times \begin{vmatrix} i^s_{1\alpha} \\ i^r_{1\alpha} \\ i^r_{2\alpha} \\ i^r_{1\beta} \\ i^r_{2\beta} \\ i^s_{1\beta} \end{vmatrix} \quad (4.1)$$

The system (4.1) comprises six voltage equations, two for the two stator windings and four for the four rotor windings. The voltage in the rotor is zero if the rotor is of the squirrel-cage type. With the energy pumped to all the three pairs of windings, the circular field

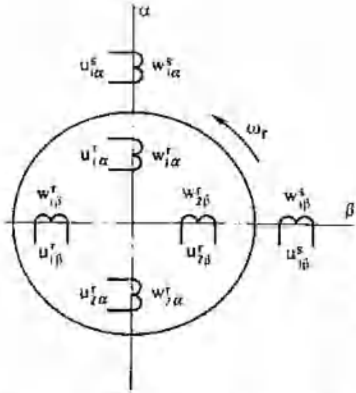


Fig. 4.1. The circular-field machine model with two pairs of phase windings on the rotor

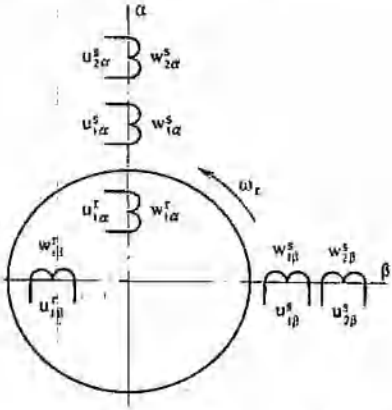


Fig. 4.2. The machine model with two pairs of phase windings on the stator and one pair of phase windings on the rotor

in the air gap being established, the voltages across the stator and rotor windings must produce magnetic fields which are stationary with respect to each other.

The torque equation for this model is

$$M_e = M_{11}\,(i^r_{1\alpha} i^s_{1\beta} - i^s_{1\alpha} i^r_{1\beta}) + M_{12}\,(i^r_{2\alpha} i^s_{1\beta} - i^s_{1\alpha} i^r_{2\beta}) \tag{4.2}$$

Equation (4.2) includes two components

$$M_e = M_{11} + M_{12} \tag{4.2a}$$

The first component is due to the interaction between the currents in the first windings on the stator and rotor, and the second is due to the interaction between the currents in the first stator windings and the second rotor windings.

A circular field in the air gap of a machine having two pairs of windings on the stator and one pair of windings on the rotor (Fig. 4.2) is defined by six voltage equations; the electromagnetic torque produced

here consists of two components

$$M_e = M_{11}(i^r_{1\alpha} i^s_{1\beta} - i^s_{1\alpha} i^r_{1\beta}) + M_{21}(i^r_{1\alpha} i^s_{2\beta} - i^s_{2\alpha} i^r_{1\beta}) \tag{4.3}$$

$$M_e = M_{11} + M_{21} \tag{4.3a}$$

The mathematical model describing the processes of energy conversion in this machine includes four voltage equations for the stator, where the rotational emf is equal to zero, and two voltage equations for the rotor.

For the machine model of Fig. 3.3 having four pairs of windings on the stator and rotor (the field in the air gap being circular) the system of equations consists of eight voltage equations. The equation for the torque includes four components:

$$M_e = M_{11}(i^r_{1\alpha} i^s_{1\beta} - i^s_{1\alpha} i^r_{1\beta}) + M_{22}(i^r_{2\alpha} i^s_{2\beta} - i^s_{2\alpha} i^r_{2\beta})$$
$$+ M_{12}(i^r_{2\alpha} i^s_{1\beta} - i^s_{1\alpha} i^r_{2\beta}) + M_{21}(i^r_{1\alpha} i^s_{2\beta} - i^s_{2\alpha} i^r_{1\beta}) \tag{4.4}$$

$$M_e = M_{11} + M_{22} + M_{12} + M_{21} \tag{4.5}$$

A further increase in the number of windings does not violate the regularities adopted in the formulation of equations. The next step following the analysis dealt with the circular field is the treatment of the processes associated with an elliptic field which consists of a forward and a backward field rotating at the same velocity but in opposite directions and differing in amplitude.

A machine displaying an elliptic field in the air gap can have several windings. The system of equations then comprises a corresponding number of voltage equations, and the torque equation includes a corresponding number of torque components.

The model of a machine with one winding on the stator and one on the rotor, the field being assumed elliptic, is an eight-winding machine model described by Eqs. (3.3). The processes of energy conversion associated with an elliptic field in the gap are given detailed treatment later in the text. Here we note only that the system of equations consists of eight voltage equations, one for each winding, and the torque equation includes four components:

$$M_e = M_{11} - M_{22} + M_{12} - M_{21} \tag{4.6}$$

The signs in Eq. (4.6) depend on the direction of rotation of the harmonics of interest. If in the analysis of a machine with an elliptic field there is a need to consider additional windings, i.e. the eddy current loops in the stator and rotor, the equations become more cumbersome. The number of voltage equations corresponds to the number of windings in the model, and the torque equation contains all the terms for pairwise interaction.

The next group of equations is the system of equations for three fields in the air gap. The mathematical model here involves 12 vol-

tage equations, and the torque equation contains nine components:

$$M_e = M_1 + M_2 + M_3 + M_{12} + M_{13} + M_{23} + M_{21} + M_{31} + M_{32} \tag{4.7}$$

If the field in the air gap has three harmonics and the stator and rotor carry several windings, the number of equations to be formulated grows. In the next steps of the analysis, the equations are set up for four, five, and n harmonics.

Thus the typical equations include the equations for circular and elliptic fields with three, four, and more harmonics. The equations become more complex with an increasing number of windings on the stator and rotor. Note that the solution of the above electromechanical equations for most electric machines requires writing corresponding programs which will enter into the library of standard programs.

4.2. Energy Conversion Involving an Elliptic Field

As mentioned above, the elliptic field consists of a forward and a backward field. An example of the motor exhibiting an elliptic field in the air gap is a single-phase motor with one winding on the stator (Fig. 4.3). The mathematical model of a single-phase motor is the model of Fig. 3.3, having two pairs of windings on the stator and two pairs on the rotor. One pair of windings, $w^s_{1\alpha}$ and $w^s_{1\beta}$, is fed with positive-sequence voltages to produce a forward field and the other pair, $w^s_{2\alpha}$ and $w^s_{2\beta}$, with negative-sequence voltages to set up a backward field. In a single-phase motor the positive- and negative-sequence voltages are the same and the amplitudes of the forward and backward fields are equal to each other.

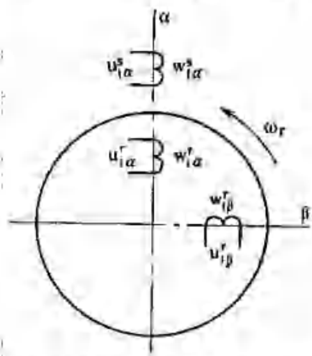


Fig. 4.3. A single-phase motor

An elliptic field is built up in symmetric and asymmetric machines when supplied from asymmetric and symmetric voltage sources respectively.

For an electric machine modeled in terms of the machine of Fig. 3.3 the system of equations (4.8) holds. This set of equations is derivable from the equations for the generalized converter on condi-

tion that the machine under study is of the unsaturated type and the interrelation between the negative- and positive-sequence variables does not exist:

$$
\begin{vmatrix} u^s_{1\alpha} \\ u^s_{2\alpha} \\ 0 \\ 0 \\ 0 \\ 0 \\ u^s_{1\beta} \\ u^s_{2\beta} \end{vmatrix}
=
\left|\begin{array}{c|c|c|c|c|c|c|c}
r^s_{1\alpha}+pL^s_{1\alpha} & 0 & pM_{11\alpha} & 0 & 0 & 0 & 0 & 0 \\
0 & r^s_{2\alpha}+pL^s_{2\alpha} & 0 & pM_{22\alpha} & 0 & 0 & 0 & 0 \\
pM_{11\alpha} & 0 & r^r_{1\alpha}+pL^r_{1\alpha} & 0 & L_{1\beta}\omega_r & 0 & M_{11\beta}\omega_r & 0 \\
0 & pM_{22\alpha} & 0 & r^r_{2\alpha}+pL^r_{2\alpha} & 0 & L_{2\beta}\omega_r & 0 & M_{22\beta}\omega_r \\
-M_{11\alpha}\omega_r & 0 & -L_{1\alpha}\omega_r & 0 & r^r_{1\beta}+pL^r_{1\beta} & 0 & pM_{11\beta} & 0 \\
0 & -M_{22\alpha}\omega_r & 0 & -L_{2\alpha}\omega_r & 0 & r^r_{2\beta}+pL^r_{2\beta} & 0 & pM_{22\beta} \\
0 & 0 & 0 & 0 & pM_{11\alpha} & 0 & r^s_{1\beta}+pL^s_{1\beta} & 0 \\
0 & 0 & 0 & 0 & 0 & pM_{22\alpha} & 0 & r^s_{2\beta}+pL^s_{2\beta}
\end{array}\right|
\times
\begin{vmatrix} i^s_{1\alpha} \\ i^s_{2\alpha} \\ i^r_{1\alpha} \\ i^r_{2\alpha} \\ i^r_{1\beta} \\ i^r_{2\beta} \\ i^s_{1\beta} \\ i^s_{2\beta} \end{vmatrix}
\qquad (4.8)
$$

To establish the equation for the torque, we should consider a model comprising two stators and two rotors coupled together

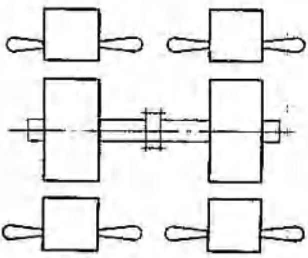


Fig. 4.4. A two-stator two-rotor machine model

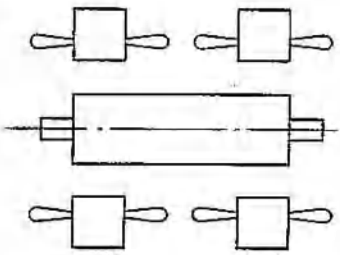


Fig. 4.5. The common-rotor machine model illustrative of the interaction between the positive- and negative-sequence voltages or currents

(Fig. 4.4) and also a model composed of two stators and a common rotor (Fig. 4.5).

As seen from the simplified two-stator two-rotor model, the positive- and negative-sequence stimuli are likely to give rise to two fields in the gap, thereby setting the rotors in motion in opposite directions; the interaction between the positive-sequence currents in the stator and the negative-sequence currents in the rotor is nonexistent.

In this case the torque is

$$M_e = M\left(i^s_{1\beta} i^r_{1\alpha} - i^s_{1\alpha} i^r_{1\beta}\right) - M\left(i^s_{2\beta} i^r_{2\alpha} - i^s_{2\alpha} i^r_{2\beta}\right) \tag{4.9}$$

or

$$M_e = M_{11} - M_{22} \tag{4.10}$$

In the common-rotor model of Fig. 4.5 there exists coupling between the positive phase sequence and the negative phase sequence, and the torque equation thus has the form

$$\begin{aligned} M_e = M\,[&(i^s_{1\beta} i^r_{1\alpha} - i^s_{1\alpha} i^r_{1\beta}) - (i^s_{2\beta} i^r_{2\alpha} - i^s_{2\alpha} i^r_{2\beta}) \\ &+ (i^s_{1\beta} i^r_{2\alpha} - i^s_{1\alpha} i^r_{2\beta}) - (i^s_{2\beta} i^r_{1\alpha} - i^s_{2\alpha} i^r_{1\beta})] \\ = {}& M_{11} - M_{22} + M_{12} - M_{21} \end{aligned} \tag{4.11}$$

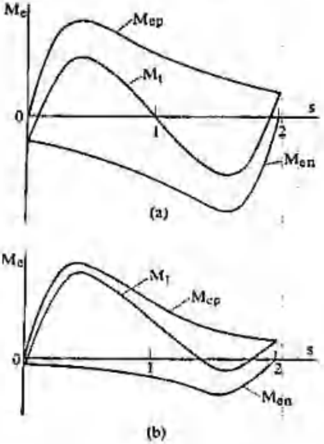
The interaction between the positive- and negative-sequence currents gives rise to pulsating torques which do not provide the mean torque component in the steady-state operation and affect the course of a transient because in the transient region the currents are decaying and the mean value of current over a cycle varies.

When $s = 1$ and the field pulsates, the resultant torque is zero, so there is no starting torque on the motor shaft (Fig. 4.6*a*). In order to start a single-phase motor, it is necessary to set the rotor in motion by an auxiliary motor or reduce the backward field; the motor then acquires a torque which keeps it going in the direction of the applied torque. The most popular method of starting a single-phase motor is to reduce the amplitude of the backward field, i.e. to transform a pulsating field into an elliptic one (Fig. 4.6*b*). Many approaches are applicable for producing an elliptic field in the air gap of a motor supplied from a single-phase circuit. One of the widespread methods utilizes an auxiliary starting winding displaced 90° in space from the main winding, the excitation currents in both windings being brought into the desired time-phase relation (Fig. 4.7).

An elliptic field in a symmetric motor is set up by impressing asymmetric voltages across the windings. With the impressed voltages being at asymmetry, the equations expressed in terms of flux linkages take on the form

$$\begin{aligned} d\Psi^s_{1\alpha}/dt &= u^s_{1\alpha} - R^s i^s_{1\alpha}, & d\Psi^s_{1\beta}/dt &= u^s_{1\beta} - R^s i^s_{1\beta} \\ d\Psi^r_{1\alpha}/dt &= R^r i^r_{1\alpha} - p\omega_r \Psi^r_{1\alpha}, & d\Psi^r_{1\beta}/dt &= R^r i^r_{1\beta} + p\omega_r \Psi^r_{1\beta} \\ d\Psi^s_{2\alpha}/dt &= \pm u^s_{2\alpha} - R^s i^s_{2\alpha}, & d\Psi^s_{2\beta}/dt &= \pm u^s_{2\beta} - R^s i^s_{2\beta} \\ d\Psi^r_{2\alpha}/dt &= -R^r i^r_{2\alpha} + p\omega_r \Psi^r_{2\beta}, & d\Psi^r_{2\beta}/dt &= -R^r i^r_{2\beta} - p\omega_r \Psi^r_{2\alpha} \end{aligned} \tag{4.12}$$

Resolving the flux linkages and currents into symmetric components, we can obtain the α-β equations expressed in terms of flux linkages and currents. It is advisable to represent flux linkages as a



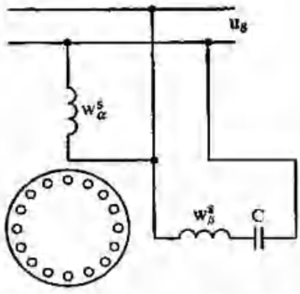


Fig. 4.7. A capacitor-start single-phase induction motor

Fig. 4.6. The mechanical characteristic of a single-phase motor
M_{ep} — torque due to positive-sequence field;
M_{en} — torque due to negative-sequence field;
M_t — total (resultant) torque

sum of the products of currents and inductances, expressed in relative units. The procedure of simulation on an analog computer will then require fewer adders because the flux linkages are the sums of currents with definite coefficients:

$$
\begin{aligned}
&\Psi^s_{1\alpha} = L^s i^s_{1\alpha} + M i^r_{1\alpha}, \ \Psi^s_{1\beta} = L^s i^s_{1\beta} + M i^r_{1\beta}\\
&\Psi^s_{2\alpha} = L^s i^s_{2\alpha} + M i^r_{2\alpha}, \ \Psi^s_{2\beta} = L^s i^s_{2\beta} + M i^r_{2\beta}\\
&\Psi^r_{1\alpha} = L^r i^r_{1\alpha} + M i^s_{1\alpha}, \ \Psi^r_{1\beta} = L^r i^r_{1\beta} + M i^s_{1\beta}\\
&\Psi^r_{2\alpha} = L^r i^r_{2\alpha} + M i^s_{2\alpha}, \ \Psi^r_{2\beta} = L^r i^r_{2\beta} + M i^s_{2\beta}
\end{aligned}
\tag{4.13}
$$

Solving the algebraic equations, which enter into the general system of equations (4.12), for currents $i^r_{1\alpha}$, $i^r_{2\alpha}$, $i^r_{1\beta}$, and $i^r_{2\beta}$ expressed in terms of flux linkages $\Psi^r_{1\alpha}$, $\Psi^r_{1\beta}$, $\Psi^r_{2\alpha}$, $\Psi^r_{2\beta}$ (4.13), we obtain the set of equations

$$
\begin{aligned}
di^r_{1\alpha}/dt &= au^s_{1\alpha} + (b/M)\Psi^r_{1\alpha} - i^r_{1\alpha}(B+C) - (p\omega_r/\sigma L^r)\Psi^r_{1\beta}\\
di^r_{1\beta}/dt &= au^s_{1\beta} + (b/M)\Psi^r_{1\beta} - i^r_{1\beta}(B+C) - (p\omega_r/\sigma L^r)\Psi^r_{1\beta}\\
di^r_{2\alpha}/dt &= au^s_{2\alpha} + (b/M)\Psi^r_{2\alpha} - i^r_{2\alpha}(B+C) + (p\omega_r/\sigma L^r)\Psi^r_{2\beta}\\
di^r_{2\beta}/dt &= \pm au^s_{2\beta} + (b/M)\Psi^r_{2\beta} - i^r_{2\beta}(B+C) - (p\omega_r/\sigma L^r)\Psi^r_{2\alpha}
\end{aligned}
\tag{4.14}
$$

$$
\begin{aligned}
d\Psi^r_{1\alpha}/dt &= -R^r i^r_{1\alpha} - p\omega_r \Psi^r_{1\beta} \\
d\Psi^r_{2\alpha}/dt &= -R^r i^r_{2\alpha} + p\omega_r \Psi^r_{2\beta} \\
d\Psi^r_{1\beta}/dt &= -R^r i^r_{1\beta} + p\omega_r \Psi^r_{1\alpha} \qquad (4.14\ cont.) \\
d\Psi^r_{2\beta}/dt &= -R^r i^r_{2\beta} - p\omega_r \Psi^r_{2\alpha}
\end{aligned}
$$

$$
\begin{aligned}
M_e = \Psi^r_{1\beta} i^r_{1\alpha} - \Psi^r_{1\alpha} i^r_{1\beta} + \Psi^r_{2\beta} i^r_{2\alpha} - \Psi^r_{2\alpha} i^r_{2\beta} + \Psi^r_{1\beta} i^r_{2\alpha} \\
- \Psi^r_{1\alpha} i^r_{2\beta} + \Psi^r_{2\beta} i^r_{1\alpha} - \Psi^r_{2\alpha} i^r_{1\beta} \qquad (4.15)
\end{aligned}
$$

where $a = M/(\sigma L^s L^r)$; $b = R^r M/(\sigma L^s L^r)$; $B = R^s/(\sigma L^s)$; $C = MR^r/(\sigma L^s L^r)$.

Substituting the expressions for currents into the flux linkage equations, we can establish the system of differential equations expressed in terms of flux linkages.

For Eqs. (4.12) through (4.15), a simulation model is set up to solve the equations on an analog computer, or a program is written to be run on a digital computer. As in the analysis involving a circular field, the objectives here are to investigate the transients, the effect of parameters, and seek the ways for optimization.

The adjustment of a simulation model is done as follows. The steps are first taken to adjust one part of the model for the forward field equations and then the other part for the backward field equations to enable the entire analog to be set up on a computer, the electromagnetic torque and angular velocity being assumed equal to zero. The computation procedure then follows to solve the behavior of an induction machine in various modes of operation.

Since the equations for an elliptic field are more difficult to solve than those for a circular field, the use of computers to deal with the former equations is a good approach to obtaining most appropriate results.

4.3. Elliptic-Field Steady-State Conditions

The steady-state equations for a symmetric machine exhibiting both a forward and a backward field in the air gap can be derived from (4.8) and (4.11) by substituting $j\omega$ for p.

Applying (1.34) for the steady state yields the set of equations:

$$
\begin{vmatrix} \dot{U}^s_\alpha \\ \dot{U}^s_\beta \\ -\dot{U}^r_\alpha \\ -\dot{U}^r_\beta \end{vmatrix} = \begin{vmatrix} r^s + jx^s & 0 & jx_m & 0 \\ 0 & r^s + jx^s & 0 & jx_m \\ jx_m & \nu x_m & r^r + jx^r & \nu x^r \\ -\nu x_m & jx_m & +\nu x^r & r^r + jx^r \end{vmatrix} \times \begin{vmatrix} \dot{I}^s_\alpha \\ \dot{I}^s_\beta \\ \dot{I}^r_\alpha \\ \dot{I}^r_\beta \end{vmatrix} \qquad (4.16)
$$

Here $\nu = \pi pn/30\omega_s$ and the voltages on the stator or rotor phase windings differ in magnitude.

Using the method of symmetric components gives

$$\dot{U}^s_\alpha = \dot{U}^s_p + \dot{U}^s_n, \quad \dot{U}^s_\beta = -j\dot{U}^s_p + j\dot{U}^s_n \tag{4.17}$$

where $\dot{U}^s_p$ and $\dot{U}^s_n$ are the positive- and negative-sequence (forward and backward) stator voltages respectively. The voltages $\dot{U}^s_p$ and $-j\dot{U}^s_p$ produce a circular forward field, while $\dot{U}^s_n$ and $j\dot{U}^s_n$ set up a backward field. For the case under study,

$$\dot{I}^s_\alpha = \dot{I}^s_p + \dot{I}^s_n, \quad \dot{I}^s_\beta = -j\dot{I}^s_p + j\dot{I}^s_n \tag{4.18}$$

where $\dot{I}^s_p$, $-j\dot{I}^s_p$ and $\dot{I}^s_n$, $j\dot{I}^s_n$ are the positive- and negative-sequence stator currents respectively.

For the rotor the following equations hold:

$$\begin{aligned} \dot{U}^r_\alpha &= \dot{U}^r_p + \dot{U}^r_n, \quad \dot{U}^r_\beta = -j\dot{U}^r_p + j\dot{U}^r_n \\ \dot{I}^r_\alpha &= \dot{I}^r_p + \dot{I}^r_n, \quad \dot{I}^r_\beta = -j\dot{I}^r_p + j\dot{I}^r_n \end{aligned} \tag{4.19}$$

where $\dot{U}^r_p$, $\dot{U}^r_n$, $\dot{I}^r_p$, $\dot{I}^r_n$ are the rotor voltages and currents of the positive and the negative sequence, respectively.

Equations (4.17) through (4.19) follow from the model of Fig. 3.3 showing four pairs of windings on the stator and rotor. Since the model represents an unsaturated machine, coupling between the negative-sequence windings does not exist. The torque can be defined by use of the common-rotor model of Fig. 4.4 or the two-stator two-rotor model of Fig. 4.5.

Considering the processes in a machine along the α axis for the positive-sequence variables, from (4.17) through (4.19) we get

$$\dot{U}^s_p = (r^s + jx^s)\dot{I}^s_p + jx_m\dot{I}^r_p \tag{4.20}$$

$$-\dot{U}^r_p = jx_m\dot{I}^s_p + \nu x_m(-j\dot{I}^s_p) + (r^r + jx^r)\dot{I}^r_p + \nu x^r(-j\dot{I}^r_p) \tag{4.21}$$

In a similar manner, for the negative-sequence variables we have

$$\dot{U}^s_n = (r^s + jx^s)\dot{I}^s_n + jx_m\dot{I}^r_n \tag{4.22}$$

$$-\dot{U}^r_n = jx_m\dot{I}^s_n + \nu x_m(j\dot{I}^s_n) + (r^r + jx^r)\dot{I}^r_n + \nu x^r(j\dot{I}^r_n) \tag{4.23}$$

As we did in deriving the equations for a circular field, here we replace the total inductive reactances x^s and x^r by the reactance of mutual induction, x_m, and the stator and rotor leakage reactances x_{σ_1} and x_{σ_2}, respectively. After transformations the equations be-

como

$$\dot{U}_p^s = (r^s + jx_{\sigma_1})\dot{I}_p^s + jx_m(\dot{I}_p^s + \dot{I}_p^r) \tag{4.24}$$

$$-\dot{U}_p^r = jx_m\dot{I}_p - j\nu x_m\dot{I}_p^s + (r^r + jx_{\sigma_2})\dot{I}_p^r$$
$$+ jx_m\dot{I}_p^r - j\nu x_m'\dot{I}_p^r - j\nu x_{\sigma_2}\dot{I}_p^r \tag{4.25}$$

Transforming Eq. (4.25) for the rotor gives us the equation of the form

$$-\dot{U}_p^r = jx_m(1-\nu)\dot{I}_p^s + jx_m(1-\nu)\dot{I}_p^r + jx_{\sigma_2}(1-\nu)I_p^r + r^r\dot{I}_p^r$$

Since

$$1 - \nu = 1 - \omega_r/\omega_s = s$$

$$-\dot{U}_p^r = jx_m s(\dot{I}_p^s + \dot{I}_p^r) + jx_{\sigma_2}s\dot{I}_p^r + r^r\dot{I}_p^r \tag{4.26}$$

Dividing both sides of (4.26) by s we get

$$-\dot{U}_p^r/s = jx_m(\dot{I}_p^s + \dot{I}_p^r) + \dot{I}_p^r(r^r/s + jx_{\sigma_2}) \tag{4.27}$$

Equations (4.24) and (4.27) for the positive-sequence voltages in the stator and rotor, respectively, describe the processes in an induction machine for the positive phase sequence. These equations follow from the equivalent circuit of Fig. 2.3. Here

$$r^r/s = r^r + r^r(1-s)/s \tag{4.28}$$

The loss in $r^r(1-s)/s$ is proportional to the useful power at the machine shaft.

A similar approach is applicable to deriving the negative-sequence equations

$$\dot{U}_n^s = (r^s + jx_{\sigma_1})\dot{I}_n^s + jx_m(\dot{I}_n^s + \dot{I}_n^r)$$

$$-\dot{U}_n^r = jx_m(1+\nu)(\dot{I}_n^s + \dot{I}_n^r) + jx_{\sigma_2}(1+\nu)\dot{I}_n^r + r^r\dot{I}_n^r \tag{4.29}$$

Since

$$1 + \nu = 1 + \omega_r/\omega_s = 2 - s$$

$$-\dot{U}_n^r = jx_m(2-s)(\dot{I}_n^s + \dot{I}_n^r) + jx_{\sigma_2}(2-s)\dot{I}_n^r + r^r\dot{I}_n^r \tag{4.30}$$

Equations (4.29) and (4.30) follow from the equivalent circuit for the negative phase sequence.

For complex equations and per-phase equivalent circuits of the induction motor, the phasor and circle diagrams may be found applicable. However, the analysis of the working processes in motors by the phasor diagrams presents difficulties because the current distribution among phases with load variation depends on the angular velocity and torque exerted on the shaft. Where the analysis involves an elliptic field, a more preferable approach is to solve Eqs. (4.8) and (4.11) rather than to proceed from the simplified diagrams. The

thing is that phasor and circle diagrams which hold and work well for a circular field are impracticable for a more general case, i.e. an elliptic field.

Note that a two-phase motor with its stator windings supplied from sources differing in frequency may have one of its phases connected to a supply line and the other to a thyristor converter or both connected to individual converters. Under these conditions, consideration should be given for pulsating fields in each winding, and the analysis of energy conversion processes should be done for at least four fields in the gap. Here the model for the two-phase motor is built up of four pairs of windings on the stator and four pairs of windings on the rotor. The system of equations comprises 16 voltage equations and the associated torque equation includes 16 torque components. If the conditions deviate from those favorable for setting up a circular field, the problem becomes more complex.

The standard programs of typical energy conversion equations are a useful tool for studies of the transient and steady-state performance of electric machines having circular and elliptic fields, fields with several harmonics, and carrying two or more windings on the stator and rotor. These programs form the basis for automatic systems of designing electric machines.

Chapter 5

Energy Conversion Involving Nonsinusoidal and Asymmetric Supply Voltages

5.1. The Equations of Electric Machines

Consider m fields in the air gap (i.e. a field containing m harmonics). A machine with a nonsinusoidal voltage at its terminals may serve as a classical example for the case under study. A general example of the nonsinusoidal voltage source for an induction motor is a semiconductor frequency converter.

Assuming the machine to be ideal (so additional harmonics in the air gap are nonexistent), it is safe to picture the field as the one faithfully reproducing the harmonic spectrum of the applied voltage. An ideal machine does not generate noise, and so for the harmonic spectrum of the magnetic B field to be defined, it is enough to expand the phase voltages into a harmonic series. The directions

of rotation and amplitudes of time harmonics depend on the number of phases in the machine and the ordinal number of each harmonic, as is obvious from the set of equations (3.1).

Assume also that each of the m harmonics making up the field in the air gap is set up by two pairs of windings arranged on the stator or rotor along the α and β axes. The model of such a machine (Fig. 5.1) has two sets of m windings on the stator and rotor along

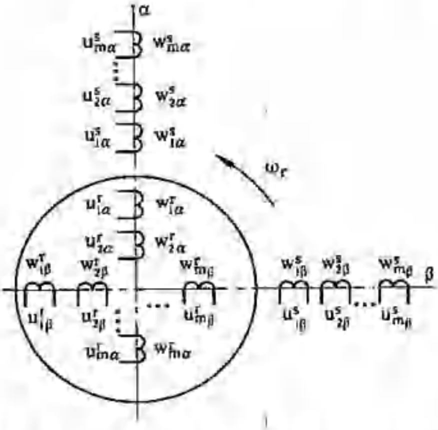


Fig. 5.1. The model of a machine for the analysis of energy conversion processes at nonsinusoidal and asymmetric supply voltages

the α and β axes rather than m and n windings on the stator and rotor, respectively. This model is analogous to the model of the generalized converter.

For the machine with m windings both on the stator and on the rotor, we derive the system of equations from (3.3). In these equations, $u^s_{1\alpha}$, $u^s_{1\beta}$ are the stator voltages at the fundamental frequency, and $u^s_{2\alpha}$, $u^s_{2\beta}$ are the stator voltages producing the field of a third harmonic. The latter voltages have a frequency $f_3 = 3f_1$ and an amplitude that corresponds to the amplitude of the field of the third harmonic. Voltages $u^s_{3\alpha}$, $u^s_{3\beta}$ correspond to the fifth harmonic of frequency $f_5 = 5f_1$, and voltages $u^s_{m\alpha}$, $u^s_{m\beta}$ are the voltages of the mth harmonic, $f_m = mf_1$. The phases and the directions of rotation of the field harmonics result from the corresponding voltages across the stator windings. If the air gap exhibits subharmonics, i.e. the fields whose frequencies are below the fundamental frequency, a part of the windings are fed with voltages at frequencies lower than

the fundamental. If the voltage contains even harmonics, a part of the windings are supplied with voltages displaying even harmonics.

For each pair of windings on the stator there is a corresponding pair of windings on the rotor. As regards an induction machine, the voltages across the rotor windings are equal to zero for a short-circuited rotor, and power is fed through the stator; $i^s_{1\alpha}$, $i^s_{1\beta}$, $i^r_{1\alpha}$, $i^r_{1\beta}$ are the stator and rotor currents of the first harmonic; $i^s_{2\alpha}$, $i^s_{2\beta}$, $i^r_{2\alpha}$, $i^r_{2\beta}$ are the stator and rotor currents of the second harmonic; and $i^s_{m\alpha}$, $i^s_{m\beta}$, $i^r_{m\alpha}$, $i^r_{m\beta}$ are the stator and rotor currents of the mth harmonic.

The equations for nonsinusoidal asymmetric supply voltages can be derived from Eqs. (3.3) of the generalized converter. The voltage and current matrices for the case at hand have the same form as in (3.4) and (3.5), but the impedance matrix is different.

The four impedance matrices arranged along the diagonal of the Z matrix have the form

$$A^s_\alpha = \begin{vmatrix} r^s_{1\alpha} + (d/dt)\,L^s_{1\alpha} & 0 & \dots & 0 \\ 0 & r^s_{2\alpha} + (d/dt)\,L^s_{2\alpha} & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & r^s_{m\alpha} + (d/dt)\,L^s_{m\alpha} \end{vmatrix} \qquad (5.1)$$

where $r^s_{1\alpha}$, $r^s_{2\alpha}$, ..., $r^s_{m\alpha}$ are the resistances the winding offers to the currents of the 1st, 2nd and mth harmonics. Disregarding current displacement, these resistances may be taken identical to a sufficient degree of accuracy. In (5.1), $L^s_{1\alpha}$, $L^s_{2\alpha}$, and $L^s_{m\alpha}$ are the total inductances of the loops carrying currents of the 1st, 2nd, and mth harmonics.

The matrix A^r_α comes from A^s_α after the replacement of the superscript s by r, and A^r_β from A^r_α with α replaced by β. The impedance matrix A^s_β comes out after substitution of β for α in A^s_α.

The submatrix A^{sr}_α has the form

$$A^{sr}_\alpha = \begin{vmatrix} (d/dt)\,M^{sr}_{11\alpha} & 0 & \dots & 0 \\ 0 & (d/dt)\,M^{sr}_{22\alpha} & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & (d/dt)\,M^{sr}_{mm\alpha} \end{vmatrix} \qquad (5.2)$$

where $M^{sr}_{11\alpha}$, $M^{sr}_{22\alpha}$, ..., $M^{sr}_{mm\alpha}$ are the mutual inductances between the stator and rotor loops, respectively, over which the harmonics of the same order complete their paths.

The matrices A_α^{rs}, A_β^{sr}, and A_β^{rs} all derive from A_α^{sr} on replacing α by β and interchanging s and r. In machines subject to saturation and other nonlinearities, the mutual inductances between the stator and rotor loops, on the one hand, and those between the rotor and stator loops, on the other, may differ in sign

$$D_\alpha = \begin{vmatrix} -L_{1\alpha}^r\omega_r & 0 & \dots & 0 \\ 0 & -L_{2\alpha}^r\omega_r & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & -L_{m\alpha}^r\omega_r \end{vmatrix} \tag{5.3}$$

$$D_\beta = \begin{vmatrix} L_{1\beta}^r\omega_r & 0 & \dots & 0 \\ 0 & L_{2\beta}^r\omega_r & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & L_{m\beta}^r\omega_r \end{vmatrix} \tag{5.4}$$

The submatrices $B_{\alpha\beta}$ and $B_{\beta\alpha}$ differ in sign and one follows from the other by interchanging the positions of α and β and of r and s.

$$B_{\alpha\beta} = \begin{vmatrix} -M_{11\alpha}^{rs}\omega_r & 0 & \dots & 0 \\ 0 & -M_{22\alpha}^{rs}\omega_r & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & -M_{mm\alpha}^{rs}\omega_r \end{vmatrix} \tag{5.5}$$

where $M_{11\alpha}^{rs}$, $M_{22\alpha}^{rs}$, ..., $M_{mm\alpha}^{rs}$ are the mutual inductances between the rotor and stator loops carrying currents of respective harmonics.

Combining the submatrices of voltages, currents, and impedances gives the equations of an electric machine supplied from a nonsinusoidal voltage source, each set of equations being written for a definite harmonic.

For a real machine having one pair of windings on the stator and one pair on the rotor, the resistance in the mathematical model may be considered equal to each other. The total inductances of the stator and rotor windings along the α and β axes may differ from each other because of the difference between the leakage fluxes (the designations of these inductances are the same as for resistances). The mutual inductances between the stator and rotor windings may be taken equal to each other.

Since the machine model is unsaturable, there is no coupling between the stator windings along the same axis. Hypothetical windings producing the field in the air gap have no coupling between each other: in the machine under study the nonsinusoidal currents flow in one winding.

As in the case of an elliptic field, here we can consider two models to derive the equation for the electromagnetic torque. The first model

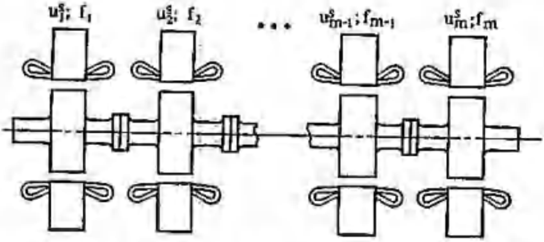


Fig. 5.2. The m-stater m-rotor machine model for determining M_e

(Fig. 5.2) has m stators and m rotors, the latter being rigidly coupled. Each stator is fed with voltages of the first and upper harmonics. For the model representing the processes of energy conversion in

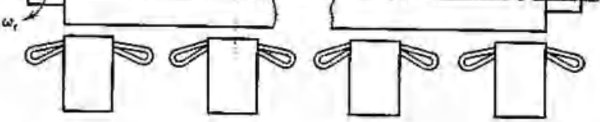


Fig. 5.3. The m-stator common-rotor machine model for determining M_e

this fashion, the torque equation contains only the products of quantities with the same indexes. The pulsating components are absent.

According to the second model (Fig. 5.3), built up of m stators and a common rotor, all harmonics interact and the torque equation includes the products of all stator currents and all rotor currents of

upper harmonics. These products contain the terms with different indexes, which produce pulsating torques.

After transformations, the mutual inductances between the stator and rotor windings being taken equal to each other, we find that

$$M_e = M\,[(i^s_{1\alpha} + i^s_{2\alpha} + \ldots + i^s_{m\alpha})(i^r_{1\beta} + i^r_{2\beta} + \ldots + i^r_{m\beta}) - (i^s_{1\beta} + i^s_{2\beta} + \ldots + i^s_{m\beta})(i^r_{1\alpha} + i^r_{2\alpha} + \ldots + i^r_{m\alpha})] \tag{5.6}$$

The electromagnetic torque defined by Eq. (5.6) is the product of nonsinusoidal currents in the stator and rotor.

Modeling permits analyzing an actual energy converter by use of four equations similar to (4.1) and a nonsinusoidal voltage source or m equations and a sinusoidal voltage source. The m-winding model is more complex, but it offers large possibilities for simulating the behaviour of the physical system on a digital computer. An analog computer is more preferable for the realization of the model for four equations in conjunction with a nonsinusoidal voltage generator as power supply.

With the nonsinusoidal asymmetric supply voltage used, both the first and upper harmonics exhibit the forward and backward fields. Therefore, the mathematical model must include the pairs of windings on the stator and rotor to represent the forward field and one more pair to represent the backward field for each harmonic. The number of fields in the gap then grows and the number of windings in the model grows accordingly; the initial equations here hold, and it remains to choose the desired harmonics and solve approximately the stated problems because computers cannot naturally take into account the interaction of all harmonics.

5.2. The Solution of Equations Involving Asymmetric Supply Voltages

Equations (5.1) through (5.6) show a more general character than the equations for a circular field, therefore the former equations require the use of analog and digital computers for their solution. In the simulation of processes on an analog computer, special feed circuits are set up, which are the analyzer networks intended for the solution of energy converter equations. Most popular are the networks designed to reproduce the modes of operation of a thyristor converter; they combine the principles of mathematical and physical modeling. The gate circuit is built up as a physical model, and the analyzer's electric equipment connected to the gate circuit represents the mathematical model of an electric machine.

Assuming that the motor under analysis is fed from a nonsinusoidal voltage supply line of infinite power (the effect of the load on the

voltage waveform being disregarded), it is possible to realize nonsinusoidal voltage generators on an analog computer and use them instead of sinusoidal voltage generators in the computer model for the solution to the equations of an energy converter (see Appendix III).

An analog computer can simulate power supply for an electric machine from a magnetic amplifier after implementing the following relations

$$u_1(t) = |U_m \sin \omega t|\, U_m \sin \omega t, \quad u_2(t) = |U_m \cos \omega t|\, U_m \cos \omega t$$

The block diagram of the model for a nonsinusoidal voltage generator is shown in Appendix III. The harmonic spectrum varies on applying to power units PU_1 and PU_2 the voltages $u_1(t) = |U_m \sin \omega t|\, U_m \sin(\omega t + \varphi)$ and $u_2(t) = |U_m \cos \omega t| \cos(\omega t + \varphi)$ as a result of changes in the phase angle φ (see Fig. A7).

A nonsinusoidal voltage generator illustrated in Fig. A8 (Appendix III) gives a step waveform of the voltage specific to that of pulses drawn from semiconductor converters. Fig. A9 shows the block diagram of an autonomous inverter with pulse-duration modulation (PDM) and pulse waveforms at the generator output.

As seen from Fig. A9, the amplifier *1* gives different-polarity sawtooth pulses u_2 which go to the input of the adder *2* to be compared with the sinusoidal voltage u_3. The control element here is a polarized relay PR_2 whose contacts shape different-polarity rectangular pulses u_5 modulated in width (duration). Applying a time-varying pulse voltage $u_p = \pm U_m \sin \omega t$ to the stationary contacts of PR_2 enables the circuit to carry out the amplitude modulation too. The pulse waveform U_p set up by the relay $1PR_2$ then corresponds to the waveform of the pulse drawn from the single-phase PDM inverter.

In Fig. 5.4 is shown the relay-diode circuit diagram with two polarized relays PR_1 and PR_2 and two diodes D_1 and D_2, which can simulate the operating modes of an amplitude-controlled thyristor drive.

The block diagrams of the models for converters, which are built up from standard computing blocks of a differential analyzer enable a rather faithful reproduction of the typical pulse waveforms at the output of a magnetic amplifier, bridge inverter, autonomous PDM inverter, and thyristor converter. These models are rather easily set up on an analog computer and are reliable over the frequency range between 0.1 and 10 Hz. The models allow for a variation of the frequency, amplitude, and harmonic spectrum of the output pulses. They enable a fairly accurate simulation of the steady-state and dynamic performance of an electromechanical EC.

Digital computers offer much greater possibilities of solving the equations associated with nonsinusoidal asymmetric voltage supply.

For the solution of Eqs. (5.1) through (5.6), the numerical Runge-Kutta technique is most popular. In handling the equations, it is of importance to make the right choice of the step of operations, for it determines the time and accuracy of computation.

The programs for the calculation of both static and dynamic characteristics of an energy converter supplied from a nonsinusoidal voltage source are written by experienced programmers. In wide use

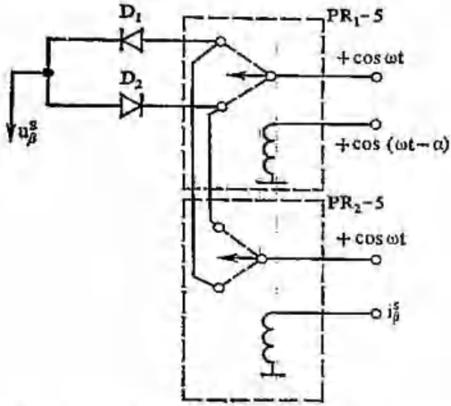


Fig. 5.4. Relay-diode circuit diagram of a thyristor cell

are standard programs for expanding the curves into a harmonic series, determining the effective values of harmonics and their initial phase. The equations are solved for both two-phase and three-phase machines with and without regard to the multiports connected to the stator and rotor.

Of great significance is the estimation of static characteristics for machines with nonsinusoidal voltage supply. By use of the voltage equations expressed, for example, in α-β coordinates, the calculation of currents is done for various angular velocities of the rotor.

For the steady-state operation, the mean value of torque over a period is

$$M_e = (1/T) \int_0^T M_e(t)\, dt \tag{5.7}$$

where T is the period of voltage of the fundamental harmonic.

If the harmonic components of the stator and rotor currents

$$i(t) = I_0 + \sum_{\mu} [I_{\mu m} \sin(\mu\omega t + \varphi_\mu)] \tag{5.8}$$

are known (where μ denotes integers), then, after substitution of M_e into Eq. (5.6) the torque due to each harmonic is found. The torque is then defined as

$$M_e = M_{e0} + \sum_{\mu} M_{e\mu} \tag{5.9}$$

where the torque due to the constant component is

$$M_{e0} = (mp/2)\, M\, (I^r_{\alpha 0} I^s_{\beta 0} - I^r_{\beta 0} I^s_{\alpha 0}) \tag{5.10}$$

The torque due to the μth harmonic in terms of the active and reactive components is given by

$$M_{e\mu} = (mp/2)\, M\, (I^r_{\alpha\mu a} I^s_{\beta\mu a} + I^r_{\alpha\mu r} I^s_{\beta\mu r} - I^r_{\beta\mu a} I^s_{\alpha\mu a} - I^r_{\beta\mu r} I^s_{\alpha\mu r}) \tag{5.11}$$

Here the subscripts a and r respectively identify the active and reactive current components for the μth harmonic along the α and β axes for the stator and rotor.

If the positive- and negative-sequence stator and rotor current components along the α and β axes are present, then, omitting the subscript μ, we get

$$\begin{aligned} \dot{I}_\alpha &= \dot{I}_p + \dot{I}_n = (\dot{I}_{p,a} + \dot{I}_{n,a}) + j\,(\dot{I}_{p,r} + \dot{I}_{n,r}) \\ &= \dot{I}_{\alpha a} + j\dot{I}_{\alpha r} \\ \dot{I}_\beta &= -j\dot{I}_p + j\dot{I}_n = (\dot{I}_{p,r} - \dot{I}_{n,r}) + j\,(\dot{I}_{n,a} - \dot{I}_{p,a}) \\ &= \dot{I}_{\beta a} + j\dot{I}_{\beta r} \end{aligned} \tag{5.12}$$

where the subscripts p and n denote the positive- and negative-sequence current components.

Substituting (5.12) in (5.11) and performing transformations, we have

$$M_{e\mu} = M_{e\mu p} - M_{e\mu n} \tag{5.13}$$

For each harmonic, the expression for torque includes the symmetric positive- and negative-sequence components.

The power at the shaft is

$$P_2 = P_{20} + \sum_{\mu} P_{2\mu} \tag{5.14}$$

where the power of the constant component, disregarding additional losses, is

$$P_{20} = (n/9.55)\, M_{e0} \tag{5.15}$$

and that for the μth component is

$$P_{2\mu} = (n/9.55)\, M_{e\mu} \tag{5.16}$$

The active power absorbed in the machine is given by

$$P_1 = P_{10} + \sum_{\mu} \operatorname{Re} S_{\mu} \tag{5.17}$$

where P_{10} is the dc power received from the bus; the addend is the ac active power; and S_{μ} is the total power of the μth harmonic, the expression for which takes the form

$$S_{\mu} = \sum_{m^s + m^r} (\dot{U}_{ph} \overset{*}{I}_{ph}) \tag{5.18}$$

which is the sum of complex powers of the stator and rotor phases.

For symmetric machines the expression for S_{μ} in terms of symmetric current and voltage components assumes the form

$$S_{\mu} = m\,(\dot{U}_p^s \overset{*}{I}{}_p^s + \dot{U}_n^s \overset{*}{I}{}_n^s + \dot{U}_p^r \overset{*}{I}{}_p^r + \dot{U}_n^r \overset{*}{I}{}_n^r) \tag{5.19}$$

The power factor is

$$\cos \varphi = P_1 / S_1 \tag{5.20}$$

In dealing with nonsinusoidal voltages, the power resulting from distortion makes the evaluation of S_1 impossible. Account then should be taken of $\cos \varphi$ for each harmonic:

$$\cos \varphi_{\mu} = P_{\mu} / S_{\mu} \tag{5.21}$$

The notion of $\cos \varphi$, introduced for the steady-state action at a sinusoidal voltage, relates the active power to the reactive power. The angle φ defines the phase shift between the voltage and current on the phasor diagram of Fig. 2.4. The concept of reactive power that holds for the steady-state conditions does not work when considering the transients at nonsinusoidal voltages.

From the dynamics viewpoint, the reactive power is definable as the power spent on producing magnetic fields. As regards transients, the active power taken from the bus transforms into the reactive power which, in turn, converts into the active power generally given off as heat in the machine and its loops where currents circulate. The description of these complex energy conversion processes by electromechanical equations presents difficulties since a few rather complicated and approximate mathematical models must be set up in handling each problem of interest. It is therefore impossible to define and measure accurately the reactive power, so there is more sense to deal with the instantaneous values of voltages and currents and their products, i.e. instantaneous total powers.

The efficiency in the motoring mode is

$$\eta = P_2/P_1 \tag{5.22}$$

and that in the generating mode is

$$\eta = P_1/P_2 \tag{5.23}$$

In the simulation studies of dynamic characteristics, a digital computer solves Eqs. (5.1) through (5.6), commonly for three to five harmonics, i.e. equations of the 13th to 21st order. The solution of equations on modern digital computers takes only a few minutes. The programs designed for the purpose and run on a computer enable the investigator to analyze the effect of the amplitudes and phases of harmonics, determine the loss due to each harmonic, and study the effect of parameters on the dynamic behavior at nonsinusoidal voltages on the terminals.

Comparing the starting curves for a motor run from sinusoidal and nonsinusoidal voltage supplies, we should point to an increased fluctuation of currents, torques, and speeds in its operation from the nonsinusoidal voltage circuit and also to the dependence of the course of a transient on the starting torque.

The studies of transients on digital computers reveal that induction motors fed from a rectangular-pulse voltage source show almost a twofold increase in the no-load current as against that observed in operation on the sinusoidal voltage. The efficiency of control motors may drop by 20 to 30 percent. The energy characteristics of general-purpose motors deviate from the nominal by 10 to 15 percent.

Consider in brief the operation of an energy converter whose windings are fed with sinusoidal voltages at different frequencies. Impress the voltage at frequency f_1 on the α phase and that at f_2 on the β phase. The air gap will exhibit four fields: the forward and the reverse field produced by the α phase and the same set of fields produced by the β phase. For the analysis of energy conversion processes, we can use the model of the four pairs of windings distributed on the stator and rotor and the system of equations of the 17th order similar to (5.1) through (5.6). Note that the energy characteristics here are much poorer than they are in the case of a circular field, but in other respects the energy conversion processes are similar to those involved in nonsinusoidal supply.

It is sometimes useful to employ the above-mentioned supply circuits for decreasing the number of static frequency converters; a machine operates with its one phase connected to the supply line and the other to a frequency converter. Electric drives are often made to operate in the condition of dynamic braking, in which case one of the motor windings receives its excitation from the dc source during the transient process.

An energy converter can be built so that its field angular velocity would vary with the amplitudes and phases of field harmonics and the rotor would pass from one synchronous speed to another with changes in the harmonic spectrum. This would enable efficient angular velocity control. But designing of such a machine involves considerable difficulties and necessitates controllable frequency converters. It is theoretically possible to vary the amplitudes of harmonics in a regular fashion to enable a linear variation of the field velocity and the desired control of EC parameters.

5.3. The Thyristor Voltage Regulator-Induction Motor System

At present there are essentially two approaches to solving the problem involved in the development of a controllable induction motor drive. The first totally relies on the use of thyristor and transistor frequency converters and the second aims at perfecting the technique of induction motor speed control by regulating the voltage with the aid of thyristors (silicon controlled rectifiers, SCR). In controlling the angular speed through changes in the slip, an electric drive shows poorer energy characteristics because the slip energy is lost as heat in the motor. But since the circuit versions here are simple in design, reliable, and feature good controllable properties, they find use in motors driving fan-type load torques, such as drives for compressors, fans, and blowers, and also in drives subject to control over a narrow speed range.

The circuit diagram under discussion is similar to that for an induction motor drive complete with saturable reactors since the adjustment of the thyristor conduction angle leads to a change and additional shift in the first harmonic of the motor current with respect to the line voltage. In other words, each pair of thyristors connected in parallel opposition (Fig. 5.5) can be regarded as a fictitious nonlinear reactance that is a function of the thyristor conduction angle and the motor parameters and slip. Thyristors designed for stator voltage regulation offer a number of advantages over saturable reactors: thyristor voltage regulators (TVRs) are practically inertialess, have a larger power gain, higher efficiency, and are smaller in size and mass. To make the use of TVRs more effective requires extensive studies into the transients, the effect of upper harmonics on losses, overvoltages, etc.

The theoretical analysis of the TVR-induction motor systems presents certain difficulties due to a number of factors. Among these we should mention the nonlinearity of current-voltage characteristics of semiconductor rectifiers in the dynamic and quasi-steady states, at which an electric drive is found to be in sequentially changing transient conditions resulting from the continuously varying switching

of the motor supply circuit. The systems of nonlinear nonhomogeneous differential equations describing an induction machine in the conditions of symmetric and asymmetric voltage supply to stator phases vary in form.

Because of the uncontrollable behavior of diodes and incompletely controllable operation of thyristors, the output characteristics of a TVR (the conduction angle, voltage waveform) prove heavily dependent on the electromagnetic transients in motor. In other words,

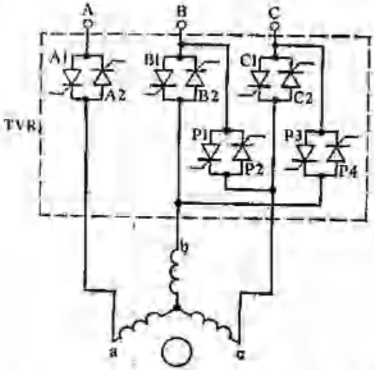


Fig. 5.5. The reversible thyristor supply circuit for an induction machine

the voltages supplied to the motor are dependent on the thyristor conduction angle, parameters and angular velocity of the motor.

A substantial deviation of the waveform of voltage at the motor terminals from the sinusoid causes high-frequency fluctuations of the electromagnetic torque and nonuniform running of the motor in the steady-state operation.

The equations of the mathematical model for the simplest versions of the system under study are essentially solved on analog computers. In solving the differential equations for an induction motor on an analog computer, special requirements are placed on the notation of equations and the choice of variables. This choice should be undertaken with due regard for both the process subject to the analysis and the factors determining the accuracy, capacity, and reliability of the model being set up. The simulation of the TVR-induction motor system on an analog computer requires special units to be built into the computer to simulate the discrete character of TVR operation. The simulation of induction motor performance with consideration for changes in the parameters due to saturation

and current displacement necessitates additional units to perform multiplication and to account for nonlinearity. The instability of analog-computer solutions due to the limited accuracy of computing amplifiers and the extreme difficulty of setting up the universal models have stimulated the evolution of techniques for the analysis of thyristor drive characteristics on digital computers.

Consider the solution to this problem on a digital computer in more detail choosing for the purpose the reversible thyristor-type induction motor drive circuit diagram of Fig. 5.5. Specifying various moments at which the trigger pulses of definite width are to be sent to the gates of thyristors, we can evolve various control circuits. Thus to obtain a nonreversible circuit diagram with two thyristor cells, the moments at which the pulses go to thyristors *C1* and *C2* should be equal to zero ($t_{C1} = t_{C2} = 0$) and the width of these pulses should be infinitely large, $\Delta_{C1} = \Delta_{C2} = \infty$. To remove reversible thyristor cells from the circuit, it is enough to keep them in the off-state since a thyristor driven into the nonconducting condition in effect breaks a subcircuit. For this the time $t_{P1} = t_{P2} = t_{P3} = t_{P4}$ should be taken infinite assuming that over the time period of interest no trigger pulses arrive at the gates of thyristors *P1*, *P2*, *P3*, and *P4*, so they stay off.

In developing an algorithm, we assume that the set of line voltages U_{AB}, U_{BC}, and U_{CA} is asymmetric. For convenience, label the line voltages with other subscripts: $U_{AB} = U_{Al}$, $U_{BC} = U_{Bl}$, and $U_{CA} = U_{Cl}$.

Setting $N = A$, B, and C, we can proceed further with the generalization for the line voltage, U_{Nl}. Line voltages vary arbitrarily with time, which enables us to consider the characteristics of the motor run on the nonsinusoidal voltage supply, too, delivered by various frequency converters. For the case where thyristors control the stator voltage, we assume that the supply line exhibits a sinusoidal asymmetric set of voltages:

$$u_{Nl} = \sqrt{2}\, U_{Nl} \sin(\omega t + \varphi_{Nl}) \tag{5.24}$$

where φ_{Nl} is the initial phase of line voltages

$$\varphi_{Nl} = \begin{Bmatrix} \varphi_{Al} = \varphi_{AB} \text{ at } N = A \\ \varphi_{Bl} = \varphi_{BC} \text{ at } N = B \\ \varphi_{Cl} = \varphi_{CA} \text{ at } N = C \end{Bmatrix} \tag{5.25}$$

To obtain the most flexible model for simulation, control over each thyristor must be separate. Thus the conduction angles of thyristors 1 and 2 are independent of each other and can vary in different fashions to enable the induction motor to run in any of the specified operating conditions.

To effect the operation of the TVR-induction motor system in various conditions, it is good practice to lock trigger pulses coming to the gate of each N_ith thyristor ($i = 1,2$) with the corresponding line voltage (Fig. 5.6). To accomplish the end, the conduction angle of thyristor N_i is counted off from the zero through which U_{Ni} passes at a certain instant. Thus for the thyristor $A1$ ($N = A$, $i = 1$) triggered to conduct current in the forward direction from the supply

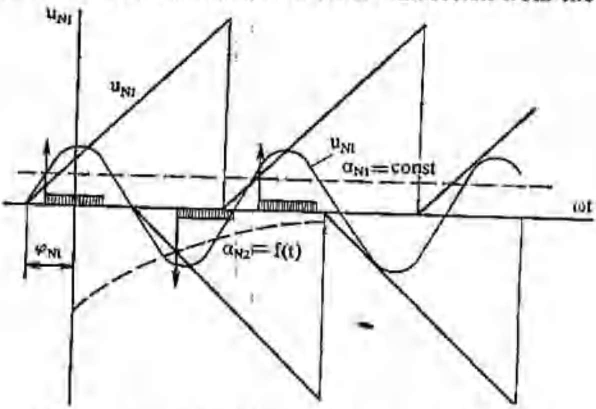


Fig. 5.6. The scheme of applying trigger pulses to drive thyristors into conduction

line into the phase winding a, the phase shift α_{A1} between the trigger pulses is counted off from the zero of the positive halfwave of line voltage u_{AB}, and for $A2$ ($N = A$, $i = 2$) conducting in the reverse direction, α_{A2} is counted off from the zero of the negative halfwave of u_{AB}.

To estimate the moments at which trigger pulses arrive at thyristors, look into the process involved in the control technique based on the comparison between the sawtooth pulse voltage u_{Ni} proportional to the phase of the corresponding line voltage and the voltage proportional to the conduction angle α_N of N_i. The equation of the sawtooth pulse voltage over the half-cycle for the N_ith thyristor (the intermediate transformations being omitted) takes on the form

$$u_{Ni} = \omega t + v_{Ni} \tag{5.26}$$

For the entire cycle, $u_{Ni} = [(\omega t + v_{Ni})/2\pi]\, 2\pi$. Here t is the running value of time; $v_{Ni} = \varphi_{Ni} = 2\pi\delta_N\,(\operatorname{sign}\varphi_{Ni}) + \pi\,(1 - i)\times$ $\times \operatorname{sign}\varphi_N$; and $\delta_N\,(\operatorname{sign}\varphi_{Ni})$ is the unit function dependent on

the sign of the initial phase of line voltages, φ_{Ni}:

$$\delta_N(\operatorname{sign}\varphi_{Ni})=\begin{cases}1 \text{ at } \varphi_{Ni}<0\\ 0 \text{ at } \varphi_{Ni}>0\end{cases}$$

As soon as the sawtooth pulse u_{Ni} grows to a value proportional to α_{Ni} appearing in the real circuit of the control device, a short negative pulse triggers the biased blocking oscillator which gives a pulse of requisite width Δ_{Ni} to the gate of the thyristor. Therefore, to determine the moments t_{Ni} at which trigger pulses go to the gates, we should solve simultaneously the equations describing the manner of variation of the conduction angles α_{Ni} and also the corresponding equations for sawtooth pulse voltages.

In the dynamic and static operating conditions, the moments at which thyristors and diodes begin to conduct are defined by nonlinear nonhomogeneous differential equations describing the joint operation of the thyristor voltage regulator and the induction motor. At an arbitrary moment of time, the system may be in one of the five states: (1) the on-state of the corresponding thyristors connected to the three phases a, b, and c; (2) the on-state of thyristor cells connected to phases b and c; (3) the on-state of thyristor cells for phases a and c; (4) the on-state of thyristor cells for phases a and b; (5) the off-state of all thyristor cells. For each time interval between the two successive actions of switching of thyristor cells, we thus obtain the solution of a partial problem; the sought-for solution involves a successive solution of a large number of various partial problems. According to the basic principle of switching, the initial values of currents and flux linkages needed for the solution of the next partial problem are taken from the preceding solution.

For the analysis of a circuit diagram with its thyristors inserted into the stator circuit, it is convenient to use the α-β coordinate notation of the differential equations for an induction motor, which contain stator currents and rotor flux linkages:

$$\begin{gathered}
u_\alpha^s=r_s i_\alpha^s+\sigma L_s\,(di_\alpha^s/dt)+(L_m/L_r)\,(d\Psi_\alpha^r/dt)\\
u_\beta^s=r_s i_\beta^s+\sigma L_s\,(di_\beta^s/dt)+(L_m/L_r)\,(d\Psi_\beta^r/dt)\\
0=-r_r\,(L_m/L_r)\,i_\alpha^s+(r_r/L_r)\,\Psi_\alpha^r+(d\Psi_\alpha^r/dt)+\omega_r\Psi_\beta^r \qquad (5.27)\\
0=-r_r\,(L_m/L_r)\,i_\beta^s+(r_r/L_r)\,\Psi_\beta^s+(d\Psi_\beta^s/dt)-\omega_r\Psi_\alpha^r\\
M_e=(3/2)\,p\,(L_m/L_r)\,(\Psi_\alpha^r i_\beta^s-\Psi_\beta^r i_\alpha^s)\\
(J/p)\,(d\omega_r/dt)=M_e-\operatorname{sign}\omega_r M_r
\end{gathered}$$

where Ψ_α^r and Ψ_β^s are the rotor flux linkage vector components along the α and β axes respectively; $\operatorname{sign}\,\omega_r M_r$ is the load torque of the dry-friction type; and σ is the leakage coefficient.

Using the relation between the instantaneous phase currents i_a^s, i_b^s, i_c^s (with the total vector components i_α^s, i_β^s) and instantaneous phase voltages u_a^s, u_b^s, u_c^s (with the total vector components u_α, u_β^s), after simple transformations we obtain

$$u_{sa} = r_s i_{sa} + \sigma L_s \frac{di_{sa}}{dt} + \frac{L_m}{L_r} \frac{d\Psi_\alpha^r}{dt}$$

$$u_{sb} = r_s i_{sb} + \sigma L_s \frac{di_{sb}}{dt} + \frac{L_m}{L_r} \left(-\frac{1}{2} \frac{d\Psi_\alpha^r}{dt} + \frac{\sqrt{3}}{2} \frac{d\Psi_\beta^r}{dt} \right) \quad (5.28)$$

$$u_{sc} = r_s i_{sc} + \sigma L_s \frac{di_{sc}}{dt} + \frac{L_m}{L_r} \left(-\frac{1}{2} \frac{d\Psi_\alpha^r}{dt} - \frac{\sqrt{3}}{2} \frac{d\Psi_\beta^r}{dt} \right)$$

From the set of equations for the line voltages of a motor, we get

$$u_{sab} = u_{sa} - u_{sb} = r_s (i_{sa} - i_{sb}) + \sigma L_s \times \frac{d(i_{sa} - i_{sb})}{dt} + \frac{L_m}{L_r} \left(\frac{3}{2} \frac{d\Psi_\alpha^r}{dt} - \frac{\sqrt{3}}{2} \frac{d\Psi_\beta^r}{dt} \right)$$

$$u_{sca} = u_{sc} - u_{sa} = r_s (i_{sc} - i_{sa}) + \sigma L_s \times \frac{d(i_{sc} - i_{sa})}{dt} + \frac{L_m}{L_r} \left(-\frac{3}{2} \frac{d\Psi_\alpha^r}{dt} - \frac{\sqrt{3}}{2} \frac{d\Psi_\beta^r}{dt} \right) \quad (5.29)$$

$$u_{sbc} = u_{sb} - u_{sc} = r_s (i_{sb} - i_{sc}) + \sigma L_s \frac{d(i_{sb} - i_{sc})}{dt} + \sqrt{3} \frac{d\Psi_\beta^r}{dt}$$

The expressions in terms of line voltages are convenient for the analysis of thyristors operated from the asymmetric voltage supply. For example, the equations for a three-phase squirrel-cage induction motor with thyristors operating in the state of three-phase conduction *ABC* have the form

$$u_{sab} = u_{AB}, \; u_{sbc} = u_{BC}, \; u_{sca} = u_{CA}$$

Noting that the sum of line voltages is zero (in the case of asymmetry too)

$$u_{AB} + u_{BC} + u_{CA} = 0$$

the sum of line currents is also equal to zero:

$$i_{sa} + i_{sb} + i_{sc} = 0$$

Omitting intermediate transformations, we write the expression for stator circuits in the form

$$u_{sa} = (2u_{AB} + u_{BC})/3 = r_s i_{sa} + \sigma L_s\,(di_{sa}/dt) + (L_m/L_r)\,(d\Psi^r_\alpha/dt) \qquad (5.30)$$

$$u_{sb} = \frac{2u_{BC} - u_{AB}}{3} = r_s i_{sb} + \sigma L_s \frac{di_{sb}}{dt} + \frac{L_m}{L_r} \times \left(-\frac{1}{2}\frac{d\Psi^r_\alpha}{dt} + \frac{\sqrt{3}}{2}\frac{d\Psi^r_\beta}{dt}\right) \qquad (5.31)$$

$$u_{sc} = \frac{-2u_{BC} + u_{AB}}{3} = r_s i_{sc} + \sigma L_s \frac{di_{sc}}{dt} + \frac{L_m}{L_r} \times \left(-\frac{1}{2}\frac{d\Psi^r_\alpha}{dt} - \frac{\sqrt{3}}{2}\frac{d\Psi^r_\beta}{dt}\right) \qquad (5.32)$$

Eq. (5.32) can be discarded since for the phase c we have

$$u_{sc} = -(u_{sa} + u_{sb}),\ i_{sc} = -(i_{sa} + i_{sb})$$

For rotor circuits the equations assume the form

$$0 = -r_r\,(L_m/L_r)\,i_{sa} + (r_r/L_r)\,\Psi^r_\alpha + d\Psi^r_\alpha/dt + \omega_r \Psi^r_\beta \qquad (5.33)$$

$$0 = -r_r\,(L_m/L_r)\,[(2i_{sb} + i_{sa})/\sqrt{3}] + (r_r/L_r)\,\Psi^r_\beta + d\Psi^r_\beta/dt - \omega_r \Psi^r_\alpha \qquad (5.34)$$

The electromagnetic torque equation is given by

$$M_e = \frac{3}{2}\,p\,\frac{L_m}{L_r}\left[\Psi^r_\alpha \frac{(2i_{sb} + i_{sa})}{\sqrt{3}} - \Psi^r_\beta i_{sa}\right] \qquad (5.35)$$

A similar approach is appropriate in deriving the equations for the induction motor with thyristors switched to other modes of operation.

Figure 5.7 shows the schematic diagram illustrative of the way of connection of the stator windings to the supply line via series-connected thyristor cells. When one of the thyristors in a thyristor cell TC is held in the conducting state (the switch is on), the thyristor cell resistance is close to zero; when it goes to the nonconducting state (the switch is off), the thyristor cell has a resistance extending to infinity.

By Kirchhoff's second law

$$u_{BC} + i_c R_C - u_{sbc} - i_b R_B = 0$$

$$u_{CA} + i_a R_A - u_{sca} - i_c R_C = 0 \qquad (5.36)$$

From the set of equations (5.36) we can determine currents and then voltages in thyristor cells:

$$u_{TA} = i_a R_a = \frac{R_A R_C\,(u_{AB} - u_{sab}) - (u_{CA} - u_{sca})\,R_A R_B}{R_A R_C + R_A R_B + R_C R_B} \qquad (5.37)$$

Similarly, we can define u_{TB} and u_{TC}. The equations for u_{TA}, u_{TB} and u_{TC} permit us to determine the voltages across thyristor cells operating in any of the states of conduction. These voltages are necessary for formulating the programs to be run on digital computers.

In developing a mathematical model for the simulation on a digital computer, it is desirable that the model should most closely approximate in structure the real circuit of the thyristor drive under

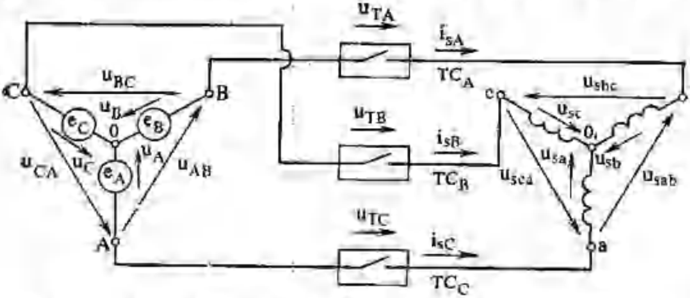


Fig. 5.7. The schematic diagram of a TVR-induction motor system

investigation. In circuits where thyristors connected to any of the phases operate independent of the other thyristors (for example, in the star-connected three-phase stator-winding network with the neutral), the following logic function corresponds to the operation of each N_ith thyristor:

$$X_{N_i}Z_{N_i}+Y_{N_i}$$

here the function X_{N_i} corresponds to the voltage across a thyristor; Z_{N_i} corresponds to the trigger pulse on the gate; and Y_{N_i} corresponds to the current through the gate; the multiplication sign denotes a logic AND ($\wedge$) operation, and the summation sign a logic OR ($\vee$) operation.

The logic functions X_{N_i}, Y_{N_i}, and Z_{N_i} are equal to unity if the corresponding voltages and currents in a thyristor are positive; they are equal to zero if these voltages and currents are negative in sign.

In three-phase networks with the isolated neutral wire, all thyristors operate in an interrelated fashion. Assume the system stays in the state of zero conduction. Find the logic function indicative of switching of the thyristors to the state of three-phase conduction ABC. Only definite triplets of thyristors can here function simultaneously—two conduct current in one direction and the third conducts

in the opposite direction:

$$\begin{aligned}A_{03} = {} & X_{A1}X_{B1}X_{C2}Z_{A1}Z_{B1}Z_{C2} + X_{A1}X_{B2}X_{C2}Z_{A1}Z_{B2}Z_{C2} \\ & + X_{A1}X_{B2}X_{C1}Z_{A1}Z_{B2}Z_{C1} + X_{A2}X_{B1}X_{C1}Z_{A2}Z_{B1}Z_{C1} \\ & + X_{A2}X_{B2}X_{C1}Z_{A2}Z_{B2}Z_{C1} + X_{A2}X_{B1}X_{C2}Z_{A2}Z_{B2}Z_{C2}\end{aligned} \tag{5.38}$$

If the system operates in the ABC mode and the gates do not change state, the logic function for this condition of operation has the form

$$\begin{aligned}A_{33} = {} & Y_{A1}Y_{B1}Y_{C2} + Y_{A1}Y_{B2}Y_{C1} + Y_{A1}Y_{B2}Y_{C2} \\ & + Y_{A2}Y_{B1}Y_{C1} + Y_{A2}Y_{B2}Y_{C1} + Y_{A2}Y_{B1}Y_{C2}\end{aligned} \tag{5.39}$$

In Eqs. (5.38) and (5.39), the functions X_{N_i} are logic functions which correspond to voltages u_{TN} and are found from the above formulas for voltages across thyristor cells. Thus, if $u_{TA} > 0$, $X_{A1} = 1$ ($X_{A2} = 0$); if $u_{TA} < 0$, $X_{A2} = 1$ ($X_{A1} = 0$); if $i_a > 0$, $Y_{A1} = 1$ ($Y_{A2} = 0$); if $i_a < 0$, $Y_{A2} = 1$ ($Y_{A1} = 0$).

If the thyristors enter the zero-conduction region and the function A_{03} is equal to zero, so that none of the thyristor triplets switches on, we need to know whether the thyristors turn on pairwise:

$$\begin{aligned}A_{02} = {} & X_{A1}X_{B2}Z_{A1}Z_{B1} + X_{A1}X_{C2}Z_{A1}Z_{C1} \\ & + X_{A2}X_{B1}Z_{A2}Z_{B1} + X_{A1}X_{C1}Z_{A2}Z_{C1} \\ & + X_{B1}X_{C2}Z_{B1}Z_{C2} + X_{B2}X_{C1}Z_{B2}Z_{C1}\end{aligned} \tag{5.40}$$

When $A_{02} = 0$, the system may go from the zero to the two-phase conduction mode. Assume, for example, trigger pulses arrive at $A1$ and $B2$ so that $u_{TA} > 0$ and $u_{TB} > 0$, but $u_{TAB} = u_{TA} - u_{TB} > 0$. The system then switches from the off-state to the state of two-phase conduction. Consequently, apart from the function A_{02} defined by Eq. (5.40), we must consider an additional logic function

$$\begin{aligned}B_{02} = {} & X_{AB}Z_{A1}Z_{B2} + X_{AC}Z_{A1}Z_{C2} + X_{BA}Z_{B1}Z_{A2} \\ & + X_{CA}Z_{C1}Z_{A2} + X_{BC}Z_{B1}Z_{C2} + X_{CB}Z_{C1}Z_{B2}\end{aligned} \tag{5.41}$$

where $X_{AB} = 1$ if $X_{BA} = 1$ ($X_{AB} = 0$). At $u_{TAB} < 0$, $X_{BA} = 1$ ($X_{AB} = 0$), and so on.

The logic function describing the condition at which the thyristors are driven on for the two-phase conduction has the form

$$\begin{aligned}A_{22} = {} & y_{A1}y_{B2} + y_{A1}y_{C2} + y_{A2}y_{B1} \\ & + y_{A2}y_{C1} + y_{B1}y_{C2} + y_{B2}y_{C1}\end{aligned} \tag{5.42}$$

Thus, from the above discussion the following conclusion can be drawn.

For the three-phase conduction mode there corresponds a logic function

$$C_3 = A_{03} + A_{23} + A_{33} \tag{5.43}$$

The logic function for the state of two-phase conduction is

$$C_2 = (A_{02} + A_{02}B_{02} + A_{22})\,\overline{C}_3 \tag{5.44}$$

where $\overline{C}_3$ stands for the negative functions of C_3; at $C_3 = 0$, $\overline{C}_3 = 1$ and at $C_3 = 1$ we have $\overline{C}_3 = 0$.

The function for the state of zero conduction is

$$C_0 = \overline{C}_3\overline{C}_2 \tag{5.45}$$

The above logical analysis enables us to pass from one set of differential equations to another depending on the state of conduction of thyristors and diodes.

The algorithm for determining the state of thyristors is so chosen as to carry out the sequential analysis for the time intervals over which the pulses on all thyristors remain invariable. The system can change state over the given length of time if a thyristor turns on (as U_T on the gate changes sign) or a thyristor cell turns off (as current changes sign). The switching moments are sought and the corresponding values of variables are found by dividing the integration step in half down to $H_{min} = 10^{-6}$ s, then the linear interpolation is performed.

As the pulses die out or new ones emerge, another criterion is set up to determine the presence of pulses on thyristors; the above-described method is then used to define the state of thyristors and to conduct the logical analysis of their operation. Based on this calculation technique, a generalized routine is written to analyze the dynamic and static performance of the given system on a digital computer for any character of changes in the conduction angles of thyristors.

5.4. Pulse Electromechanical Energy Converters

Electromechanical dc and ac systems belong to the continuous type. There are discrete electromechanical systems where the conversion of energy proceeds by way of the pulses of electromagnetic power. The last few decades have seen a considerable advancement in the area of discrete electromechanical systems.

Pulse energy converters operate both in the motoring mode (step motors) and in the generating mode (pulse generators). Step motors transform voltage pulses into discrete angular motions or stepwise linear displacement and pulse generators produce powerful current pulses fed to electrophysical setups. Step motors have a small power, usually up to a few hundreds of watts, and pulse generators are

generally built to give a high output, up to tens of megawatts in a pulse.

A step motor is made complete with a commutating arrangement and designed to carry a load on the rotor shaft. For each motor there is a definite switching frequency at which its rotor follows in step a varying field in the air gap. This frequency is known as *a response frequency* determined by the entire system comprising the semiconductor commutator (pulse generator) and the step motor, and also by the load on the shaft.

Most step motors are multipolar multiphase synchronous machines in which either groups of stator windings or each winding receives unipolar or two-polarity pulses. The rotor revolves nonuniformly within the entire angle of revolution as it follows the stepwise displacement of the magnetic field. The motor incorporates special means or provision is made in the motor design to fix the rotor in a definite position to register the response. Changing the order of arrival of pulses, it becomes possible to alter the sense of rotation of the rotor and thus sum up the positive and negative pulses in the form of angular deflections.

Step electromechanical systems, like all electric machines, are convertible. They can act as sources of low-power pulses. An important task is to produce high-power pulses up to 100 kJ with a steep leading edge and a high repetition rate, or pulses of a definite waveform.

Pulse generators must store energy in the form of the kinetic energy of rotating members and in the form of the energy of the magnetic field. On the one hand, a pulse generator must be capable of controlling the energy stored and, on the other, must have a small time constant to produce desired pulses. These are conflicting requirements from the viewpoint of electromechanics.

Difficulties often arise in designing a magnetic-field type pulse generator adequate to the requirements imposed on it. It therefore makes sense to resort to pulse generators of the electric-field and electromagnetic-field types. Superconducting magnetic systems offer much promise as devices for storing large energies in the magnetic field.

The approaches and equations applied in the analysis of ordinary energy converters also hold for the studies of electromechanical energy conversion processes in pulse energy converters. The form of equations for pulse voltages remains the same as for sinusoidal voltages. In the simulation of step motors it is common to simplify equations so as to take into account the character of load and the internal resistance of the supply source.

Continuous sinusoidal supply voltages can be thought of as an infinitely long train of pulses varying in amplitude. As the voltage deviates from the sinusoid, the rotor revolves at a nonuniform angu-

lar speed; the motor having a definite form of the field displacing in a definite manner is a step motor that translates control signals into the desired angles of revolution. So, as mentioned above, the same equations and the same laws hold for describing conventional pulse generators and motors. Also, the approaches discussed in the present book work for the analysis of the performance of these machines.

Chapter 6
Multiwinding Machines

6.1. The Equations of Multiwinding Machines

All electric machines can be regarded as multiwinding (multiloop) machines. The magnetic cores of electric machines generally develop eddy currents which should be taken into account in the analysis of electromechanical energy conversion processes. Most of the real machines have a few windings. An electric machine having one winding (i.e. two phase windings) on the stator and one winding (i.e. two phase windings) on the rotor is a primitive machine adopted in the analysis practice, the construction of which calls for a rather large number of constraints.

Synchronous machines have a damper winding and a field winding on the rotor. Disregarding eddy current loops, the model of a synchronous machine carries one winding (two phase windings) on the stator and two windings (four phase windings) on the rotor.

A dc machine has windings of commutating poles (commutating windings) and a compensating winding; its field winding may consist of a coil of shunt (parallel) excitation and a coil of series excitation. With allowance made for eddy current loops, the dc machine is a three- to five-winding machine. A cross-field control generator has a control winding consisting of a few coils.

Industry produces induction machines with two windings on the stator and two or three windings on the rotor. These are multiple-speed machines with pole-pair changers, which constitute a wide class of double squirrel-cage machines.

A deep-slot machine can also be classed with multiwinding machines if we consider that a few parallel-connected conductors over the slot height form several loops. In the process of starting of an induction motor, the current spreads out nonuniformly over the slot height because of its displacement. This fact can be taken into account by solving equations for n parallel-connected conductors placed on the rotor.

A stator winding is often built up of a few parallel-connected conductors. Solving the equation for a multiwinding machine, we can allow for the nonuniform current distribution over the conductors, if this is the case.

Electromagnetic converters, i.e. transformers, are built with two or three windings. They are also multiwinding energy converters if eddy current loops are taken into account.

In Sec. 3.2 were given the equations for a hypothetical *m-n* winding machine. As a particular case, we can use these equations to formulate the equations for any multiwinding machine met with in practice. In the equations describing the processes of energy conversion in multiwinding machines, mutual inductances determine the link between windings, each winding being assumed to exhibit a definite magnetic leakage. The electromagnetic torque is defined as the products of currents entering into the equations. Under these conditions, Eqs. (3.3) through (3.12) are the equations of multiwinding machines.

6.2. The Equations of Synchronous Machines

Synchronous machines, both salient-pole and nonsalient-pole types, should be treated as multiwinding machines having an armature win-

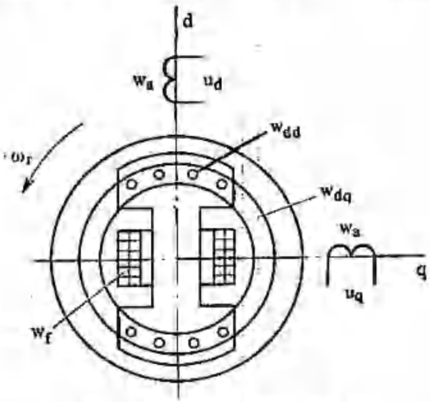


Fig. 6.1. A synchronous machine
w_a — armature winding; w_{dd} and w_{dq} — d-and q-axis damper windings; w_f — field winding

ding, field winding, and damper winding as shown in Fig. 6.1. In the simplest case, a synchronous machine is a three-winding machine, its model being represented by six phase windings.

It is customary to express the equations for a synchronous machine in the system of d-q coordinates rigidly connected to the rotor structure. For a machine with transformed windings we write down the following equations

$$\begin{aligned}
u_d &= d\Psi_d/dt - \Psi_q\omega_r + r_a i_d \\
u_q &= d\Psi_q/dt + \Psi_d\omega_r + r_a i_q \\
u_f &= d\Psi_f/dt + r_f i_f \\
0 &= d\Psi_{dd}/dt + r_{dd} i_{dd} \\
0 &= d\Psi_{dq}/dt + r_{dq} i_{dq}
\end{aligned} \tag{6.1}$$

Here r_a and r_f are the resistances of the armature and field windings respectively; r_{dd} and r_{dq} are the resistances of the damper winding, i.e. two phase windings along the direct and the quadrature axis respectively; u_d and u_q are the voltages across the armature winding, i.e. two phase windings along the d and q axes respectively; and u_f is the voltage in the field winding. The magnetic flux linkages for the windings are given by

$$\begin{aligned}
\Psi_d &= L_d i_d + M_{ad} i_f + M_{ad} i_{dd} \\
\Psi_q &= L_q i_q + M_{aq} i_{dq} \\
\Psi_f &= L_f i_f + M_{ad} i_d + M_{ad} i_{dd} \\
\Psi_{dd} &= L_{dd} i_{dd} + M_{ad} i_d + M_{ad} i_f \\
\Psi_{dq} &= L_{dq} i_{dq} + M_{aq} i_q
\end{aligned} \tag{6.2}$$

Here L_d, L_q, L_f, L_{dd}, and L_{dq} are the inductances of the armature, field, and damper windings respectively; and M_{ad} and M_{aq} are the mutual inductances of the windings along the d and q axes. The assumption is that the mutual inductances between the windings lying along the same axis are identical, while the leakage inductances of windings are different.

The electromagnetic torque is defined in terms of currents and flux linkages

$$M_e = \Psi_d i_q - \Psi_q i_d \tag{6.3}$$

or in terms of currents

$$M_e = M\,(i_f i_q + i_q i_{dd} - i_d i_{dq}) \tag{6.4}$$

The equality (6.4) is written for a nonsalient-pole machine. For a salient-pole machine, the equality includes a component to allow for the difference between the permeances along the d and q axes. The products of damper winding currents and currents i_d and i_q give rise to an asynchronous torque. In the steady-state conditions these products are responsible for rotor hunting.

The equations for a synchronous machine can be derived from the equations for the generalized energy converter if we assume that the armature winding rotates and the field and damper windings are stationary (Fig. 6.2). It is advantageous that the mathematical model should have a minimum number of rotating windings since the simulation will then require a fewer multipliers. The transformation of windings to the armature winding should be done in the same way as for an induction machine.

In modeling synchronous machines it is more judicious to use per-unit units. The basic quantities are taken to be the angular velocity $\omega_b = \omega_s$, stator current $i_b = I_{m,ph}$, and voltage $u_b = u_{m,ph}$. We then have the flux linkage $\Psi_b = u_b/\omega_b$, impedance $z_b = u_b/i_b$, and inductance $L_b = z_b/\omega_b = u_b/i_b\omega_b$. Here $M_{ad} = x_{ad}$, $M_{aq} = x_{aq}$, $L_d = x_d$, $L_q = x_q$, $L_f = x_f$, $L_{dd} = x_{dd}$, and $L_{dq} = x_{dq}$.

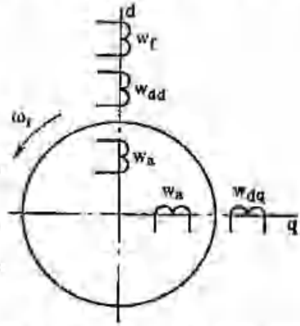


Fig. 6.2. The model of a synchronous machine

In modeling permanent-magnet synchronous machines, the degree of excitation of a magnet is defined as the product of current I_m times the mutual inductance of the armature winding and the fictitious current of the magnet. Permanent magnets are represented by an equivalent inertialess and lossless loop supplied from a dc source (see Appendix 1, Fig. A6).

What complicates the simulation of synchronous machines is a saturation-induced change in the parameters with load. In salient-pole machines, the displacement of the field axis with respect to the axis of poles adds still more to the complexity of the model.

The transients were given treatment first in synchronous machines. As far back as the late 1920s Park and Gorev formulated Eqs. (6.1) which now bear the names of Park and Gorev. The need for the study of transients in synchronous machines arose in the course of development of power supply systems; the aim was to investigate the effect of the emergency operation of one machine on the stability of parallel operation of other machines. Since computers came into being much later, it was impossible to solve Eqs. (6.1) for the case of a varying angular velocity. The only way out was to simplify the initial equations to describe the main events determining the behaviour of a machine in conditions considered most important from the practical viewpoint. In performing the analysis, it was necessary to define a set of parameters characterizing the machine

operation. The focus of attention was largely on transients in the conditions of suddenly applied short circuits. With three-phase or asymmetric short circuits placed on stator windings, the surges of currents in the windings may exceed 10 to 15 times the nominal values.

In solving Eqs. (6.1) without the use of a computer, the armature winding resistance is usually taken equal to zero to make flux linkages constant. This assumption facilitates the solution to the problem, but leads to an inconsistency of the analysis and to a number of contradictions.

A short-circuit transient process includes several stages. If a machine has a damper winding, the first stage is determined by a direct-axis subtransient inductive reactance

$$x_d'' = x_{\sigma a} + \frac{1}{1/x_{ad} + 1/x_{\sigma f} + 1/x_{\sigma dd}}$$

where $x_{\sigma a}$ is the leakage inductive reactance of the armature winding; x_{ad} is the direct-axis inductive reactance of the armature; $x_{\sigma f}$ is the leakage inductive reactance of the field winding; and $x_{\sigma dd}$ is the direct-axis leakage inductive reactance of the damper winding.

The peak short-circuit current here is

$$I_{am} = E_m/x_d''$$

where E_m is the peak phase emf.

The second stage of the short-circuit transient starts at the instant when the armature flux passes through the damper winding and begins to traverse the field winding. This condition of the machine is defined by the direct-axis transient reactance

$$x_d' = x_{\sigma a} + \frac{1}{1/x_{ad} + 1/x_{\sigma f}}$$

The steady-state short-circuit current is a function of the direct-axis inductive reactance

$$x_d = x_{\sigma a} + x_{ad}$$

and is equal to

$$I_{sc}^{st} = E_m/x_d$$

The effective value of short-circuit current is found from I_{sc}^{st}, the transient and subtransient short-circuit current. The decay of the transient current is defined by the direct-axis transient time constant T_d' and that of the subtransient current by the direct-axis subtransient time constant T_d''; here $T_d' > T_d''$.

On the curve of the short-circuit current we can single out a periodic and an aperiodic component. The aperiodic component dies away with the armature time constant T_a which depends on the armature inductive reactance and armature resistance.

The aperiodic components of armature winding currents produce a field that is stationary with respect to the armature, therefore quadrature-axis loops also take part in the transient process. Consideration is then given for the transient and subtransient inductive reactances along the q axis.

The q-axis subtransient inductive reactance is

$$x''_q = x_{\sigma a} + \frac{1}{1/x_{aq} + 1/x_{\sigma dq}}$$

where x_{aq} is the q-axis armature reactance; and $x_{\sigma dq}$ is the q-axis leakage reactance of the damper winding.

The q-axis transient reactance is

$$x'_q = x_{\sigma a} + x_{aq} = x_q$$

The aperiodic component of the armature current oscillates at a double frequency between the currents

$$E_m/x''_d \text{ and } E_m/x''_q$$

This analysis of complex processes in a synchronous machine calls for many assumptions. Nevertheless, it discloses well the physical processes and gives sufficiently accurate results.

In the analysis of the static and dynamic stability of the parallel operation of synchronous machines, wide use is made of linearized incremental equations. The increments of variables are taken linear and the treatment is given to the modes of small oscillations.

The study of static stability on the basis of small harmonic perturbations is justifiable since in the problems involved here account should be taken of the parameters of the supply line and electric machines and transformers operated into the same network together with the synchronous machine under analysis.

The creation of turbogenerators of a unit power of 1.2 to 1.5 mln kW in the last years and the emergence of more complex power systems have raised new problems relating to the study of transients in synchronous machines. There is a need for a more rigorous analysis of transients in asynchronous conditions, at restarting of alternators, during rough synchronization, and in other emergency conditions of operation of synchronous machines in the power systems.

As regards the analysis of transients in synchronous machines, of much interest is the investigation of torsional vibrations of turbogenerator shafts under various emergency conditions with consideration for transients in the power system. In the analysis, the electromagnetic torque and the moment of inertia are taken to be distributed along the rotor length. The equation of motion is then solved simultaneously with the voltage equations.

A most judicious approach to investigating the dynamics of synchronous machines is to use computers for the solution of equations

in conjunction with traditionally adopted techniques. The guideline for the analysis of salient-pole and nonsalient-pole machines with three rotor windings and two stator windings should envisage the use of universal routines to be designed for solving the steady-state and transient performance on digital computers. It is of much importance to test and make the right choice of models obtained by the method of experimental design. This will open up new possibilities for studying the performance of synchronous machines.

6.3. The Equations of Direct Current Machines

Direct current machines are multiwinding machines. Most schematic diagrams of real dc motors and generators can be transformed to a simplified model such as illustrated in Fig. 6.2. In the model of Fig. 6.3,

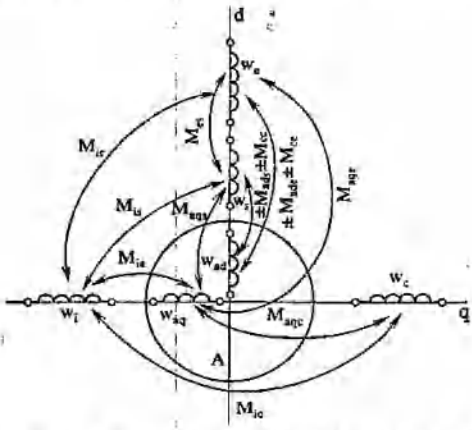


Fig. 6.3. Electromagnetic couplings in a dc machine

the armature winding is shown to consist of two symmetric windings w_{ad} and w_{aq}; the field winding is seen to comprise a separate excitation winding w_e and a series excitation winding w_s on the stator along the d axis. Shown also on the figure are a compensating winding w_c and an interpole (commutating) winding w_i on the stator along the q axis.

It is of convenience to express the equations for dc machines in the d-q coordinates, just like the equations for synchronous machines. The simulation on an analog computer involves the representa-

tion of problem variables by direct currents and voltages. Nonlinear couplings make the modeling of a dc machine a rather complicated problem. These couplings arise from saturation, q- and d-axis transient armature reactances, commutating armature reactions, and also as a result of eddy current influences. It is impossible to allow accurately for all the above factors. Therefore, in the study of dc machines, the open-circuit characteristic is taken linear and the

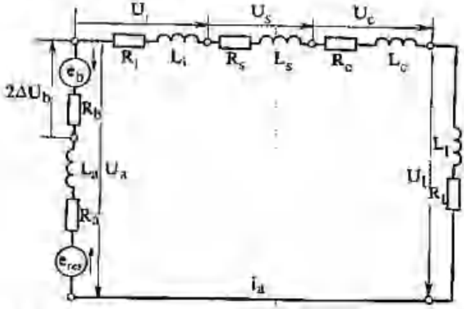


Fig. 6.4. The equivalent circuit for a dc machine

parameters constant. But such assumptions do not afford the desired accuracy in view of the asymmetry of field (stator) windings.

Apart from the mutual inductances M_{ie}, M_{is}, M_{ia}, M_{es}, and M_{ic} between the windings of Fig. 6.3, the electromechanical equations must contain mutual inductances M_{aqs}, M_{aqe}, and M_{aqc} due to the q-axis transient reactions of the series excitation winding w_s, separate excitation winding w_e, and compensating winding w_c; M_{ads} and M_{ade} due to the direct-axis transient armature reactions; and M_{ec} and M_{co} due to the commutating reaction of the armature.

Using the circuit model of Fig. 6.4 and electromagnetic interactions between the windings, for the armature and excitation (field) windings of a generator we write down the following equations

$$-u_l = -e_{res} + 2\Delta u_b + R'_a i_a + L'_a\,(d/dt)\,i_a - M_{ea}\,(d/dt)\,i_e$$
$$u_e = R'_e\,i_e + L_e\,(d/dt)\,i_e - M_{ep}\,(d/dt)\,i_a \qquad (6.5)$$

Here u_l is the instantaneous value of load voltage; e_{res} is the resultant emf of rotation; Δu_b is the voltage drop across a brush contact; $R'_a = R_a + R_s + R_i + R_c$ is the armature circuit resistance equal to the sum of resistances of the armature, series, interpole, and compensating windings respectively; i_a is the current in the armature circuit; i_e is the current in the excitation winding; L'_a is the total

armature circuit inductance equal to

$$L_a' = L_a + L_s + L_i + L_c + 2M_{is} - M_{ia} + 2M_{ic} \pm 2M_{ads} \pm 2M_{cs} - 2M_{aqc} + M_{aqs}$$

The term M_{ea} in (6.5) is the coefficient for the back mutual inductance between the armature and excitation circuits: $M_{ea} = M_{ed} - M_{aqe}$. Here M_{ed} is the coefficient for the direct mutual inductance between the armature and excitation circuits:

$$M_{ed} = M_{es} \pm M_{ade} \pm M_{ce} - M_{ie} + M_{aqe}$$

For a dc generator the angular velocity can be considered constant. This enables us to disregard the equations of motion and limit the analysis to the solution of voltage equations.

In the analysis of the dynamic performance of a machine, it is of importance to determine correctly the emf e_{res} as a function of currents and to take into account the magnetizing and demagnetizing forces. This can be done by use of the B-H curve, transient response, and relations

$$M_e = C_m i_a \Phi_{res}, \quad e_{res} = C_e \omega \Phi_{res} \tag{6.6}$$

where C_e and C_m are the coefficients that account for the characteristic features of the machine under analysis; and Φ_{res} is the resultant flux determined from the transient response.

The analysis of transient characteristics on an analog computer requires constructing a complicated mathematical model or necessitates the simulation on a digital computer.

Although the steady-state equations for a dc machine are the simplest, the studies on the dynamic performance of an asymmetric machine involve the solution of cumbersome nonlinear equations.

In Appendix I (see Figs. A5 and A6) are shown the block diagrams of models for the solution of equations of dc machines.

As mentioned earlier, dc machines differ from synchronous machines in that they have commutators to rectify the alternating emf generated in the armature windings. A mechanical frequency converter, or commutator, keeps a rigid tie between the angular velocity and electric frequency, while a semiconductor frequency converter may afford a flexible tie.

Despite the fact that synchronous machines have much in common with dc machines, the theory of either of the two classes continued to develop separately for many years; commutation processes were given treatment in conjunction with operating processes in a machine. In considering the processes of energy conversion in the air gap of a dc machine, it is quite justifiable to employ phasor diagrams and equivalent circuits after reducing the multiphase armature winding to a two-phase winding. In its classical design, the dc machine is a salient-pole machine with a stationary field winding. However, con-

trolled-rectifier commutator machines widely use stationary ac windings. Nonsalient-pole dc machines with a compensating winding sometimes find use in practice. The generalized approach to studying synchronous and dc machines will promote further the theory of electric machines.

6.4. The Double Squirrel-Cage Induction Motor. The Effect of Eddy Currents

We restrict our attention to the equations of the machine with one stator winding and two rotor windings. The mathematical model of this type is applicable to the analysis of a wide class of machines with a double squirrel-cage (short-circuited) rotor and to the consideration of the effect of eddy currents in the rotor.

Let us define what one should understand under eddy currents in a cage rotor. In the practice of design of a motor, account is generally taken of the action of one short-circuited winding made up of bars and end rings. The calculation technique deals with an idealized physical process since it disregards the fact that the rotor is current-conducting and the winding is in electrical contact with the core. This approach, generally permissible, does not work where there is a need for an accurate calculation of the starting characteristics of a motor whose rotor current frequency becomes comparable to the supply line frequency or even twice as high in plug reversing.

In high-frequency motors designed for voltages at 400 to 1 000 Hz the effect of eddy currents is noticeable. If we remove the cage winding from the rotor core, thus leaving the slots empty, and then connect the stator to the line, the rotor will run at a steady speed even with the sheet-steel laminations insulated from one another.

Strapping the sheet-steel laminations that form a core with copper bars by tightly embedding them into the slots, we make up a rotor structure, yet without the end rings, i.e. with the cage open-circuited. The motor so built develops a substantial torque, comes up to a steady speed, and can carry a load, about one-third of the rated load. In the first case considered above, the rotor keeps going under the action of eddy currents induced in each steel lamination of the core; in the second case, the rotor turns by the action of eddy currents in steel-bar-steel loops. Such loops exist in any cage rotor and have a pronounced effect since the rotor core laminations are usually not isolated from one another and all bars are solidly connected to end rings, commonly by aluminum flow brazing.

Thus, as regards its equivalent circuit, a single squirrel-cage motor is similar to a transformer with one primary and one secondary, but only if the rotor cage winding has no electrical contact with sheet-steel laminations and the rotor core is made up of thoroughly insulated sheets. Where this is not the case, the motor circuit model

must be analogous to the circuit model of a multiwinding transformer, in which the number of secondaries is equal to the number of loops under study, or to the circuit model of a transformer with two secondaries if the properties of all eddy current loops in the rotor are amenable to generalization to those of an integral eddy current winding.

Theoretically, the generalized electromechanical energy converter permits considering all the variety of eddy current loops if every loop, including the loops in individual laminations of the stacked core, can be treated as an independent winding with its own parameters.

Consider the equations of an energy converter having one stator winding and two rotor windings. In the transformed coordinate system u, v revolving in space at an arbitrary velocity ω_c, the equations for the case of interest have the form

$$\begin{aligned}
u_{u_1} &= d\Psi_{u_1}/dt - \omega_c\Psi_{v_1} + R_s i_{u_1}\\
u_{v_1} &= d\Psi_{v_1}/dt + \omega_c\Psi_{u_1} + R_s i_{v_1}\\
0 &= d\Psi_{u_2}/dt - (\omega_c - \nu)\,\Psi_{v_2} + R_{1r} i_{u_2}\\
0 &= d\Psi_{v_2}/dt + (\omega_c - \nu)\,\Psi_{u_2} + R_{1r} i_{v_2}\\
0 &= d\Psi_{u_3}/dt - (\omega_c - \nu)\,\Psi_{v_3} + R_{2r} i_{u_3}\\
0 &= d\Psi_{v_3}/dt + (\omega_c - \nu)\,\Psi_{u_3} + R_{2r} i_{v_3}
\end{aligned} \tag{6.7}$$

$$i_{u_1} = D_{u_1}/D, \quad i_{u_2} = D_{u_2}/D, \quad i_{u_3} = D_{u_3}/D \tag{6.8}$$

where

$$D = \begin{vmatrix} x_s & x_0 & x_0 \\ x_0 & x_{r_1} & (x_0 + x') \\ x_0 & (x_0 + x') & x_{r_2} \end{vmatrix}$$

$$D_{u_1} = \begin{vmatrix} \omega_0\Psi_{u_1} & x_0 & x_0 \\ \omega_0\Psi_{u_2} & x_{r_1} & (x_0 + x') \\ \omega_0\Psi_{u_3} & (x_0 + x') & x_{r_1} \end{vmatrix}$$

$$D_{u_2} = \begin{vmatrix} x_s & \omega_0\Psi_{u_1} & x_0 \\ x_0 & \omega_0\Psi_{u_2} & (x_0 + x') \\ x_0 & \omega_0\Psi_{u_3} & x_{r_2} \end{vmatrix}$$

$$D_{u_3} = \begin{vmatrix} x_s & x_0 & \omega_0\Psi_{u_1} \\ x_0 & x_{r_1} & \omega_0\Psi_{u_2} \\ x_0 & (x_0 + x') & \omega_0\Psi_{u_3} \end{vmatrix}$$

$$i_{v_1} = D_{v_1}/D, \quad i_{v_2} = D_{v_2}/D, \quad i_{v_3} = D_{v_3}/D \tag{6.9}$$

where

$$D_{v_1} = \begin{vmatrix} \omega_0 \Psi_{v_1} & x_0 & x_0 \\ \omega_0 \Psi_{v_2} & x_{r_1} & (x_0 + x') \\ \omega_0 \Psi_{v_3} & (x_0 + x') & x_{r_1} \end{vmatrix}$$

$$D_{v_2} = \begin{vmatrix} x_s & \omega_0 \Psi_{v_1} & x_0 \\ x_0 & \omega_0 \Psi_{v_2} & (x_0 + x') \\ x_0 & \omega_0 \Psi_{v_3} & x_{r_2} \end{vmatrix}$$

$$D_{v_3} = \begin{vmatrix} x_s & x_0 & \omega_0 \Psi_{v_1} \\ x_0 & x_{r_1} & \omega_0 \Psi_{v_2} \\ x_0 & (x_0 + x') & \omega_0 \Psi_{v_3} \end{vmatrix}$$

The system (6.7) includes the voltage equations for the stator and for the first and the second rotor cage; the systems (6.8) and (6.9) include the current equations for windings on the u and v axes respectively. In these equations, Ψ_{u_1} and Ψ_{v_1} are the flux linkages of the stator along the u and v axes; Ψ_{u_2} and Ψ_{v_2} are the flux linkages of the first rotor cage along the u and v axes; Ψ_{u_3} and Ψ_{v_3} are the flux linkages of the second rotor cage along the u and v axes; R_s (R_{1r}, R_{2r}) stands for resistances of the phase involving the stator and rotor windings; x_s (x_{r_1}, x_{r_2}) stands for inductive reactances corresponding to the total inductance of the phase involving the stator and rotor windings; x_0 is the reactance of mutual induction for a corresponding pair of windings, which is due to the main field of the machine; x' is the reactance of mutual induction for rotor cages; and $\nu = d\theta/dt$ is the angular velocity of the rotor.

The analysis of the most popular three-phase induction machine with two rotor windings calls for the formulation of the relations between self- and mutual inductances of windings and use of the design parameters applied in the theory of electric machines. The relations are obtainable from the comparison of the electromechanical equations for the steady state (ω_r is constant) with the classical equations for a three-phase double-cage motor:

$$\begin{aligned} x_s &= \omega_0 (L_1 - M_1) \\ x_1^r &= \omega_0 (L_2 - M_2) \\ x_2^r &= \omega_0 (L_3 - M_3) \\ x_0 &= \omega_0 (3/2) M_0 \\ x' &= \omega_0 (3/2) (M_{23} - M_0) \end{aligned} \qquad (6.10)$$

where L_1 is the inductance of the stator phase; L_2 is the inductance of the rotor phase (first cage); L_3 is the inductance of the rotor phase (second cage); M_1 is the mutual inductance between stator windings;

M_2 is the mutual inductance between the rotor windings of the first cage: M_3 is the mutual inductance between the rotor windings of the second cage; and M_{23} is the mutual inductance between the rotor windings located on the same axis.

Setting $\omega_c = \omega_r$, $\omega_c = \omega_0$ or $\omega_c = 0$ allows us to consider the converter in the most preferable system of coordinates. The equations expressed in the α-β coordinate system have the form

$$
\begin{aligned}
\frac{di^s_\alpha}{dt} &= \frac{u^s_\alpha}{L^s} - \frac{R^s}{L^s} i^s_\alpha - \frac{M}{L^s}\frac{di^r_{1\alpha}}{dt} - \frac{M}{L^s}\frac{di^r_{2\alpha}}{dt} \\
\frac{di^s_\beta}{dt} &= \frac{u^s_\beta}{L^s} - \frac{R^s}{L^s} i^s_\beta - \frac{M}{L^s}\frac{di^r_{1\beta}}{dt} - \frac{M}{L^s}\frac{di^r_{2\beta}}{dt} \\
\frac{di^r_{1\alpha}}{dt} &= -\frac{R^r_1}{L^r_1} i^r_{1\alpha} - \nu\left(\frac{M}{L^r_1} i^s_\beta + i^s_{1\beta} + \frac{M^r}{L^r_1} i^r_{2\beta}\right) - \frac{M}{L^r_1}\frac{di^s_\alpha}{dt} - \frac{M^r}{L^r_1}\frac{di^r_{2\alpha}}{dt} \\
\frac{di^r_{1\beta}}{dt} &= -\frac{R^r_1}{L^r_1} i^r_{1\beta} + \nu\left(\frac{M}{L^r_1} i^s_\alpha + i^r_{1\alpha} + \frac{M^r}{L^r_1} i^r_{2\alpha}\right) \\
&\quad - \frac{M}{L^r_1}\frac{di^s_\beta}{dt} - \frac{M^r}{L^r_1}\frac{di^r_{2\beta}}{dt} \\
\frac{di^r_{2\alpha}}{dt} &= -\frac{R^r_2}{L^r_2} i^r_{2\alpha} - \nu\left(\frac{M^r}{L^r_2} i^s_\beta + i^r_{2\beta} + \frac{M}{L^r_2} i^s_{1\beta}\right) \\
&\quad - \frac{M}{L^r_2}\frac{di^s_\alpha}{dt} - \frac{M^r}{L^r_2}\frac{di^r_{1\alpha}}{dt} \\
\frac{di^r_{2\beta}}{dt} &= -\frac{R^r_2}{L^r_2} i^r_{2\beta} + \nu\left(\frac{M}{L^r_2} i^s_\alpha + i^r_{2\alpha} + \frac{M^r}{L^r_2} i^r_{1\alpha}\right) \\
&\quad - \frac{M^r}{L^r_2}\frac{di^s_\beta}{dt} - \frac{M^r}{L^r_2}\frac{di^s_{1\beta}}{dt}
\end{aligned}
\tag{6.11}
$$

The equation of motion is

$$(d\nu/dt) = [1/(J/p)][(mp/2)M(i^s_\beta i^r_{1\alpha} - i^s_\alpha i^r_{1\beta} + i^s_\beta i^r_{2\alpha} - i^s_\alpha i^r_{2\beta}) - M_r] \tag{6.12}$$

In Eqs. (6.11) and (6.12), M is the mutual inductance between the stator and rotor windings, and M^r is the mutual inductance between rotor windings.

The inductances of stator and rotor windings are

$$L^s = M + l^s_\sigma, \ \ L^r_1 = M + l^r_{1\sigma}, \ \ L^r_2 = M + l^r_{2\sigma} \tag{6.13}$$

where l^s_σ, $l_{1\sigma}^r$, and $l^r_{2\sigma}$ are the leakage inductances of stator and rotor windings.

It can be shown that for the steady state, differential equations

(6.11) convert to complex equations for the phase of a double-cage induction machine. Replacing in voltage equations (6.11) the differential operator by $j\omega$, we obtain for the steady-state performance

$$\begin{aligned}
\dot{U}^s_\alpha &= R^s\dot{I}^s_\alpha + j\omega L^s\dot{I}^s_\alpha + j\omega M\dot{I}^r_{1\alpha} + j\omega M\dot{I}^r_{2\alpha} \\
\dot{U}^s_\beta &= R^s\dot{I}^s_\beta + j\omega L^s\dot{I}^s_\beta + j\omega M\dot{I}^r_{1\beta} + j\omega M\dot{I}^r_{2\beta} \\
-\dot{U}^r_{1\alpha} &= R^r_1\dot{I}^r_{1\alpha} + j\omega L^r_1\dot{I}^r_{1\alpha} + j\omega M\dot{I}^s_\alpha + M\dot{I}^s_\beta\nu + L^r_1\dot{I}^r_{1\beta}\nu \\
&\quad + M^r\dot{I}^r_{2\beta}\nu + j\omega M^r\dot{I}^r_{2\alpha} \\
-\dot{U}^r_{1\beta} &= R^r_1\dot{I}^r_{1\beta} + j\omega L^r_1\dot{I}^r_{1\beta} + j\omega M\dot{I}^s_\beta - M\dot{I}^s_\alpha\nu \\
&\quad - L^r_1\dot{I}^r_{1\alpha}\nu - M^r\dot{I}^r_{2\alpha}\nu + j\omega M^r\dot{I}^r_{2\beta} \qquad (6.14) \\
-\dot{U}^r_{2\alpha} &= R^r_2\dot{I}^r_{2\alpha} + j\omega L_2\dot{I}^r_{2\alpha} + j\omega M\dot{I}^s_\alpha + M\dot{I}^s_\beta\nu \\
&\quad + L^r_2\dot{I}^r_{2\beta}\nu + M^r\dot{I}^r_{1\beta}\nu + j\omega M^r\dot{I}^r_{1\alpha} \\
-\dot{U}^r_{2\beta} &= R^r_2\dot{I}^r_{2\beta} + j\omega L^r_2\dot{I}^r_{2\beta} + j\omega M\dot{I}^s_\beta - M\dot{I}_\alpha\nu \\
&\quad - L^r_2\dot{I}^r_{2\alpha}\nu - M^r\dot{I}^r_{1\alpha}\nu + j\omega M^r\dot{I}^r_{1\beta}
\end{aligned}$$

Substitute relations (6.13) into (6.14), next multiply and divide the terms in rotor circuit equations for the emf of rotation by the angular frequency of the supply line voltage. We finally get

$$\begin{aligned}
\dot{U}^s_\alpha &= R^s\dot{I}^s_\alpha + jx_1\dot{I}^s_\alpha + jx_0\dot{I}^s_\alpha + jx_0\dot{I}^r_{1\alpha} + jx_0\dot{I}^r_{2\alpha} \\
\dot{U}^s_\beta &= R^s\dot{I}^s_\beta + jx_1\dot{I}^s_\beta + jx_0\dot{I}^s_\beta + jx_0\dot{I}^r_{1\beta} + jx_0\dot{I}^r_{2\beta} \\
-\dot{U}^r_{1\alpha} &= R^r_1\dot{I}^r_{1\alpha} + jx_2\dot{I}^r_{1\alpha} + jx_0\dot{I}^r_{1\alpha} + jx_0\dot{I}^s_\alpha + x_0\dot{I}^s_\beta\nu' \\
&\quad + (x_2 + x_0)\dot{I}^r_{1\beta}\nu' + jx'\dot{I}^r_{2\alpha} + jx_0\dot{I}^r_{2\alpha} + x'\dot{I}^r_{2\beta}\nu' + x_0\dot{I}^r_{2\beta}\nu' \\
-\dot{U}^r_{1\beta} &= R^r_1\dot{I}^r_{1\beta} + jx_2\dot{I}^r_{1\beta} + jx_0\dot{I}^r_{1\beta} + jx_0\dot{I}^s_\beta - x_0\dot{I}^s_\alpha\nu' \qquad (6.15) \\
&\quad - (x_2 + x_0)\dot{I}^r_{1\alpha}\nu' + jx'\dot{I}^r_{2\beta} + jx_0\dot{I}^r_{2\beta} - x'\dot{I}^r_{2\alpha}\nu' - x_0\dot{I}^r_{2\alpha}\nu' \\
-\dot{U}^r_{2\alpha} &= R^r_2\dot{I}^r_{2\alpha} + jx_3\dot{I}^r_{2\alpha} + jx_0\dot{I}^r_{2\alpha} + jx_0\dot{I}^s_\alpha + x_0\dot{I}^s_\beta\nu' \\
&\quad + (x_3 + x_0)\dot{I}^r_{2\beta}\nu' + jx'\dot{I}^r_{1\alpha} + jx_0\dot{I}^r_{1\alpha} + x'\dot{I}^r_{1\beta}\nu' + x_0\dot{I}^r_{1\beta}\nu' \\
-\dot{U}^r_{2\beta} &= R^r_2\dot{I}^r_{2\beta} + jx_3\dot{I}^r_{2\beta} + jx_0\dot{I}^r_{2\beta} + jx_0\dot{I}^s_\beta - x_0\dot{I}^s_\alpha\nu' \\
&\quad - (x_3 + x_0)\dot{I}^r_{2\alpha}\nu' + jx'\dot{I}^r_{1\beta} + jx_0\dot{I}^r_{1\beta} - x'\dot{I}^r_{1\alpha}\nu' - x_0\dot{I}^r_{1\alpha}\nu'
\end{aligned}$$

In Eqs. (6.15) we put

$x_1 = \omega l^s_\sigma$, $x_2 = \omega l^r_{1\sigma}$, $x_3 = \omega l^r_{2\sigma}$, $x_0 = \omega M$, $\nu' = \nu/\omega$, $\nu = d\theta/dt$

where x' is the reactance of mutual induction between the rotor windings.

Considering that

$$\dot{I}^s_\alpha = j\dot{I}^s_\beta, \quad \dot{I}^r_{1\alpha} = j\dot{I}^r_{1\beta}, \quad \dot{I}^r_{2\alpha} = j\dot{I}^r_{2\beta} \tag{6.16}$$

the processes in the symmetric mode of operation can only be treated for one phase of the machine. So, substituting relations (6.16) into (6.14) for one phase yields

$$\begin{aligned}
\dot{U}^s &= R^s\dot{I}^s + jx_1\dot{I}^s + jx_0(\dot{I}^s + \dot{I}^r_1 + \dot{I}^r_2)\\
-\dot{U}^r_1 &= R^r_1\dot{I}^r_1 + jx_2\dot{I}^r_1(1-\nu') + jx_0(1-\nu')\dot{I}^r_1 + jx_0(1-\nu')\dot{I}^s\\
&\quad + jx'(1-\nu')\dot{I}^r_2 + jx_0(1-\nu')\dot{I}^r_2\\
-\dot{U}^r_2 &= R^r_2\dot{I}^r_2 + jx_3\dot{I}^r_2(1-\nu') + jx_0(1-\nu')\dot{I}^r_2 + jx_0(1-\nu')\dot{I}^s\\
&\quad + jx'(1-\nu')\dot{I}^r_1 + jx_0(1-\nu')\dot{I}^r_1
\end{aligned} \tag{6.17}$$

After transforming the rotor windings to the stator winding and introducing the magnetizing current $\dot{I}_0 = \dot{I}^s + \dot{I}^{r\prime}_1 + \dot{I}^{r\prime}_2$ in Eqs. (6.14), we have

$$\begin{aligned}
\dot{U}^s &= R^s\dot{I}^s + jx_1\dot{I}^s + jx_0\dot{I}_0\\
0 &= R^{r\prime}_1\dot{I}^{r\prime}_1 + jx'_2\dot{I}^{r\prime}_1 s + jx_0 s\dot{I}_0 + jx' s\dot{I}^{r\prime}_2\\
0 &= R^{r\prime}_2\dot{I}^{r\prime}_2 + jx'_3\dot{I}^{r\prime}_2 s + jx_0 s\dot{I}_0 + jx' s\dot{I}^{r\prime}_1
\end{aligned} \tag{6.18}$$

Here s is the slip.

By performing appropriate transformations, from Eqs. (6.18) we obtain the following system of equations for the classical circuit model of a double-cage induction motor:

$$\begin{aligned}
U^s &= I_0 z_0 + I^s z_1\\
0 &= \dot{I}_0 z_0 + \dot{I}^r_1 z'_2 + \dot{I}^{r\prime}_1 R^{r\prime}_1 (1-s)/s + jx'\dot{I}^{r\prime}_2\\
0 &= \dot{I}_0 z_0 + \dot{I}^{r\prime}_2 z'_3 + \dot{I}^{r\prime}_2 R^{r\prime}_2 (1-s)/s + jx'\dot{I}^{r\prime}_1\\
\dot{I}_0 &= \dot{I}^s + \dot{I}^{r\prime}_1 + \dot{I}^{r\prime}_2
\end{aligned} \tag{6.19}$$

In solving the equations for the machine with two rotor windings on an analog computer, it is advisable to use the model expressed in terms of currents since this model is most preferable to the analysis of machines with varying parameters. A more stable model expressed in terms of flux linkages becomes impracticable for the purpose since a change that is to be made in any one of the inductive reactances calls for recalculating all the coefficients in the equations and rearranging the same number of gain factors on the model.

The equations below correspond to the model of an induction machine with two windings on the rotor

$$i^s_\alpha = \frac{(1/L^s)\, U^s_{\alpha\,\max}\cos\omega_0 t - (R^s/L^s)\, i^s_\alpha}{p} - \frac{M}{L^s}\, i^r_{1\alpha} - \frac{M}{L^s}\, i^r_{2\alpha}$$

$$i^s_\beta = \frac{(1/L^s)\, U^s_{\beta\,\max}\sin\omega_0 t - (R^s/L^s)\, i^s_\beta}{p} - \frac{M}{L^s}\, i^r_{1\beta} - \frac{M}{L^s}\, i^r_{2\beta}$$

$$i^r_{1\alpha} = \frac{-(R^r_1/L^r_1)\, i^r_{1\alpha} - \nu B}{p} - \frac{M}{L^r_1}\, i^s_\alpha - \frac{M}{L^r_1}\, i^r_{2\alpha}$$

$$i^r_{1\beta} = \frac{-(R^r_1/L^r_1)\, i^r_{1\beta} + \nu A}{p} - \frac{M}{L^r_1}\, i^s_\beta - \frac{M}{L^r_1}\, i^r_{2\beta}$$

$$i^r_{2\alpha} = \frac{-(R^r_2/L^r_2)\, i^r_{2\alpha} + \nu D}{p} - \frac{M}{L^r_2}\, i^r_{1\alpha} - \frac{M}{L^r_2}\, i^s_\alpha$$

$$i^r_{2\beta} = \frac{-(R^r_2/L^r_2)\, i^r_{2\beta} + \nu C}{p} - \frac{M}{L^r_2}\, i^r_{1\beta} - \frac{M}{L^r_2}\, i^s_\beta$$

$$B = i^r_{1\beta} + (M/L^r_1)\, i^r_{2\beta} + (M/L^r_1)\, i^s_\beta$$

$$A = i^r_{1\alpha} + (M/L^r_1)\, i^r_{2\alpha} + (M/L^r_1)\, i^s_\alpha$$

$$D = i^r_{2\beta} + (M/L^r_2)\, i^r_{1\beta} + (M/L^r_2)\, i^s_\beta$$

$$C = i^r_{2\alpha} + (M/L^r_2)\, i^r_{1\alpha} + (M/L^r_2)\, i^s_\alpha$$

$$M_e = (mp/2)\, M\, (i^s_\beta i^r_{1\alpha} - i^s_\alpha i^r_{1\beta}) + (mp/2)\, M\, (i^s_\beta i^r_{2\alpha} - i^s_\alpha i^r_{2\beta})$$

$$\frac{d\nu}{dt} = \frac{1}{J/p}(M_e - M_r) \qquad (6.20)$$

An analog computer is suitable for the analysis of a machine with two windings on the rotor only if the winding parameters do not vary. In studying the processes within a wide range of changes in the slip, it is advantageous to choose the parameters for the slip which conforms to the initial stage of the transient. Where the purpose is to determine the functional relations between the static and dynamic characteristics describing the transient, it makes sense to apply the experiment planning technique to the model. A digital computer offers the possibility of solving the behavior of a machine with varying parameters of the windings.

To start with the solution of equations, we need first to determine the parameters of loops. The parameters of the integral eddy-current loop can be defined proceeding from the identity of its parameters with those of the solid rotor of the same size. Experimen-

tal investigations (the short-circuit test at different frequency of the stator circuit supply voltage, the construction of torque curves for a motor) attest to the identity of the parameters of a rotor having open-circuited cage bars embedded in slots (an integral eddy current loop) with the parameters of a solid rotor of the same size. What accounts for this fact is that the bars placed into slots add to electric conduction of current through individual laminations.

To define the parameters of an integral eddy current loop acting jointly with the main loop, we should apply the expression

$$z_3'^* = z_3' (I_3'/I_3'^*)^{(x-1)/2x} \tag{6.21}$$

which is the first equation establishing the relation between the impedance of the loop and the current through it when this loop acts separately (z_3', I_3') and together with the main loop ($z_3'^*$, $I_3'^*$). Here x is the order of the $B = kH^{1/x}$ parabola used to approximate the main B-H curve for a ferromagnetic material.

The second equation for the impedance $z_3'^*$ and current

$$I_3'^* = \frac{U x_\mu z_2'}{(z_1 + z_3'^*)[x_\mu (z_2' + z_1) + z_2' z_1] - z_1^2 (z_2' + x_\mu)} \tag{6.22}$$

is written using the equivalent circuit. The parameters z_1 and z_2' of the stator and the main cage respectively are taken from the calculation data. The value of x_μ is found from the U-shaped curve (this value is close to the calculated value).

Experimental investigations and calculations confirm that the results of experiments compare more favorably with the calculated results if account is taken of rotor eddy currents. The disparity between the electromagnetic torques with and without regard to eddy currents for the A03-24-4 motor comes to about 10% at 50 Hz and to 16% at 400 Hz. The calculations reveal that neglecting the effect of eddy currents introduces a greater calculation error for motors designed to operate at higher frequencies since a more tangible share of eddy currents affects the electromagnetic torque.

Any induction motor should be treated as a multiloop system. The eddy currents in a rotor can be allowed for by adding the integral eddy current loop to the equivalent circuit. The effect of this loop need be given due consideration in the dynamic and steady-state analyses of motors operating within a wide range of changes in the slip and also motors supplied from sources of increased frequency.

Eddy current loops in a core stacked of sheet-steel laminations affect but little the electromagnetic torque of the motor. However, it is the loops formed by the cage bars and rotor core that produce the driving torque. An open-circuited cage corresponds to the integral eddy current loop and is identical to a solid rotor.

The α-β model expressed in terms of currents is most suitable for the analysis of an energy converter with two rotor windings on an analog computer. The program for the digital-computer solution of differential equations of a machine with two rotor windings permits studying the dynamic behavior of the machine having both constant parameters of the windings and parameters functionally varying with time. It is also advisable to use this program for the study of double-cage motors. The equations obtained in this case are cumbersome and become still more so with the addition of a winding on

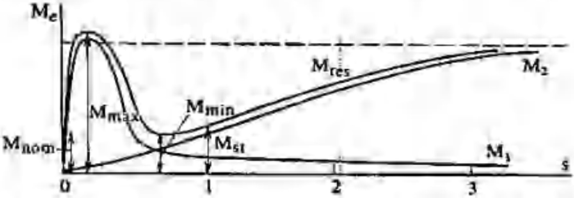


Fig. 6.5. Induction-motor torque characteristic allowing for the effect of the eddy current loop

M_{st}, M_{min}, M_{max}, M_{nom} and M_{res} – starting, minimum, maximum, nominal and resultant torques respectively

the stator to allow for the effect of eddy currents. Imagine the complexity of equations and the transformations involved for a larger number of windings.

The analysis of the equations for an induction machine with two rotor loops leads us to the conclusion that all the variety of mechanical characteristics, M_{res}, reduces to the two-winding motor characteristic M_1 disregarding eddy current loops and to the characteristic M_2 of a motor with a solid rotor (Fig. 6.5).

6.5. The Induction Machine Model Including Stator and Rotor Eddy Currents

As mentioned above, the effect of eddy current loops must be allowed for in solving the dynamic and steady-state behavior of motors operating over a wide range of slip changes and also motors built for voltages at increased frequencies.

The mathematical model of a machine carrying two stator windings and two rotor windings and having a circular field in the air gap is the model of Fig. 3.3. Considering that the mutual inductance between the stator and rotor windings results from M_1 and the mutual inductance between rotor windings from M^r, after appropriate

transformations we get

$$\frac{di_{1\alpha}^s}{dt}=\frac{u_{1\alpha}^s}{L_1^s}-\frac{R_1^s}{L_1^s}i_{1\alpha}^s-\frac{M}{L_1^s}\frac{di_{1\alpha}^r}{dt}-\frac{M}{L_1^s}\frac{di_{2\alpha}^s}{dt}-\frac{M}{L_1^s}\frac{di_{2\alpha}^r}{dt}$$

$$\frac{di_{2\alpha}^s}{dt}=-\frac{R_2^s}{L_2^s}i_{2\alpha}^s-\frac{M}{L_2^s}\frac{di_{1\alpha}^r}{dt}-\frac{M}{L_2^s}\frac{di_{1\alpha}^s}{dt}-\frac{M}{L_2^s}\frac{di_{2\alpha}^r}{dt}$$

$$\frac{di_{1\alpha}^r}{dt}=-\frac{R_1^r}{L_1^r}i_{1\alpha}^r-\nu\left[\frac{M}{L_1^r}i_{1\beta}^s+\frac{M}{L_1^r}i_{1\beta}^s+i_{1\beta}^r+\frac{M^r}{L_1^r}i_{2\beta}^r\right]$$
$$-\frac{M}{L_1^r}\frac{di_{1\alpha}^s}{dt}-\frac{M}{L_1^r}\frac{di_{2\alpha}^s}{dt}-\frac{M^r}{L_1^r}\frac{di_{2\alpha}^r}{dt}$$

$$\frac{di_{2\alpha}^r}{dt}=-\frac{R_2^r}{L_2^r}i_{2\alpha}^r-\nu\left[\frac{M}{L_2^r}i_{1\beta}^s+\frac{M}{L_2^r}i_{2\beta}^s+i_{2\beta}^r+\frac{M_1^r}{L_2^r}i_{1\beta}^r\right]$$
$$-\frac{M}{L_2^r}\frac{di_{1\alpha}^s}{dt}-\frac{M}{L_2^r}\frac{di_{2\alpha}^s}{dt}-\frac{M^r}{L_2^r}\frac{di_{1\alpha}^r}{dt} \tag{6.23}$$

$$\frac{di_{1\beta}^s}{dt}=\frac{u_{1\beta}^s}{L_1^s}-\frac{R_1^s}{L_1^s}i_{1\beta}^s-\frac{M}{L_1^s}\frac{di_{1\beta}^r}{dt}-\frac{M}{L_1^s}\frac{di_{2\beta}^s}{dt}-\frac{M}{L_1^s}\frac{di_{2\beta}^r}{dt}$$

$$\frac{di_{2\beta}^s}{dt}=-\frac{R_2^s}{L_2^s}i_{2\beta}^s-\frac{M}{L_2^s}\frac{di_{1\beta}^r}{dt}-\frac{M}{L_2^s}\frac{di_{1\beta}^s}{dt}-\frac{M}{L_2^s}\frac{di_{2\beta}^r}{dt}$$

$$\frac{di_{1\beta}^r}{dt}=-\frac{R_1^r}{L_1^r}i_{1\beta}^r+\nu\left[\frac{M}{L_1^r}i_{1\alpha}^s+\frac{M}{L_1^r}i_{1\alpha}^s+i_{1\alpha}^r+\frac{M}{L_1^r}i_{2\alpha}^r\right]$$
$$-\frac{M}{L}\frac{di_{1\beta}^s}{dt}-\frac{M}{L_1^r}\frac{di_{2\beta}^s}{dt}-\frac{M_2}{L_1^r}\frac{di_{2\beta}^r}{dt}$$

$$\frac{di_{2\beta}^r}{dt}=-\frac{R_2^r}{L_2^r}i_{2\beta}^r+\nu\left[\frac{M}{L_2^r}i_{1\alpha}^s+\frac{M}{L_2^r}i_{2\alpha}^s+i_{2\alpha}^r+\frac{M^r}{L_2^r}i_{1\alpha}^r\right]$$
$$-\frac{M}{L_2^r}\frac{di_{1\beta}^s}{dt}-\frac{M}{L_2^r}\frac{di_{2\beta}^s}{dt}-\frac{M^r}{L_2^r}\frac{di_{1\beta}^r}{dt}$$

$$\frac{d\nu}{dt}=\frac{1}{J/p}\left[\frac{mp}{2}M\left(i_{1\beta}^s i_{1\alpha}^r-i_{1\alpha}^s i_{1\beta}^r+i_{1\beta}^s i_{2\alpha}^r-i_{1\alpha}^s i_{2\beta}^r\right.\right.$$
$$\left.\left.+i_{2\beta}^s i_{1\alpha}^r-i_{2\alpha}^s i_{1\beta}^r+i_{2\beta}^s i_{2\alpha}^r-i_{2\alpha}^s i_{2\beta}^r\right)-M_r\right] \tag{6.24}$$

Replacing p by $j\omega$ from (6.23) we can obtain the equations for the steady-state operation

$$\dot{u}_1^s=\dot{I}_1^s z_1^s+\dot{I}_m z_m$$
$$\dot{u}_2^s=0=\dot{I}_2^s z_2^s+\dot{I}_m z_m$$
$$-\dot{u}_1^r=0=\dot{I}_1^{r\prime} z_1^{r\prime}+\dot{I}_m z_m+\dot{I}_1^r R_1^{r\prime}(1-s)/s+jx'\dot{I}_2^{r\prime} \tag{6.25}$$
$$-\dot{u}_2^r=0=\dot{I}_2^{r\prime} z_2^{r\prime}+\dot{I}_m z_m+\dot{I}_2^{r\prime} R_2^{r\prime}(1-s)/s+jx'\dot{I}_1^{r\prime}$$
$$\dot{I}_m=\dot{I}_1^s+\dot{I}_2^s+\dot{I}_1^{r\prime}+\dot{I}_2^{r\prime}$$

In Eq. (6.25), the impedances of α and β windings are taken equal, and the currents and voltages bear indexes to identify the first and second stator and rotor windings.

Solving (6.23) and (6.24) on a digital computer, we can estimate the effect of eddy current loops on the dynamic and static modes of operation of an induction machine.

As found from investigations, the effect of a stator eddy-current loop during the period of starting a 7-kW motor is greater than that of a rotor eddy-current loop, but both loops have an equal effect on the impact starting current in the stator winding. Applying the experiment planning technique to the analysis of motors of various powers and with different numbers of poles, we can evaluate the effect of eddy current loops in the stator and rotor on the dynamic and static characteristics of motors.

Where a few loops are involved in the process of energy conversion, of much importance is an accurate determination of the winding parameters, for which purpose a frequency method is advantageous. The parameters of stator eddy-current loops can be found from the value of iron loss. The calculation method that more fully allows for manufacturing factors gives better results. The effect of eddy currents on the characteristics of a machine is accounted for by the interaction of all coefficients entering into Eqs. (6.23) and (6.24). The stator and rotor steel sheet thickness, steel grade, and manufacturing operations are chosen after the analysis of (6.23) and (6.24) and also after consideration of economic factors.

In Eq. (6.23), the voltages on the second loops in the stator and rotor may be other than zero. If the voltage impressed across the stator winding is the same and thus the field in the air gap is circular, the problem reduces to the study of current distribution among the parallel branches of windings. A small discrepancy between the inductive reactances and resistances of parallel branches causes a nonuniform distribution of currents and thus affects the performance of the machine in the steady-state and transient conditions.

6.6. The Effect of Manufacturing Factors on Electric Machine Performance

In the theory of energy converters it is customary to consider the air gap uniform, though this is not the case in a real machine for a variety of manufacturing reasons. The air gap nonuniformity may arise from the eccentricity of a rotor with respect to a stator (Fig. 6.6*a*), rotor ellipticity (Fig. 6.6*b*), rotor and stator conicity (Fig. 6.6*c*), and misalignment (Fig. 6.6*d*). These factors are responsible for additional losses, vibrations, and various errors, so they must be taken into account in the mathematical analysis of energy conversion processes.

For the case shown in Fig. 6.6a, b, a multipolar machine structure can be broken down along its length into a few elementary machines with different air gaps and into m machines around the gap circumference. For the case of Fig. 6.6c, d, the structure can be broken apart

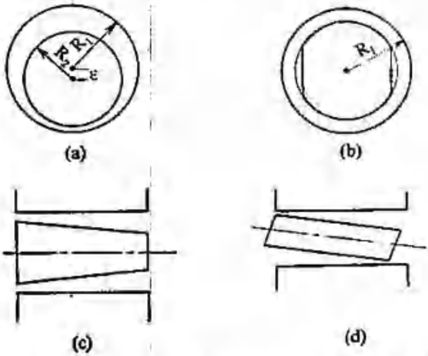


Fig. 6.6. Gap nonuniformity due to manufacturing factors

into n machines with different air gaps along its length. If the multipolar machine of Fig. 6.6a, b has parallel branches, the current distribution over the elementary machines becomes nonuniform.

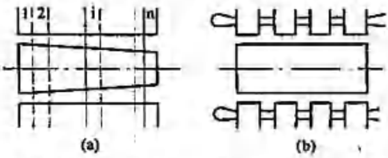


Fig. 6.7. Typical nonuniformities over the machine length
(a) tapered rotor; (b) stator divided into pole cores

For the case of Fig. 6.6c, d, the elementary machines are connected in series and the voltages are distributed nonuniformly.

Consider a machine with a nonuniform air gap (Fig. 6.7) and divide it into n pieces throughout its length. A usual approach to designing a machine with its core divided into pieces over a uniform air gap is similar to the case under study since the extreme core portions and the middle core portions operate under different conditions.

As a first approximation, assume that the machine has n stators and a common rotor. Supposing that there is no link between n stators, the voltage equations then take on the form

$$\begin{vmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_i \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{vmatrix} = \begin{vmatrix} \square & & & & & & & \\ & \square & & & & & & \\ & & \cdot & & & & & \\ & & & \cdot & & & & \\ & & & & \square & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & & \square \end{vmatrix} \times \begin{vmatrix} i_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_i \\ \cdot \\ \cdot \\ \cdot \\ i_n \end{vmatrix} \tag{6.26}$$

In Eqs. (6.26), u_i and i_i are the voltage and current matrices of the ith machine. In the impedance matrix of (6.26), the squares denote the impedance matrices of a machine with a circular field. Each impedance matrix contains corresponding parameters. The electromagnetic torque here is equal to the sum of products of currents in each elementary machine

$$M_e = M_1 + M_2 + \ldots + M_i + \ldots + M_n \tag{6.27}$$

Torque equation (6.27) for the common-rotor machine model includes pairwise products of currents in the n stators and the rotor apart from the products of currents in each elementary machine. In the common-rotor model, the rotor establishes the link between n machines.

For the series-connected elementary machines operating in the steady-state conditions, the line voltage is

$$\dot{U}_l = \dot{U}_1 + \dot{U}_2 + \ldots + \dot{U}_n \tag{6.28}$$

and for the parallel-connected machines, the line current is

$$\dot{I}_l = \dot{I}_1 + \dot{I}_2 + \ldots + \dot{I}_n \tag{6.29}$$

where $\dot{U}_1$, $\dot{U}_2$, . . ., $\dot{U}_n$ and $\dot{I}_1$, $\dot{I}_2$, . . ., $\dot{I}_n$ are the voltages and currents in elementary machines.

If, apart from the linkage between the elementary pieces due to the rotor current, we consider the linkage resulting from the machine

saturation, the impedance matrix in (6.26) will be filled completely:

$$
\begin{vmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{vmatrix}
=
\begin{vmatrix}
\underline{\square} & \square & \cdots & \square & \cdots & \square \\
\square & \underline{\square} & \cdots & \square & \cdots & \square \\
\vdots & \vdots & & \vdots & & \vdots \\
\square & \square & \cdots & \underline{\square} & \cdots & \square \\
\vdots & \vdots & & \vdots & & \vdots \\
\square & \square & \cdots & \square & \cdots & \underline{\square}
\end{vmatrix}
\times
\begin{vmatrix} i_1 \\ i_2 \\ \vdots \\ i_i \\ \vdots \\ i_n \end{vmatrix}
\qquad (6.30)
$$

The torque will then contain not only the products of currents in the stator and rotor but also the products of currents with different signs:

$$
M_e = M_1 + M_2 + \ldots + M_i + \ldots + M_n + M_{12} + \ldots + M_{1i} + \ldots + M_{1n} + \ldots + M_{(n-1)n} \qquad (6.31)
$$

Equations (6.30) and (6.31) are similar in structure to the equations for an *m-n* winding machine. The equations for the generalized energy converter permit the analysis of energy conversion processes in electric machines with due regard for manufacturing factors.

Machining the stator and rotor can also affect the characteristics of an energy converter. This factor can be allowed for by considering the differences between the parameters of elementary machines or the presence of eddy current loops, as is done in Secs. 6.4 and 6.5.

A set of various manufacturing factors commonly determine the performance of a machine, and consideration for each factor in the set makes the analysis a very difficult problem. As noted above, the equations for the *m-n* winding machine permit the study of most of these factors, each separately and a few simultaneously in one combination or another.

Equations (6.26) and (6.27) apply to the analysis of print-winding machines in which the field distribution at the end portions around the periphery differs from that close to the center of the air gap.

Chapter 7

Models of Electric Machines with Nonlinear Parameters

7.1. The Analysis of Electric Machines with Nonlinear Parameters

As noted earlier, the equations of electromechanical energy conversion with constant coefficients are nonlinear equations since they contain the products of variables. The analytical solutions to these equations do not exist if ω_r undergoes changes. Consider the effect of nonlinear coefficients in the electromechanical equations on the processes of energy conversion in electric machines, namely, the coefficients L, M, l_σ, r_1, r_2, and J and independent variables u, f, and M_r. All coefficients entering into the equations can be nonlinear. The resistance of a rotor changes with current displacement, and that of a stator with heat. The inductive reactance depends on saturation. The moment of inertia in some drives is a function of the angular speed.

The parameters depend on voltages, load, and other factors, but in general they are functions of time as is clear from Eqs. (7.1), (7.2), and (7.3):

$$\begin{vmatrix} u_\alpha^s \\ u_\alpha^r \\ u_\beta^r \\ u_\beta^s \end{vmatrix} = \begin{vmatrix} r_1(t) + \frac{d}{dt}L_1(t) & \frac{d}{dt}M(t) & 0 & 0 \\ \frac{d}{dt}M(t) & r_2(t) + \frac{d}{dt}L_2(t) & L_2(t)\,\omega_r & M(t)\,\omega_r \\ -M(t)\,\omega_r & -L_2(t)\,\omega_r & r_2(t) + \frac{d}{dt}L_2(t) & \frac{d}{dt}M(t) \\ 0 & 0 & \frac{d}{dt}M(t) & r_1(t) + \frac{d}{dt}L_1(t) \end{vmatrix} \times \begin{vmatrix} i_\alpha^s \\ i_\alpha^r \\ i_\beta^r \\ i_\beta^s \end{vmatrix} \tag{7.1}$$

$$M_e = M(t)\,(i_\beta^s i_\alpha^r - i_\alpha^s i_\beta^r) \tag{7.2}$$

$$d\omega_r/dt = [p/J(t)]\,[M_e - M_r(t)] \tag{7.3}$$

The model of an energy converter, as shown in Fig. 7.1, corresponds to Eqs. (7.1), (7.2), and (7.3).

$$
\begin{vmatrix}
u^s_{1\alpha} \\ u^s_{2\alpha} \\ \cdot \\ u^s_{m\alpha} \\ u^r_{1\alpha} \\ u^r_{2\alpha} \\ \cdot \\ u^r_{m\alpha} \\ u^r_{1\beta} \\ u^r_{2\beta} \\ \cdot \\ u^r_{m\beta} \\ u^s_{1\beta} \\ u^s_{2\beta} \\ \cdot \\ u^s_{m\beta}
\end{vmatrix}
=
\begin{vmatrix}
r^s_{1\alpha} + \frac{d}{dt} L^s_{1\alpha} & \cdots & \frac{d}{dt} M^s_{1m\alpha} & \frac{d}{dt} M^{sr}_{11\alpha} & \cdots & \frac{d}{dt} M^{sr}_{1m\alpha} \\
\frac{d}{dt} M^s_{21\alpha} & \cdots & \frac{d}{dt} M^s_{2m\alpha} & \frac{d}{dt} M^{sr}_{21\alpha} & \cdots & \frac{d}{dt} M^{sr}_{2m\alpha} \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
\frac{d}{dt} M^s_{m1\alpha} & \cdots & r^s_{m\alpha} + \frac{d}{dt} L^s_{m\alpha} & \frac{d}{dt} M^{sr}_{m1\alpha} & \cdots & \frac{d}{dt} M^{sr}_{mm\alpha} \\
\frac{d}{dt} M^{rs}_{11\alpha} & \cdots & \frac{d}{dt} M^{rs}_{1m\alpha} & r^r_{1\alpha} + \frac{d}{dt} L_1 & \cdots & \frac{d}{dt} M^r_{m\alpha} \\
\frac{d}{dt} M^{rs}_{21\alpha} & \cdots & \frac{d}{dt} M^{rs}_{2m\alpha} & \frac{d}{dt} M^r_{21\alpha} & \cdots & \frac{d}{dt} M^r_{2m\alpha} \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
\frac{d}{dt} M^{rs}_{m1\alpha} & \cdots & \frac{d}{dt} M^{rs}_{mm\alpha} & \frac{d}{dt} M^r_{m1\alpha} & \cdots & r^r_{m\alpha} + \frac{d}{dt} L^r_{m\alpha} \\
-M_{11\alpha}\omega_r & \cdots & -M_{1m\alpha}\omega_r & -L^r_{1\alpha}\omega_r & \cdots & -M^r_{1m\alpha}\omega_r \\
-M_{21\alpha}\omega_r & \cdots & -M_{2m\alpha}\omega_r & -M^r_{21\alpha}\omega_r & \cdots & -M^r_{2m\alpha}\omega_r \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
-M_{m1\alpha}\omega_r & \cdots & -M_{mm\alpha}\omega_r & -M^r_{m1\alpha}\omega_r & \cdots & -L^r_{m\alpha}\omega_r \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
0 & \cdots & 0 & 0 & \cdots & 0
\end{vmatrix}
$$

The nonlinearity of the parameters of an energy converter operating at sinusoidal voltage is responsible for the emergence of a harmonic spectrum in the air gap.

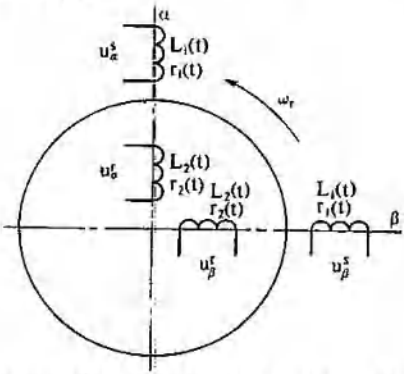


Fig. 7.1. The model of an electric machine with nonlinear parameters

The mathematical model of the processes of energy conversion in the air gap of a symmetric machine having two stator windings and

$$
\left.\begin{matrix}
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
L^r_{1\beta}\omega_r & \cdots & M^r_{m\beta}\omega_r & M_{11\beta}\omega_r & \cdots & M_{1m\beta}\omega_r \\
M^r_{21\beta}\omega_r & \cdots & M^r_{2m\beta}\omega_r & M_{21\beta}\omega_r & \cdots & M_{2m\beta}\omega_r \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
M^r_{m1\beta}\omega_r & \cdots & L^r_{m\beta}\omega_r & M_{m1\beta}\omega_r & \cdots & M_{mm\beta}\omega_r \\
r^r_{1\beta}+\frac{d}{dt}L^r_{1\beta} & \cdots & \frac{d}{dt}M^r_{1m\beta} & \frac{d}{dt}M^{rs}_{11\beta} & \cdots & \frac{d}{dt}M^{rs}_{1m\beta} \\
\frac{d}{dt}M^r_{21\beta} & \cdots & \frac{d}{dt}M^r_{2m\beta} & \frac{d}{dt}M^{rs}_{21\beta} & \cdots & \frac{d}{dt}M^{rs}_{2m\beta} \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
\frac{d}{dt}M^r_{m1\beta} & \cdots & r^r_{m\beta}+\frac{d}{dt}L^r_{m\beta} & \frac{d}{dt}M^{rs}_{m1\beta} & \cdots & \frac{d}{dt}M^{rs}_{mm\beta} \\
\frac{d}{dt}M^{sr}_{11\beta} & \cdots & \frac{d}{dt}M^{sr}_{1m\beta} & r^s_{1\beta}+\frac{d}{dt}L^s_{1\beta} & \cdots & \frac{d}{dt}M^s_{1m\beta} \\
\frac{d}{dt}M^{sr}_{21\beta} & \cdots & \frac{d}{dt}M^{sr}_{2m\beta} & \frac{d}{dt}M^s_{21\beta} & \cdots & \frac{d}{dt}M^s_{2m\beta} \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\
\frac{d}{dt}M^{sr}_{m1\beta} & \cdots & \frac{d}{dt}M^{sr}_{mm\beta} & \frac{d}{dt}M^s_{m1\beta} & \cdots & r^s_{m\beta}+\frac{d}{dt}L^s_{m\beta}
\end{matrix}\right\| \times \left\|\begin{matrix}
i^s_{1\alpha} \\ i^s_{2\alpha} \\ \cdot \\ i^s_{m\alpha} \\ i^r_{1\alpha} \\ i^r_{2\alpha} \\ \cdot \\ i^r_{m\alpha} \\ i^r_{1\beta} \\ i^r_{2\beta} \\ \cdot \\ i^r_{m\beta} \\ i^s_{1\beta} \\ i^s_{2\beta} \\ \cdot \\ i^s_{m\beta}
\end{matrix}\right\| \tag{7.4}
$$

two rotor windings and operating from the symmetric supply voltage source is the m-n winding machine model of Fig. 3.2.

Each harmonic of the field can be set up on the model by choosing a pair of windings on the stator or rotor and applying to their terminals appropriately phase-shifted sinusoidal voltages of corresponding amplitudes and frequencies.

Voltage equations (7.4) describe the model of Fig. 3.2. The torque equation follows from (3.12) by substituting m for n.

The nonlinearity of at least one of the coefficients in the electromechanical equations gives rise to an infinite spectrum of field harmonics, and the equations become similar to those for the generalized m-n winding energy converter. However, the model here has the same number of windings both on the stator and on the rotor, and the determination of links between harmonics (between fictitious windings in the model) differs with each parameter. With a change of the load or voltage on the terminals of an energy converter, the couplings between harmonics (mutual inductances in the equations) undergo changes too.

Thus, the analysis of an electric machine with nonlinear parameters is possible by use of the two notations for the electromechanical equations. Specifying the parameters as functions of currents or

time, or other factors, we can implement these functions on the nonlinear units of an analog computer or realize them in the form of tables on a digital computer and then solve Eqs. (7.1) to (7.3). A second approach is to choose the required number of harmonics in the model of the m-winding machine and solve Eqs. (7.4) on an analog or digital computer using constant coefficients or coefficients varying with load, temperature, voltage, etc. (see below).

The equations for the harmonics to be chosen in each case are cumbersome, but they offer large possibilities for the study of systems. For example, they permit us to determine the effect of each harmonic on the torque being produced, to consider the products of currents due to various harmonics, to vary coupling, etc. Eqs. (7.1) to (7.3) are more readily solvable but they do not allow for an easy assessment of many peculiarities.

The equations with nonlinear coefficients are not amenable to an accurate solution as is seen from the analysis of the system of equations (7.4). However, taking into account some nonlinearities and solving the equations on a digital computer, it is possible to obtain the result to a desired accuracy.

7.2. The Effect of Saturation

For most rotating machines, the operating point lies on the nonlinear branch of the B-H curve. The saturation of an energy converter varies with voltage, frequency, and load, thereby affecting the machine's output characteristics. With the saturation being taken into account, in a first approximation the magnetization M is taken to depend on the magnetizing current or time. If $M = f_1(t)$, then $L = f_2(t)$ since $L = M + l_{\sigma}$. We may assume here that the leakage inductance l_{σ} is independent of saturation because the leakage flux ends on itself in the air and accounts for a small share of the working flux. We may also make one more assumption that L and M vary in the same manner

$$L^s(t) = M(t) + l_{\sigma s}, \quad L^r(t) = M(t) + l_{\sigma r} \tag{7.5}$$

Then

$$\begin{aligned} \Psi^s_\alpha &= L^s(t)\, i^s_\alpha + M(t)\, i^r_\alpha, \quad \Psi^s_\beta = L^s(t)\, i^s_\beta + M(t)\, i^r_\beta \\ \Psi^r_\alpha &= L^r(t)\, i^r_\alpha + M(t)\, i^s_\alpha, \quad \Psi^r_\beta = L^r(t)\, i^r_\beta + M(t)\, i^s_\beta \end{aligned} \tag{7.6}$$

To simplify the model for the solution of equations with nonlinear coefficients L and M and thus to cut down the number of products, let us introduce new variables

$$i_{m\alpha} = i^s_\alpha + i^r_\alpha, \quad i_{m\beta} = i^s_\beta + i^r_\beta \tag{7.7}$$

At $M(t)$, the equations then assume the form

$$
\begin{aligned}
u_\alpha^s &= [R^s + (d/dt)\, l_{\sigma s}]\, i_\alpha^s + (d/dt)\, M(t)\, i_{m\alpha} \\
u_\beta^s &= [R^s + (d/dt)\, l_{\sigma s}]\, i_\beta^s + (d/dt)\, M(t)\, i_{m\beta} \\
u_\alpha^r &= [R^r + (d/dt) l_{\sigma r}]\, i_\alpha^r + (d/dt)\, M(t)\, i_{m\alpha} \\
&\quad + \omega_r\, [L^r i_\beta^r + M(t)\, i_{m\beta}] \\
u_\beta^r &= [R^r + (d/dt)\, l_{\sigma r}]\, i_\beta^r + (d/dt)\, M(t)\, i_{m\beta} \\
&\quad - \omega_r\, [L^r i_\alpha^r + M(t)\, i_{m\alpha}]
\end{aligned}
\tag{7.8}
$$

The torque equation is written as

$$M_e = pM(t)\, [i_{m\beta} i_\alpha^r - i_{m\alpha} i_\beta^r] \tag{7.9}$$

Considering a nonlinear change in the leakage inductances, we should transform the equations in view of the fact that

$$L^s(t) = M + l_{\sigma s}(t), \quad L^r(t) = M + l_{\sigma r}(t) \tag{7.10}$$

The equation for u_α^s then takes on the form

$$u_\alpha^s = \left[R^s + \frac{d}{dt}\, l_{\sigma s}(t)\right] i_\alpha^s + M \frac{d}{dt}\, i_{m\alpha} \tag{7.11}$$

In a similar way we transform the equations for u_α^r, u_β^r, and u_α^s, u_β^s. The torque equation is given by (7.9).

The analysis of (7.1) through (7.4), (7.6) and (7.11) on analog and digital computers reveals that the leakage inductive reactances have a greater effect on the impact currents, impact torques, and starting time than the reactance of mutual induction. The pattern of variations of M and l_σ has a smaller effect on the dynamic characteristics. The values of the parameters at the initial stage of the transient process play a dominant part. In a first approximation, therefore, we can disregard variations of M and l_σ, and solve the equations with constant coefficients, substituting into them the saturation values of the parameters, which determine the static characteristics at the end of the transient.

At its starting, a machine first draws power from the line (during one or two periods) necessary to accelerate from rest, then the machine and the line exchange energy. Depending on the combination of parameters, the rotor may reach the speed in excess of the synchronous speed (at a small moment of inertia) or slowly gain the steady-state velocity (at a large moment of inertia). Motors supplied from hf voltage sources and motors with large moments of inertia have similar starting characteristics.

In Sec. 7.1 we have discussed Eqs. (7.4) for a saturated machine, which are set up to define an infinite series of harmonics in the air gap. Consideration of the interrelations between harmonics presents a complicated problem. Let us illustrate the way of determining

these interrelations by an example of a transformer. With a harmonic voltage applied to the transformer input, the inductances of windings can be written as functions of time

$$L_1(t) = L_0 + L_1 \cos(\omega t + \alpha_1) + L_2 \cos(2\omega t + \alpha_2) + \dots \quad (7.12)$$

$$M(t) = M_0 + M_1 \cos(\omega t + \alpha_1) + M_2 \cos(2\omega t + \alpha_2) + \dots \quad (7.13)$$

The transformer equations are considered here as the equations with periodic coefficients. These equations do not however reflect

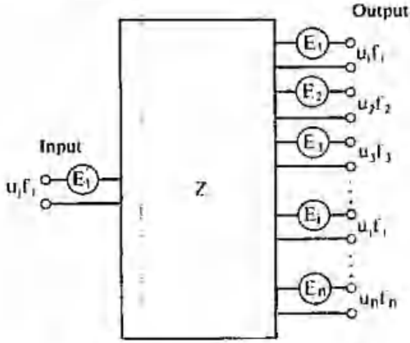


Fig. 7.2. The nonlinear transformer as a linear multiport

fully the processes in the nonlinear transformer because in the circuits with nonlinear parameters there exists an interaction between the components of the harmonic series.

The nonlinear (saturated) transformer is a generator of upper harmonics. Such a transformer can be represented as a linear multiport (Fig. 7.2) with a sinusoidal voltage of one frequency (u_1, f_1) supplied to its input terminals and a spectrum of harmonics at its output terminals:

$$u_1, f_1;\ u_2, f_2;\ \dots;\ u_i, f_i;\ \dots;\ u_n, f_n$$

In a nonlinear transformer one or few windings that receive energy exhibit a sinusoidal emf of one frequency; the transfer of this energy and its transformation then occurs not only at the fundamental, but also at upper, lower, and fractional harmonics. In an ideal transformer the sum of incoming energies is equal to the sum of outcoming energies at all frequencies.

The output parameters of an n-port with harmonic voltage sources can be defined by the Z matrix and complex amplitudes of voltages at open-circuited output terminals $E_1, E_2, \ldots, E_l, \ldots, E_n$. The assumption is that any of the windings features a sinusoidal emf and carries currents of only one frequency since the ideal filters inserted into each winding block the currents at other frequencies.

A transformer, as viewed from its output terminals, can be described by the matrix

$$\begin{vmatrix} e_1 \\ e_2 \\ \cdot \\ e_l \\ \cdot \\ e_n \end{vmatrix} = \begin{vmatrix} r_1+\frac{d}{dt}L_1 & \frac{d}{dt}M_{12} & \ldots & \frac{d}{dt}M_{1l} & \ldots & \frac{d}{dt}M_{1n} \\ \frac{d}{dt}M_{21} & r_2+\frac{d}{dt}L_2 & \ldots & \frac{d}{dt}M_{2l} & \ldots & \frac{d}{dt}M_{2n} \\ \cdot & \cdot & & \cdot & & \cdot \\ \frac{d}{dt}M_{l1} & \frac{d}{dt}M_{l2} & \ldots & r_l+\frac{d}{dt}L_l & \ldots & \frac{d}{dt}M_{ln} \\ \cdot & \cdot & & \cdot & & \cdot \\ \frac{d}{dt}M_{n1} & \frac{d}{dt}M_{n2} & \ldots & \frac{d}{dt}M_{nl} & \ldots & r_n+\frac{d}{dt}L_n \end{vmatrix} \times \begin{vmatrix} i_1 \\ i_2 \\ \cdot \\ i_l \\ \cdot \\ i_n \end{vmatrix} \tag{7.14}$$

The impedance matrix has the form

$$z = \begin{vmatrix} z_{11} & z_{12} & \ldots & z_{1l} & \ldots & z_{1n} \\ z_{21} & z_{22} & \ldots & z_{2l} & \ldots & z_{2n} \\ \cdot & \cdot & & \cdot & & \cdot \\ z_{l1} & z_{l2} & \ldots & z_{ll} & \ldots & z_{ln} \\ \cdot & \cdot & & \cdot & & \cdot \\ z_{n1} & z_{n2} & \ldots & z_{nl} & \ldots & z_{nn} \end{vmatrix} \tag{7.15}$$

The square matrix (7.15) describing the internal sources of harmonic voltages will be called *the noise matrix of a transformer*. In matrices (7.14) and (7.15) the terms that have a physical meaning are the equivalent-multiport winding impedances $z_{11}, z_{22}, \ldots, z_{ll}, \ldots, z_{nn}$ lying on the principal diagonals. The remaining z-matrix terms $M_{12}, M_{21}, \ldots, M_{lm}, \ldots, M_{ln}$, which describe the interaction of harmonics in a saturated system, will be termed *the coefficients of coupling* between the harmonics of different frequences. This interaction of harmonics is only present in a nonlinear system.

The analytical determination of the coefficients of coupling between harmonics presents great difficulties, for the calculation procedure necessitates the analytical expression of the magnetization curve. It is therefore more advantageous to resort to the graphical

method of evaluating the interaction between harmonics. Referring to the B-H graph for the sinusoidal voltage of the fundamental (Fig. 7.3), the area of triangle OAC with a curvilinear side OC is proportional to the mutual induction between the windings. But if there is no saturation, the third harmonic is one-third of the first in amplitude, the fifth is one-fifth, etc. Where the B-H relation is linear, upper harmonics are absent. Proceeding from the above, we estimate approximately the coefficient of coupling between the first and the third harmonic:

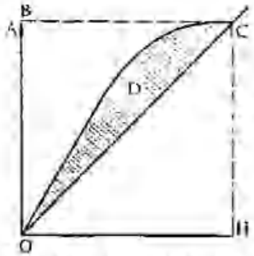


Fig. 7.3. Determining the coefficients of coupling between harmonics

$$M_{13} = (1/3)\, M_{11} D/\Delta OAC = (1/3)\, kM_{11} \tag{7.16}$$

where D is the area shown hatched in the figure; ΔOAC is the area of the rectangular triangle OAC; and M_{11} is the mutual inductance between the windings of a nonsaturated transformer.

Defining the interrelation between the first and the third, the first and the ith harmonic, we can find that

$$M_{1i} = (1/i)\, M_{11} D/\Delta OAC = (1/i)\, kM_{11} \tag{7.17}$$

The coefficient of coupling between the ith and the $(i-1)$th harmonic is

$$M_{(i-1)\,i} = (1/i)\,[1/(i-1)]\, kM_{11} \tag{7.18}$$

The analysis of the saturated transformer as a linear multiport with internal sources enables us to determine the interchange power of each harmonic separately and also the available power of the transformer over the entire spectrum of harmonics. *The interchange power* is the peak value of the power at the output of a source whatever the changes in the output current or voltage. Considering each source as a oneport network, we get the interchange power

$$P_i = (1/2)\, E_i E_i^* /(z_{ii} + z_{ii}^*) \tag{7.19}$$

where E_i^* is the complex conjugate of the effective (rms) voltage E_i; z_{ii}^* is the conjugate-matched impedance of a load supplied from the ith oneport of impedance z_{ii}.

The available power of a multiport can be found as the total output power regarded as a function of currents in all pole pairs.

Considering the saturated transformer as a linear noise-generating multiport, we can readily visualize the working processes in frequency multipliers and dividers. The equations thus derived are convenient for simulation on computers. We have given here an

example of the transformer to illustrate how to determine the interaction between harmonics, though the discussion certainly relates to rotating machines too.

There are a few methods for the analysis of energy conversion processes. They give approximate solutions to the problems stated, but on the whole ensure the desired accuracy.

7.3. The Effect of Current Displacement in the Slot

The study of the effect of current displacement (skin effect) in the slot on the dynamic characteristics of an energy converter is of much practical significance. A change in the angular velocity of a rotor causes a change in the rotor current frequency. This affects

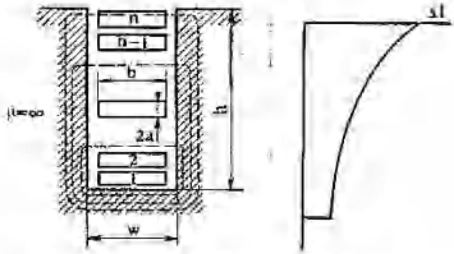


Fig. 7.4. Illustrating current displacement in the slot of an energy converter
The figures 1, 2, ..., n denote conductors

the current density distribution over the height of a conductor embedded in the slot (Fig. 7.4). The current in a conductor or conductors in parallel varies over the slot height because of the difference between the inductive reactances of conductors lying at the slot bottom and nearer to the air gap. The amplitude and phase of currents then vary too. The distribution of ΔI over the slot height is given in Fig. 7.4.

The calculation practice uses the coefficient k_r to account for an increase in the resistance due to current displacement. It depends on frequency, the type of winding, slot height h and width w, the material of elementary conductors, their number and dimensions $2a$ and b. In use is also the coefficient k_x to account for the variation in leakage inductive reactance with current displacement. Both k_r and k_x vary nonlinearly with rotor angular speed. The variation of these coefficients in relative units for a deep slot is shown in Fig. 7.5.

On defining the pattern of changes in the slot resistance and inductive reactance, we can solve the electromechanical equations on an analog or digital computer.

In the analysis involving the equations with parameters dependent on currents or time, use is made of the following equations expressed in terms of currents and solved for current derivatives:

$$
\begin{aligned}
i^s_\alpha &= (1/L^s\overline{p})\,u^s_\alpha - (R^s/L^s\overline{p})\,i^s_\alpha - (M/L^s)\,i^r_\alpha \\
i^s_\beta &= (1/L^s\overline{p})\,u^s_\beta - (R^s/L^s\overline{p})\,i^s_\beta - (M/L^s)\,i^r_\beta \\
i^r_\alpha &= (R^r/\overline{p})\,\sigma i^r_\alpha - (1/\overline{p})\,p\omega_r f - M\sigma i^s_\alpha \\
i^r_\beta &= (R^r/\overline{p})\,\sigma i^r_\beta + (1/\overline{p})\,p\omega_r q - M\sigma i^s_\beta \\
M_e &= pM\,(i^s_\beta i^r_\alpha - i^s_\alpha i^r_\beta) \\
d\omega_r/dt &= (p/J)\,(M_e - M_r)
\end{aligned}
\tag{7.20}
$$

Here $1/\overline{p}$ is the integration symbol; $\sigma = 1/L^r$; $f = i^r_\beta + M\sigma i^s_\beta$; $q = i^r_\alpha + M\sigma i^s_\alpha$.

A computing device permits considering separately an increase in the resistance and decrease in the inductive reactance of the rotor with a change in its angular velocity, and also the interaction between these quantities.

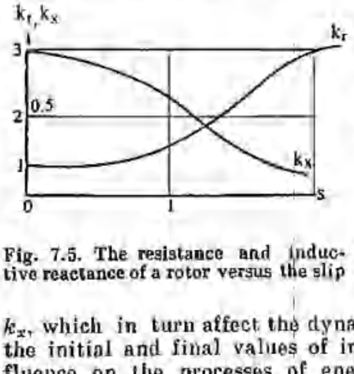


Fig. 7.5. The resistance and inductive reactance of a rotor versus the slip

The study of nonlinear variations in impedances shows that changes in rotor resistance have the greatest effect on the dynamics of induction motors at starting. The time of starting, impact currents and torques decrease with current displacement in rotor slots.

The shape of slots affects the character of variations in k_r and k_x, which in turn affect the dynamic characteristics. However, it is the initial and final values of impedances that exert a greater influence on the processes of energy conversion. The character of changes in rotor impedance (the pattern of nonlinear impedance variation with time) plays a secondary role. Computing devices enable the solution of problems for various sets of linear and nonlinear parameters and various patterns of changes in k_r and k_x with time.

It is to be noted that the problem of considering the current in a slot comes to the solution of multiwinding-rotor machine equations, the subsequent simplification of which can give the equations for a double-squirrel cage machine. The double-squirrel cage rotor of an induction

machine requires more labor for its manufacture and has a larger diameter in comparison with a rotor with deep dew-drop or bottle-shaped slot, so in designing new versions of induction machines the preference is given to the latter rotor. By choosing a proper shape of the slot, it is possible to bring the dynamic characteristics of a deep-slot machine close to those for a double-cage machine.

In a real induction machine, apart from current displacement, the magnetic core saturation and eddy currents greatly affect the processes at starting. The analysis of these factors in combination can give the equations for a multiwinding machine with nonlinear parameters.

An approach aimed at reducing the effect of current displacement on the operation of synchronous and dc machines in steady-state conditions is to transpose conductors and decrease their cross section. In induction machines the effect of current displacement is taken advantage of for improving the dynamic characteristics.

7.4. Energy Conversion Problems Involving Independent Variables

The independent variables in electromechanical equations are commonly the voltage and moment of resistance M_r (torque). It should be kept in mind that these equations also contain the voltage frequency which determines the current frequency. In the general case, both voltage (and frequency) and torque may change simultaneously. In most cases, however, the study of the effect of torque on the dynamic and static characteristics involves invariable voltages with the torque at the shaft also kept constant.

In considering complex electromechanical systems which consist of many electric machines, it is necessary to reduce the number of equations for describing the simplified energy conversion processes. The researcher must certainly have a thorough insight into what assumptions he must introduce and what features he can neglect to make the analysis simpler but adequate enough. Look at the effects of voltages and frequences on the processes of energy conversion in electric machines.

The investigation of transients at varying frequencies and voltages on the terminals of a motor is of much significance. This is particularly the case for autonomous electromechanical systems, where there is a need to obtain the optimal course of transients by varying the voltages and frequencies, and also for motors at starting in the conditions at which the motor powers and supply powers are comparable.

The processes of electromechanical energy conversion at varying frequencies and voltages are described by the systems of equations (4.1) through (4.3) and (3.3) through (3.12).

The analysis of dynamics of induction machines on an analog computer at varying frequencies and voltages calls for a special supply network. The periodic functions $\sin \omega t$ and $\cos \omega t$ of a varying frequency can be found from the solution of two equations

$$dx/dt = \omega y, \quad dy/dt = -\omega x \tag{7.21}$$

For the stabilization of the voltage amplitude proportional to $\sin \omega t$ and $\cos \omega t$, the computer model should have an additional circuit for the solution of the equation

$$\sin^2 \omega t + \cos^2 \omega t - 1 = 0 \tag{7.22}$$

Should the voltage amplitude undergo changes, the feedback path provides for the compensation of errors.

The supply network at the varying voltage amplitude and constant frequency is made adequate from the solution of the equation

$$d^2x/dt^2 + \omega x = 0 \tag{7.23}$$

The problems being stated may involve voltage amplitudes and frequency that are functions of the effective values of the magnetizing current, flux linkages, and rotor speed. The model for the solution of equations of an induction motor with constant parameters is set up with consideration for the above factors.

Consider transients at a varying supply voltage and constant frequency. The pattern of voltage changes is recorded on nonlinearity units. The voltage is made to vary between the limiting values equal to about 0.8 and 1.2 of the nominal value U_n. The analysis of oscillograms taken at constant parameters can reveal that for small-power and medium-power motors, the currents and torques show maxima during the first one or two periods when the voltage still changes little. The character of voltage variations has therefore a weak effect on the time of starting of these machines. In high-power motors, the peak currents and torques are evident during the first eight to twelve periods, so the voltage changes here have a more pronounced effect on the course of transients. This is also the case with motors operated from hf voltage sources and with motors having a large moment of inertia. The results of studies show that the impact currents and torques depend on the character of an initial change in the supply voltage. The derivative $\alpha = du/dt$ or $du/d\Psi$ therefore characterizes the course of a transient.

A change in the supply voltage exerts a greater effect on the time of starting of low-power and medium-power motors. A decrease in voltage exerts a greater effect than an increase in voltage at starting. It should be noted that in motors where the currents and torques reach peak values in one or two periods, the nonlinearity of mutual inductance (saturation) has a smaller effect on transients than its decrease.

It is of interest to study the dynamics of induction motors with nonlinear parameters, in which the frequency and voltage change simultaneously. This problem is only solvable on digital computers.

From the results of the analog-computer analysis of a motor with constant parameters we can conclude that a transient occurring at a varying voltage and constant frequency f differs in character from a transient proceeding at a varying f and constant U. The course of

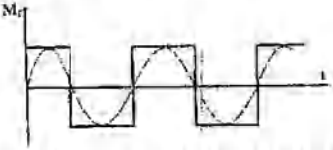


Fig. 7.6. Impact and sinusoidal loads

a transient heavily depends on the initial values of frequency and voltage. Transients at constant U and constant f represent a particular case.

In widespread use are energy converters designed to operate at a constant voltage and periodically varying load (drives of crushers, rolling mills, etc.). The character of changes in M_r can be most different. In a machine operating at an impact load, the air gap contains harmonics with peak amplitudes, but the gap is free from these harmonics at a sinusoidal load (Fig. 7.6). If an energy converter is comparable in power with the line, the impact load distorts the voltages and currents in the EC, and the upper harmonics arise in the line and affect the operation of other devices.

The independent variables U and M_r and dependent variables i and ω_r can interchange places. For example, if we control current in a generator (in a generator-motor system called the current drive), the frequency and voltage will undergo changes at the output. In sources with semiconductor elements, the controlled variable is current rather than voltage.

7.5. The Analysis of Operation of a Real Electric Machine

Let us analyze the processes of electromechanical energy conversion in a real induction machine which is the most general type of EC since here $\omega_r \neq \omega_s$. A symmetric induction machine has its windings arranged in slots of the magnetic core stacked of sheet-steel lamina-

tions. In a machine operated on sinusoidal voltage supply, the air gap contains the spectrum of harmonics associated with magnetizing forces, saliency, nonlinearity of resistances and inductive reactances, manufacturing factors, etc. In general, it may have any kind of harmonics.

The induction machine is a multiwinding electric machine. In the analysis of this machine, one should consider that the stator and rotor have eddy current loops and the parallel branches of stator windings may operate in different conditions. Besides, it is to be kept in mind that the active structure of the machine is built up of core sections, the operating conditions of which are different on the extremities and in the middle.

Obviously, the accurate mathematical description of the processes of energy conversion in a real machine cannot be given because each of the sources of space harmonics produces an infinite spectrum of harmonics, and the number of such sources in a real EC runs into a few tens.

The most general tool for the description of energy conversion in a conventional induction machine is the system of equations (3.3) through (3.12) with constant and nonlinear coefficients.

Although the description of processes in an EC is always approximate, computing facilities can provide a sufficient accuracy in the solution of most problems encountered in electromechanics. It should be remembered that experimental investigations too can give only approximate data.

For a machine with a sinusoidal voltage at its input, it is possible to use Eqs. (3.3) through (3.12) with constant coefficients under the assumption that the machine parameters are independent of load.

In a saturated machine the magnetizing current contains odd harmonics. Because of the shift between the windings in space and between currents in time, these harmonics produce fields in the air gap, that travel at different slips with respect to the rotor. Based on the model of the generalized energy converter, the model of a saturated machine can be set up with m pairs of windings on the stator and rotor along the α and β axes, in which the applied voltages produce a field in the air gap such as that found in a real machine (see Sec. 7.2).

Assuming that there is no interrelation between the harmonics, we can transform Eqs. (3.3) through (3.12) into Eqs. (5.1) and (5.2) and construct a machine model (see Fig. 5.1) with m windings on the stator and rotor. Such a model corresponds to an ideal machine supplied from a nonsinusoidal asymmetric voltage source (see Sec. 5.1).

Thus, considering a machine with variable parameters and sinusoidal voltages at the terminals, we can represent it as a machine with constant parameters and nonsinusoidal voltages at the input. The equations for a saturated machine differ from those for an in-

duction machine in that the former contain coefficients $M^s_{13\alpha}$, $M^s_{1m\alpha}$, $M^s_{13\beta}$, ..., $M^{sr}_{1m\beta}$, $M^{rs}_{13\beta}$, ..., $M^{rs}_{1m\beta}$, and also other coefficients to account for the ferromagnetic couplings between harmonics.

Let us take a look at the effect of a nonsinusoidal distribution of the magnetizing force on the spectrum of harmonics in the air gap and give the mathematical description of the processes under the assumption that the machine of interest is unsaturable and the voltage is sinusoidal. If we assume that the number of harmonics μ is infinite and the air gap is smooth, then the air-gap field repeats the pattern of distribution of the magnetizing force. Knowing the spectrum of harmonics in the air gap, namely, their amplitudes and phases, we apply the equations for the generalized energy converter and set up the mathematical model of the machine to describe the energy conversion processes. The assumption here is that the stator and rotor carry the same number of fictitious windings, which corresponds to the chosen number of harmonics.

The problem being stated must cover two to four harmonics, and its solution cannot certainly be accurate. The resistances of fictitious windings may be taken equal to the resistances of actual windings. The mutual inductances associated with upper harmonics of the magnetizing force may be taken approximately equal to one-third of the fundamental for the third harmonic, to one-fifth for the fifth harmonic, etc. The coefficients of coupling between harmonics, M_{ni}, cannot be higher than the mutual inductances between upper harmonics, M_{ii}. The coefficients entering into electromechanical equations depend on load. Thus the equations for the machine with a nonsinusoidal mmf distribution are the same as those for a saturated machine. The equations differ from each other by the values of coefficients and the amplitudes of harmonics.

For a nonsaturated machine, the mmf distribution is sinusoidal, the air gap is smooth, though nonuniform due to manufacturing factors (misalignment of the rotor with respect to the stator, ellipticity, conicity, etc.). The air-gap nonuniformity is responsible for the appearance of the spectrum of harmonics in the air gap.

If the gap is nonuniform both in the axial and in the radial direction, for analyzing the processes the machine is broken into m pieces in the axial direction, and the machine model is constructed with m stators and a common rotor, such as illustrated in Fig. 5.3. It can be assumed here that the line voltage distributes itself uniformly among m machines and each elementary machine differs little from the other in parameters. Even if we assume the parameters to be identical, the problem at hand will be insuperably difficult because each elementary machine has a spectrum of harmonics due to eccentricity. The harmonic spectrum contains space harmonics which interact and affect each other. As mentioned earlier, the analysis

must reduce to the investigation of processes involving a specified number of harmonics.

Considering the air-gap nonuniformity, we again arrive at Eqs. (3.3) through (3.12) whose solution necessitates that we should correctly specify parameters and determine the amplitudes and phases of harmonics. The air-gap nonuniformity due to saliency also gives rise to a definite spectrum of harmonics. The mathematical description here is the same as for the other types of nonuniformity.

So, in a real unsaturated machine there are infinite sets of space harmonics arising from a nonsinusoidal mmf distribution and air-gap nonuniformity due to saliency and eccentricity. In a saturated machine there appears another spectrum due to nonlinear self- and mutual inductances and also heterodyne frequencies. But a major portion of these harmonics are practically harmless since they have infinitely small amplitudes, and only a small portion of harmonics in this spectrum affect the machine performance.

It is easy to see that Eqs. (3.3) through (3.12) describe the processes of energy conversion at a nonsinusoidal supply voltage in a saturated machine with due regard for other space harmonics.

In the analysis of the processes in a real machine, the researcher should, first, have a clear idea of the fact that the solution to the problem can be approximate, second, set a definite limit on the number of equations (number of harmonics) to be dealt with, and, third, perform the most complex procedure, namely, define the amplitudes and phases of the harmonics in question and also the parameters for the electromechanical equations. This done, the researcher can solve the equations on a computer and obtain an approximate solution. From the engineer's viewpoint, the solution can be considered accurate because the obtained results can compare well with the results of the experiment on a real machine; in other words, the measurement errors can be of the same order as the ones introduced in the results obtained from the solution of the equations.

Despite the complexity of the processes in the gap of an electric machine and the different causes responsible for the emergence of harmonics, the mathematical description of energy conversion processes can be given by using one and the same set of equations for an m-n winding machine.

The model of a machine with m windings on the stator and n windings on the rotor permits the researcher to formulate the equations for any case of electromechanical energy conversion and solve these equations on computers.

At present almost all problems involved in the analysis of energy conversion processes are amenable to the solution to a definite accuracy.

Chapter 8 Asymmetric Energy Converters

8.1. Types of Asymmetry in Electric Machines

The theory of electromechanical energy conversion generally deals with symmetric machines. However, most electric machines, or even almost all real machines, are asymmetric if manufacturing factors are taken into account, because it is impossible to attain the same parameters for each phase.

Asymmetric machines can show electrical, spacial, and magnetic asymmetry. Electrical asymmetry results from a difference between the resistances or inductive reactances of machine phases. To machines with this type of asymmetry belong induction motors having various phase-shifting elements and operating from a single-phase power line.

Spacial asymmetry appears as a result of shift in space of the neighboring phase winding axes through an angle other than an angle of $2\pi/m$ electrical radians. The machines with this type of asymmetry include motors having shaded poles, synchro motors, etc.

Magnetic asymmetry arises from a nonuniform air gap and, sometimes from an asymmetric magnetic core.

Some machines may simultaneously display three types of asymmetry. An example is a single-phase induction motor with a short-circuited shading loop (turn, or core on the pole). The above-mentioned three types of asymmetry have to do with the principle of action of the machines. It is also of interest to investigate symmetric machines in which asymmetry arises from the effect of various manufacturing factors.

Of much importance in the theory of energy converters is the investigation of symmetric machines with asymmetric voltages on their terminals. The asymmetric response of synchronous machines, transformers, and induction machines in the steady-state and transient conditions deserves particular attention for the study of power system performance.

Worthy of notice is the most general case concerned with the mathematical description of energy conversion in asymmetric machines operating at asymmetric voltages on their terminals.

The theory of asymmetric energy converters is given treatment in quite a few books on the subject, though its further development is essential, for the class of these machines covers a great variety of types.

The method of symmetric components and the theory of rotating fields are the main tools for the study of asymmetric ECs.

The mathematical description of energy conversion in symmetric ECs represents a particular case of the analysis of asymmetric ECs. For this reason it is not always judicious to extend the achievements in the theory of symmetric machines into the area of asymmetric machines. On the whole, whatever the complexity of asymmetric machines and however diverse the asymmetric conditions in which they operate, the analysis essentially involves the study of the air-gap field. For the solution of the problems stated, the researcher must of course thoroughly define the air gap in which the magnetic field stores energy and then give the mathematical description of energy conversion processes.

In an asymmetric machine operating on symmetric voltage supply, the air gap contains both a forward (positive-sequence) and a backward (negative-sequence) field. In three-phase and multiphase machines, zero-sequence fields appear under certain conditions. *Asymmetry is responsible for the buildup of a zero-sequence field in the air gap. The study of energy conversion in asymmetric machines in a first approximation reduces to solving the electromechanical equations for the two fields in the air gap.*

8.2. Electrical and Magnetic Asymmetry

The torque in a symmetric machine results from the products of stator and rotor currents along different reference axes. In this machine, the stator and rotor current products along the same axes

$$i_\alpha^s i_\alpha^r - i_\beta^s i_\beta^r \tag{8.1}$$

do not give rise to the torque since the sum of terms in (8.1) is zero.

In an asymmetric machine, the current products along the same axis do produce the torque which is definable on the assumption that the mutual inductances between the stator and rotor phase windings are identical:

$$M_e = (m/2)\, M\, (i_\beta^s i_\alpha^r - i_\alpha^s i_\beta^r + i_\alpha^s i_\alpha^r - i_\beta^s i_\beta^r) \tag{8.2}$$

In determining the torque in an asymmetric machine with consideration for a difference between the mutual inductances along the machine axes, it is more judicious to define the torque in terms of flux linkages.

In the general case, the mathematical description of energy conversion in an asymmetric multiphase multipolar machine involves the solution of Eqs. (3.3) through (3.12) for the generalized energy converter. In going from a simple to a more complex mathematical analysis, it makes sense to consider some particular cases involved in the study of asymmetric machines.

Consider a two-phase machine in which $w^s_\alpha \neq w^s_\beta$, the number of slots per pole and the number of slots per phase are different, and the conductors differ in cross section. These conditions promote magnetic asymmetry which shows up as a different saturation along each reference axis of the machine. Both electrical and magnetic types of asymmetry are responsible for the difference between the parameters along the machine axes (Fig. 8.1).

If the machine rotor is symmetric, we need to convert one stator winding to the other and handle the transformed equations for the rotor winding.

Introduce the conversion factor

$$1/k = M_{Bb}/M_{Aa} = (w_B k_B / w_A k_A) \tag{8.3}$$

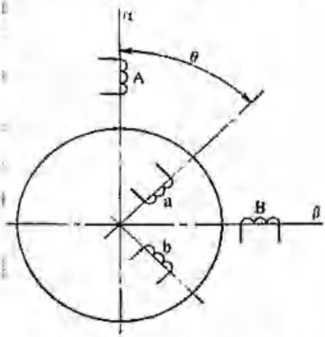


Fig. 8.1. The model of an energy converter with electrical asymmetry

where M_{Bb} and M_{Aa} are the mutual inductances between the stator and rotor windings shown in Fig. 8.1; and k_B and k_A are the factors that account for asymmetry between phases B and A.

Define the mutual inductances along the α axis

$$M_\alpha = M_{Aa} = M$$

and along the β axis

$$M_\beta = M_{Bb} = kM$$

Hence,

$$M_{Aa} = M_{aA} = M \cos\theta, \quad M_{Bb} = M_{bB} = kM \cos\theta \tag{8.4}$$

$$M_{Ab} = M_{bA} = M \sin\theta, \quad M_{Ba} = M_{aB} = kM \sin\theta$$

This done, set up the voltage equations. First, express the flux linkages as

$$\begin{aligned} \Psi_A &= L^s_\alpha i^s_\alpha + M\,(\cos\theta\ i_a - \sin\theta\ i_b) \\ \Psi_B &= L^s_\beta i^s_\beta + kM\,(\sin\theta\ i_a + \cos\theta\ i_b) \\ \Psi_a &= L^r i_a + M \cos\theta\ i_a + kM \sin\theta\ i_b \\ \Psi_b &= L^r i_b - M \sin\theta\ i_a + kM \cos\theta\ i_b \end{aligned} \tag{8.5}$$

The electromagnetic torque can be defined as a partial derivative of the total stored electromagnetic energy with respect to the geo-

metric angle:

$$M_e = p\partial w_{em}/\partial\theta \tag{8.6}$$

$$\frac{\partial w_{em}}{\partial\theta}=\frac{1}{2}\left\{\frac{\partial l^s_{\sigma\alpha}}{\partial\theta}(i^s_\alpha)^2+\frac{\partial l^s_{\sigma\beta}}{\partial\theta}(i^s_\beta)^2\right.$$
$$\left.+\frac{\partial l^r_\sigma}{\partial\theta}[(i^r_\alpha)^2+(i^r_\beta)^2]+2M(ki^s_\beta i^r_\alpha-i^s_\alpha i^r_\beta)\right\} \tag{8.7}$$

Here

$$\partial l^s_{\sigma\alpha}/\partial\theta = (\partial l^s_{\sigma\alpha}/\partial s)(\partial s/\partial\theta)$$
$$= -(\partial l^s_{\sigma\alpha}/\partial s)(a_r/\omega_s\omega_r) \tag{8.8}$$

where $a_r = d\omega_r/dt$ is the angular acceleration of the rotor.

Similarly,

$$\partial l^s_{\sigma\beta}/\partial\theta = -(\partial l^s_{\sigma\beta}/\partial s)(a_r/\omega_s\omega_r)$$
$$(\partial l^r_\sigma/\partial\theta) = -(\partial l^r_\sigma/\partial s)(a_r/\omega_s\omega_r) \tag{8.9}$$

Considering that

$$\partial l^s_{\sigma\alpha}/\partial s = 0 \text{ and } \partial l^s_{\sigma\beta}/\partial s = 0$$

the torque equation becomes

$$M_e=\frac{1}{2}p\left\{-\frac{\partial l^r_\sigma}{\partial s}\frac{a_r}{\omega_s\omega_r}[(i^r_\alpha)^2+(i^r_\beta)^2]+2M(ki^s_\beta i^r_\alpha-i^s_\alpha i^r_\beta)\right\} \tag{8.10}$$

Expression (8.10) contains two components, of which the first is a function of the change in the rotor winding leakage inductance and the second is a function of the change in the air gap energy. The energy stored up in the rotor leakage field with a change in acceleration takes part in energy conversion. If it accounts for a large share, the leakage field energy adds to the torque and should be taken into consideration in the analysis of the transient. In the steady-state conditions at which $a_r = 0$, this energy component exerts no effect on the machine torque.

If we disregard a change in the leakage field energy, Eq. (8.10) will assume the form

$$M_e = pM(ki^s_\beta i^r_\alpha - i^s_\alpha i^r_\beta) \tag{8.11}$$

An asymmetric machine is often made complete with phase-shifting elements inserted into one of its phases. These are commonly capacitors and resistors. The voltage equation for the phase incorporating a capacitor of capacitance C has the form

$$u^s_\beta = u^s_\alpha - (1/C)\int i^s_\beta\, dt \tag{8.12}$$

With the capacitance C and series resistor r_s added to the circuit,

$$u^s_\beta = u^s_\alpha - (1/C)\int i^s_\beta\, dt - r_s i^s_\beta \tag{8.13}$$

On inserting the starting capacitance C_{st} and operating capacitance C_{op} in the circuit, the voltage equation becomes of the form

$$u^s_\beta = u^s_\alpha - [1/(C_{st} + C_{op})] \int i^s_\beta \, dt \tag{8.14}$$

The models for the solution of equations of asymmetric machines are similar to those set up for ordinary machines. In the analysis of energy converters furnished with capacitors or series resistors, the

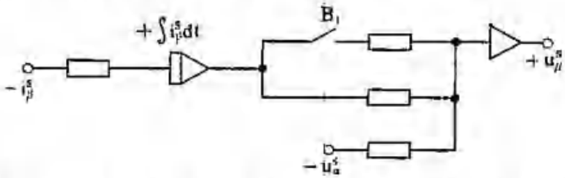


Fig. 8.2. The setup involving the starting and operating capacitances for the solution of equations on an analog computer

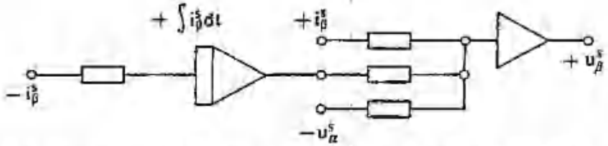


Fig. 8.3. The setup for the solution of Eq. (8.13) on an analog computer

model must incorporate operational amplifiers. For example, in solving Eq. (8.12) for a motor with C_{st} or C_{op}, the model has to include an additional setup B_1 to cut out C_{st} (Fig. 8.2). The model for (8.13) with C and r_s should be fitted with an attachment such as in Fig. 8.3.

With the mathematical model set up on an analog computer, we can investigate the effect of the parameters of the machine and its phase-shifting elements on the static and dynamic characteristics.

8.3. Spacial Asymmetry

To build up a circular field in an energy converter, the windings must be at certain angles in space with respect to each other. In a two-phase machine the angle between the windings is equal to 90 electrical degrees, and in an m-phase machine it is equal to $2\pi/m$. In the general case, the angles between the windings can take any values, thereby causing spacial asymmetry in a machine. Any asym-

metry, the spacial one included, gives rise to a negative-sequence field in the air gap.

Imagine that one of the windings in a two-phase machine gradually turns from its original location at $\delta = 90^\circ$ to a position at which $\delta = 0$ (Fig. 8.4). As the winding goes on moving, the air gap field converts from the circular to the elliptic field and then to the pulsating one at $\delta = 0$. As the windings move with respect to

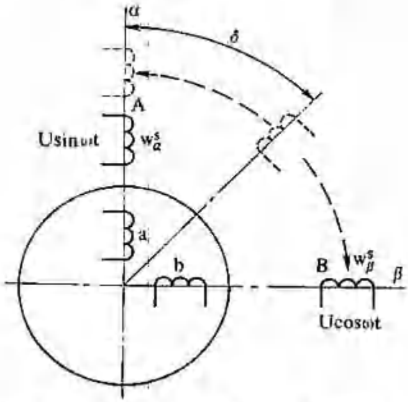


Fig. 8.4. Illustrating the motion of windings with respect to each other that results in the conversion of the circular field to the pulsating one

each other, the amplitudes of the positive-sequence and negative-sequence field components of the elliptic field undergo changes. If the winding axes are coincident, we can superpose one winding on the other and construct the model of a single-phase machine.

At spacial asymmetry,

$$L_{Aa} = L_{aA} = M \cos\theta, \quad L_{Ab} = L_{bA} = M \sin\theta$$
$$L_{Bb} = L_{bB} = M \sin(\delta - \theta), \quad L_{Ba} = L_{aB} = M \cos(\delta - \theta) \tag{8.15}$$

For asymmetric windings shifted in space, the flux linkages of phase assume the form

$$\begin{aligned}
\Psi_A &= L_A i_A + L_{AB} i_B + M(\cos\theta i_a - \sin\theta i_b)\\
\Psi_B &= L_B i_B + L_{BA} i_A + kM[\cos(\delta - \theta)\, i_a + \sin(\delta - \theta) i_b]\\
\Psi_a &= L_a i_a + M\cos\theta i_A + kM\cos(\delta - \theta)\, i_B\\
\Psi_b &= L_b i_b - M\sin\theta i_A + kM\sin(\delta - \theta)\, i_B
\end{aligned} \tag{8.16}$$

In transforming the set of equations to the α and β coordinate axes, we should remember that

$$i_A = i^s_\alpha, \quad i_B = i^s_\alpha \cos\delta + i^s_\beta \sin\delta$$

Referring to (8.10), the electromagnetic torque of an asymmetric machine at an arbitrary angle of δ is given by

$$\begin{aligned} M_e = pM\,[&- i^s_\alpha i^r_\beta (1 + k\cos^2\delta) + 0.5k \sin 2\delta\, (i^s_\alpha i^r_\alpha \\ &- i^s_\beta i^r_\beta) + k \sin^2\delta\, i^r_\alpha i^s_\beta] \end{aligned} \tag{8.17}$$

The feature common to all asymmetric machines is that they display an elliptic field. Setting up the models for the equations of asymmetric machines on a computer, we can analyze both the transient and steady-state performance of the energy converters. The analysis of static characteristics enables us to compare the potentialities of amplitude control with those of phase control, clarify the effect of phase-shifting elements and machine parameters, etc.

In comparison with symmetric machines, asymmetric machines in dynamic operation show a greater nonuniformity of the angular velocity, higher peaks of the torque, lower no-load speeds, and longer transient times.

The positive-sequence and negative-sequence fields present in the air gap of a machine impair its static and dynamic characteristics as against those of a machine with a circular field. The analysis that disregards the effect of upper harmonics in an elliptic field gives errors to within 10-15 percent. In the presence of two field components comparable in amplitude, these errors are greater for an elliptic field than for a circular field if upper harmonics and eddy currents are not taken into consideration.

The dynamic behavior of salient-pole synchronous machines at asynchronous starting depends on the rotor position at the instant of switching the machine into the supply circuit.

8.4. Single-Phase Motors

Single-phase motors operate from single-phase power supply systems and find wide use in domestic appliances. Rarer uses include traction drives.

Examine an ideal single-phase motor whose air gap exhibits only a positive-sequence and a negative-sequence field. Such a motor has a uniform air-gap structure, a distributed sinusoidal winding, and is free from saturation. In a real motor, this type of winding is impossible to build up, so the air gap always contains a spectrum

of upper harmonics. Along with ordinary space harmonics, the reflected waves of the magnetic field appear in a single-phase motor because the single-phase winding occupies only a portion of the pole pitch in contrast to two-phase, three-phase, and multiphase windings. Arranging the single-phase winding in all slots is economically impracticable.

In an ideal single-phase motor, the positive-sequence and negative-sequence stator voltages are equal and their amplitudes come to half the amplitude of the impressed voltage. The parameters of the model of Fig. 3.3 for the positive-sequence and negative-sequence components can be the same. The processes of energy conversion in the motor under study are describable by Eqs. (4.8) through (4.11) under the conditions specified above.

If the positive-sequence and the negative-sequence fields are equal in amplitude, the motor does not develop a starting torque.

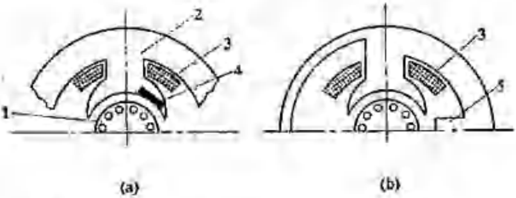


Fig. 8.5. Single-phase induction motors with a shorted shading loop on the pole (*a*) and asymmetric magnetic system (*b*)

It is then necessary to reduce the negative-sequence field and thus to produce the difference between the torques due to both fields at $S = 1$, the difference being the starting torque. One of the steps taken to reduce the negative-sequence field and bring the air gap field closer to the circular field pattern is to use an additional winding shifted in space with respect to the main one and incorporate a device to secure the time shift between the currents in the two windings. A capacitor is best suited for the purpose. A single-phase motor having two windings one of which contains a starting capacitor is known as a capacitor motor whose circuit diagram is shown in Fig. 4.7.

Of the asymmetric motors, the simplest in design and most popular is a single-phase motor with shaded poles (with a shorted shading loop on the pole), such as illustrated in Figs. 8.5 and 8.6. Despite the fact that the motor design is simple, the mathematical description of energy conversion processes in this motor is most complex.

In a short-circuited loop *4* (see Fig. 8.6) arranged on a pole *2*, a change in the flux produced by a winding *3* gives rise to a current shifted in time with respect to the field winding current. Since the windings are displaced in space (see Figs. 8.5 and 8.6*a*) and the currents are shifted in time, a traveling field appears in the gap. This is an elliptic field with a rather large negative-sequence component. The interaction between the stator currents and the currents in the short-circuited rotor *1* provides for the starting and driving torques.

For the model of Fig. 8.6*a*, energy conversion equations can be set up, given the parameters of windings w_α^s and w_β^s and also the

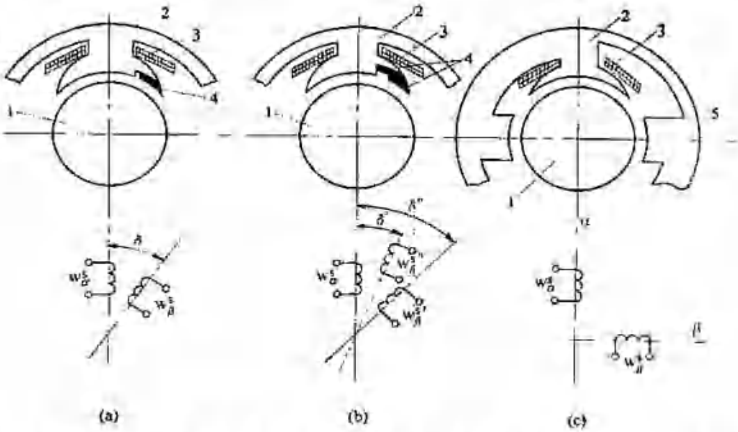


Fig. 8.6. Induction motors with one shorted l oop on the pole (*a*), a few shorted loops on the pole (*b*), and asymmetric magnetic system (*c*)

positive-sequence and negative-sequence voltages in the circuit model with windings spaced 90° apart.

A single-phase motor with two or more closed loops on the pole (see Fig. 8.6*b*) is more difficult to analyze than the motor with a single loop, for we then need to formulate and solve the equations for an asymmetric multiwinding machine with an elliptic field. Here $w_\beta^{s'}$ and $w_\beta^{s''}$ are equivalent windings shifted through angles δ' and δ'' respectively.

Eddy currents exert a considerable effect on the characteristics of asymmetric single-phase motors. These currents can be put to use so that a motor will have a sufficient starting torque. In Fig. 8.6*c*

is shown the motor with an asymmetric magnetic system. In the laminations of poles 5 there appear eddy currents due to a change of the flux in the single-phase winding placed on the longitudinal axis of the machine. These currents are in time displacement with respect to currents in the field winding 3, and eddy current loops are in quadrature with each other (see Fig. 8.1). In comparison with motors with shaded poles, single-phase motors with an asymmetric magnetic system can have better energy characteristics and are well adaptable to driving household fans.

In salient-pole machines the magnetic field energy concentrates within the pole pitch. The difference between permeances in the area under the poles and in the space between the poles (between the rotor and pole pieces) causes the appearance of reflected waves which worsen the characteristics of single-phase motors.

8.5. The Electric Machine as an Element of the System

Electric machines generally serve as functional units of electromechanical systems. If an energy converter operates from or into the bus of infinite power, we can treat the processes disregarding

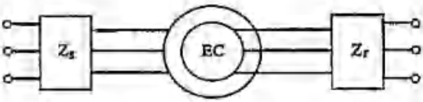


Fig. 8.7. The simple representation of an electromechanical system

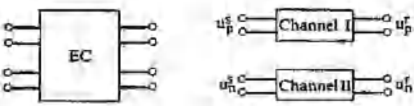


Fig. 8.8. The two-phase energy converter as a two-channel fourport showing the positive-sequence and negative-sequence voltages u_p^s, u_p^r and u_n^s, u_n^r respectively

the parameters of elements connected to the stator and rotor circuits (Fig. 8.7). In the figure, Z_s and Z_r are asymmetric multiports representing the elements connected to the stator and rotor circuits respectively.

In the analysis of a two-phase machine as an element of the system, it is convenient to represent it as a two-channel fourport (Fig. 8.8). A three-phase machine can be treated as a threeport network.

Knowing the parameters of the circuit model for the positive and the negative sequence, we can determine the positive-sequence and negative-sequence currents using the method of symmetric components. Given the parameters of fourports connected to the stator and rotor of an asymmetric machine, we can estimate the positive-sequence and negative-sequence currents with due regard for the parameters of multiport networks. Disregarding the processes of conversion associated with multiports, it is possible to solve problems for estimating the characteristics of an asymmetric energy converter supplied from a nonsinusoidal source via asymmetric multiports connected to the stator and rotor. This type of studies enables the analyst to compile tables for various asymmetric connection networks and formulate expressions for describing the steady-state performance.

We cannot, in the space available, consider the complex equations for the internal impedances of electric circuit elements connected to the stator and rotor circuit, and only note in passing that in some cases fairly large errors may arise if Z_s and Z_r are not taken into consideration.

A useful approach to the study of an asymmetric machine is to reduce the machine to a symmetric one with a twoport that includes an impedance ΔZ_α^s or ΔZ_α^r and thus allows for asymmetry. A powerful tool for the study of the behavior of an electric machine as part of the electromechanical system is the tensor analysis first employed for the purpose by Gabriel Kron.

Thus, the problems involved in the analysis of asymmetric machines supplied from a nonsinusoidal voltage source call for the formulation and solution of equations with due regard for eddy currents, asymmetric windings on the stator and rotor, etc. In the system analysis relying on multiterminal representation, it is possible to treat as multiports any objects connected to the mechanical and thermal terminals of an electric machine and thus go to a more detailed description of energy conversion processes. In solving technical problems, however, the researcher must not complicate the mathematical model. His objective is to handle the task in the allotted time and to the specified accuracy so as to give the answer satisfying the customer's requirements. The art of attacking the problems in the original way over the shortest time period naturally comes from the skill and experience of the engineer.

Chapter 9

The Equations for Electric Machines of Various Designs

9.1. The Mathematical Models of Energy Converters with a Few Degrees of Freedom

As is known, the electromechanical energy converters with one degree of freedom are electric machines having one rotating member, namely, a rotor. The energy converters with two degrees of freedom are electric machines in which both the rotor (rotors) and stator are rotating members (Fig. 9.1). These are double-rotation machines described by the equations

$$\begin{vmatrix} u_\alpha^s \\ u_\alpha^r \\ u_\beta^r \\ u_\beta^s \end{vmatrix} = \begin{vmatrix} r_\alpha^s + (d/dt)\,L_\alpha^s & (d/dt)\,M & L_\beta^r\omega_s & M\omega_s \\ (d/dt)\,M & r_\alpha^r + (d/dt)\,L_\alpha^r & L_\beta^r\omega_r & M\omega_r \\ -M\omega_r & -L_\alpha^r\omega_r & r_\beta^r + (d/dt)\,L_\beta^r & (d/dt)\,M \\ -M\omega_s & -L_\alpha^r\omega_s & (d/dt)\,M & r_\beta^s + (d/dt)\,L_\beta^s \end{vmatrix} \times \begin{vmatrix} i_\alpha^s \\ i_\alpha^r \\ i_\beta^r \\ i_\beta^s \end{vmatrix}$$

$$M_e = pM\,(i_\alpha^r i_\beta^s - i_\alpha^s i_\beta^r) \tag{9.1}$$

$$(J_r/p)\,(d\omega_r/dt) = M_e - M_{rr} \tag{9.2}$$

$$(J_s/p)\,(d\omega_s/dt) = M_e - M_{rs} \tag{9.3}$$

$$\omega = \omega_r + \omega_s \tag{9.4}$$

Equations (9.1) through (9.4) for an electric machine with two degrees of freedom differ from equations for a conventional machine (with one degree of freedom) in that the voltage equations contain the terms defining the emf of rotation of stator and rotor windings. Eqs. (9.2) and (9.3) include moments of inertia of the rotor and stator, J_r and J_s, and resisting torques on the rotor and stator, M_{rr} and M_{rs}. The system becomes determinate under definite conditions (9.4) set up for the rotor and stator velocities.

Energy conversion in the machine of Fig. 9.1 occurs in the air gap and the electromagnetic torque on the rotor and stator causes these members to rotate in opposite directions. In the steady-state operation, the distribution of velocities depends on the load torques

exerted on the rotor and counter-rotor. Under overload conditions, one of the rotors stops running and the other, load-free rotor, begins to accelerate. On switching an energy converter, in which both the rotor and stator are able to rotate, into the supply circuit, ω_r and

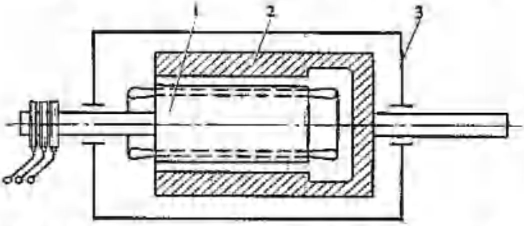


Fig. 9.1. The machine with two degrees of freedom
1 — inner rotor; 2 — outer rotor; 3 — stator

ω_s that set in at no load become functions of the moments of inertia, J_r and J_s. With a large increase in one of the moments of inertia, a rotor with a lower moment of inertia starts accelerating. In the energy converter of the above type, it does not matter where the energy that the air gap receives comes from since either of the rotating members of the machine has a contact arrangement. Such a machine has limited applications, though some of its features deserve consideration.

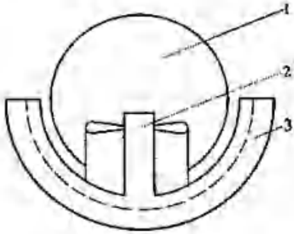


Fig. 9.2. The energy converter with a spherical rotor
1—spherical rotor; 2 — first stator; 3 — second stator

Let us turn our attention to an electric machine with three degrees of freedom (Fig. 9.2). In this machine, the rotor in the form of a sphere is kept suspended by the action of superconducting loops above two semicircular stators, one being turned 90° with respect to the other; the machine windings produce two traveling fields. Depending on the torques produced by the stators, the sphere can rotate in a three-dimensional space.

An electric machine with three degrees of freedom is described by eight voltage equations which can be represented in the form of a matrix

$$[u] = [Z]\,[I]$$

The impedance matrix for two immobile stators has four rows where the emfs of rotation are equal to zero. If we neglect the magnetic

coupling between the two stators, the voltage equations for this machine become similar to the voltage equations for a machine with an elliptic field. There are three equations of motion:

$$(J_{rx}/p)\,(d\omega_{rx}/dt) = M_{ex} - M_{rx} \tag{9.5}$$

$$(J_{ry}/p)\,(d\omega_{ry}/dt) = M_{ey} - M_{ry} \tag{9.6}$$

$$(J_{rz}/p)\,(d\omega_{rz}/dt) = M_{ez} - M_{rz} \tag{9.7}$$

Here J_{rx}, J_{ry}, and J_{rz} are the rotor's moments of inertia along the x, y, and z axes; ω_{rx}, ω_{ry}, and ω_{rz} are the rotor velocities along the x, y, and z axes; M_{ex}, M_{ey}, and M_{ez} are electromagnetic torques along the x, y, and z axes; and M_{rx}, M_{ry}, and M_{rz} are resisting torques along the x, y, and z axes.

As we did for the machine with two degrees of freedom, here we need to introduce one more equation in order that the system of equations for the machine under study should be determinate:

$$\omega_{rx} + \omega_{ry} + \omega_{rz} = \omega \tag{9.8}$$

Thus, twelve equations, namely, eight voltage equations, three equations of motion, and one velocity equation describe the processes of energy conversion in a machine with three degrees of freedom.

For a symmetric machine at $M_{rx} = M_{ry} = M_{rz} = M_r$, Eqs. (9.5) through (9.7) become simpler. When $M_{ex} = M_{ey} = M_{ez}$ and $J_{rx} = J_{ry} = J_{rz}$, the velocities along the axes are equal to

$$\omega_{rx} = \omega_{ry} = \omega_{rz} = \omega/3 \tag{9.9}$$

Machines with a spherical rotor find application in navigation devices. If one of the stators is made to revolve about the rotor, the machine so designed shows four degrees of freedom. If two stators revolve independently about the rotor, the machine will have five degrees of freedom. If we rigidly connect two stators and allow them to revolve about the spherical rotor, the machine will have six degrees of freedom.

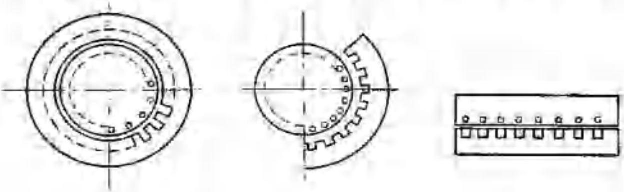
Based on the equations for a machine with three degrees of freedom, it is easy to increase the number of equations and thus describe a hypothetical machine with n degrees of freedom. Such a machine can be thought to carry a few windings on the stator and rotor, operate from nonsinusoidal supply, and exhibit nonlinearities. For its description, we would need to derive an infinite number of voltage equations and equations of motion, which would be the most general equations of electromechanical energy conversion.

Regarding typical equations, we should note that the above machines are describable by the even and the odd set of equations, while conventional machines need the odd set of equations for their description.

In an electric machine with n degrees of freedom, the electromechanical energy conversion degenerates because the angular velocity tends to zero as n approaches infinity, so that the energy converter becomes an electromagnetic converter.

9.2. Linear Energy Converters

There are many design versions of electric drives in which actuators execute a reciprocating motion by virtue of the mechanical transformation of rotational motion of electric machines. It is to be-



(a) (b) (c)

Fig. 9.3. Converter types
(a) conventional; (b) segmental; (c) linear

thought that an energy converter has good characteristics if it displays electromechanical resonance, i.e. its design is such that only a standing wave appears in the air gap and reflected waves are not present. Linear energy converters are low-performance devices, so their uses are justifiable only where a rotating machine with a mechanical converter is unacceptable.

A linear EC is a design extension of a conventional converter (Fig. 9.3a). Evidently, if we first build an EC with a segmental stator (Fig. 9.3b) and then increase the segment radius to allow it to go to infinity, a linear motor will result, such as shown in Fig. 9.3c. Linear converters find rare uses for work in the generating mode, though there are the cases of application of linear converters in practice as generators to transform the energy of the reciprocating motion of a diesel engine's or steam engine's rod into electric energy.

Linear motors come in asynchronous or synchronous types depending on whether they operate from an ac or dc source. The design versions are no less diverse than those of ordinary motors. Various constructions are adaptable to perform the function of a rotor, such as a steel sheet of infinite length, a car moving along the stator, or a magnetic liquid.

Maxwell's equations and the model such as in Fig. 9.4a form the basis for the mathematical description of energy conversion proces-

ses in linear motors. The plots of permeability μ and its derivative versus space coordinate $\lambda = \partial\mu/\partial x$ are shown in Fig. 9.4*b*.

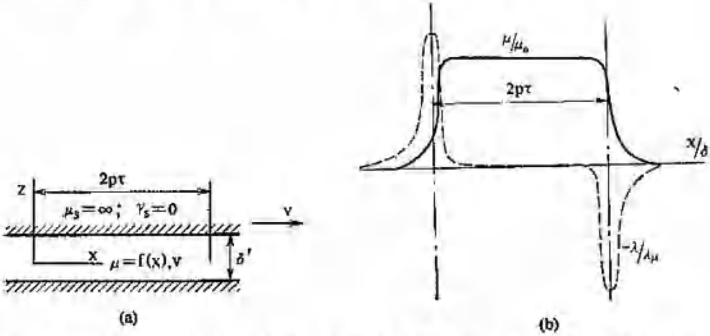


Fig. 9.4. A linear motor model (*a*) and the plot of μ and its derivative versus λ (*b*)

The instantaneous electromagnetic force acting on the working member of a linear motor is given by

$$f_{em} = \int_v BI_2 \, dv \tag{9.10}$$

The integration over the volume comes to multiplying the width $2a$ of the working member by the value of the normalized air gap, δ':

$$f_{em} = 2a\delta' \int BI_2 \, dx \tag{9.11}$$

where B is the magnetic flux density in the air gap; and I_2 is the secondary current in the area of the field structure.

The instantaneous value of the electromagnetic power is

$$P_{em} = \int_v EI_2 \, dv \tag{9.12}$$

Designing linear motors involves many difficulties because the unusual construction along with the reflected waves present in the air gap makes the determination of the field pattern in a real machine a rather complicated problem.

The design procedure for linear motors often follows the same guideline as for conventional motors, using the coefficients to account for poorer energy characteristics due to edge effects and other specific features of operation. The voltage equations are set up in

the same manner as for an ordinary asymmetric machine with due regard for the coefficients depending on the design of the linear motor. The driving force is found proceeding from the assumption that the powers in the rotational and translational motion are equal:

$$M\ (1/p)\ 2\pi f\ (1 - s_r) = F 2\tau f\ (1 - s_t) \qquad (9.13)$$

where s_r and s_t are the slips in rotational and translational motion respectively.

Such an approach certainly gives very approximate results, but it can prove valid in tentative calculations and also in the calculation of multipolar machines.

Linear motors have recently found use in high-speed transport facilities riding on a magnetic cushion. In the transport systems of

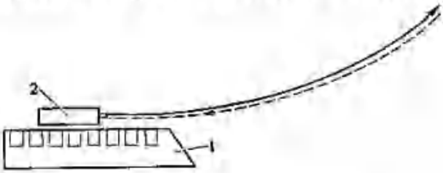


Fig. 9.5. An electric catapult
1 — stator; *2* — rotor

this type, the stator is a long line extending into tens and hundreds of kilometers and the rotor is a car suspended above the line bed. In designing magnetic-cushion transport vehicles, the engineer has to solve the problems of control and stabilization (levitation) of a passenger car, let alone the problem of decreasing the cost of such a transport system.

In the area of linear motors there are yet many complex problems that await their solution, of which the most complex one comes to the following. As far back as the middle 1930s electric catapults were built with the aim to impart an additional acceleration to flying vehicles (Fig. 9.5). While in ground transport systems the gap between the bed and the car must be kept accurate to a high degree, in catapulting the gap is made to vary (the rotor flies into space and the parameters in equations undergo changes). In the latter case there is a need for calculating the driving force and acceleration. More difficult problems arise in an attempt to bring the rotor back and take off the definite amounts of energy from it to effect the desired deceleration.

Although they are not devoid of shortcomings, linear motors enjoy use in graph plotters, manipulators of metal pieces, pushers, and in other electric drives. Reversing the motion of a linear motor gives

an oscillatory-motion motor. The analysis of linear motors enables us to extend the results to determine the relation between electric machines and apparatus in which the driving elements mainly execute linear displacements with varying parameters of electric circuits.

9.3. Energy Converters with Liquid and Gaseous Rotors

The stator of a linear motor can be built in the form of a pipe inside of which a traveling field can be set up. In the pipe filled with a magnetic liquid or gas (a moving conductor), the magnetic

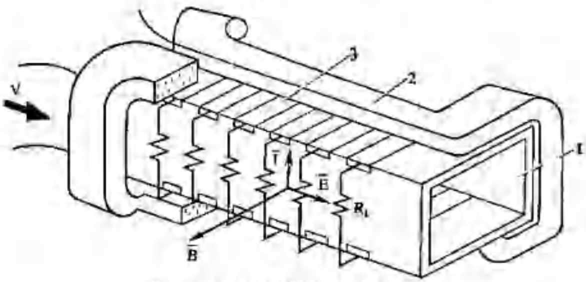


Fig. 9.6. An MHD generator

field will influence the motion of the fluid. If an ionized gas (a plasma) or a magnetic liquid is driven through the channel, we obtain an energy converter called a magnetohydrodynamic (MHD) generator (Fig. 9.6).

An MHD generator converts the mechanical (kinetic) energy of plasma particles to electric energy as the conducting plasma flows through a channel *1* in which magnetic coils *2* placed alongside the pipe produce a magnetic field $\overline{B}$. The conductivity of the hot gas grows with the addition of an easily ionized "seed" alkali metal such as potassium. The motion of the plasma at a speed v in the magnetic field induces a voltage on electrodes *3* and gives rise to a current I that flows in an external circuit R_1. The load current completes its path across the channel and produces the armature (plasma) reaction, thereby distorting the exciting field and the longitudinal voltage component—the Hall voltage. The Hall emf $\overline{E}$ is in a direction normal to the plane $\overline{BI}$.

The process of MHD power generation can be of the open cycle if the working medium passes through the channel only once, or of the closed cycle if the medium is made to flow through the generator

repeatedly. An MHD generator uses an inverter to change direct current collected on the electrodes to alternating current. The MHD channel is built up of segments, each being insulated from the other. Electrodes operate in heavy conditions and their life determines the service life of the generator. The pulse and short-time modes of operation show promise as regards the life expectancy.

Magnetic hydrodynamics that studies the motion of liquid and gaseous conducting media in a magnetic field belongs to electromechanics, for the interaction of a high-velocity conducting stream with a magnetic field causes the conversion of the kinetic energy of the stream into electric energy.

A conducting medium that moves in an external magnetic field in a direction normal to the plane $\overline{Bv}$ induces an emf, so that an electric energy of direct current and of low voltage can be taken off the electrodes. As in conventional energy converters, in MHD generators the load field exerts an influence on the external field, with the result that emfs appear which affect both the motion of the medium as a whole and the motion of individual portions of the stream. A change in the external $\overline{B}$ field also causes energy conversion (see Fig. 1.13). The laws of electromechanics also hold for MHD generators, so these converters certainly belong to electric machines.

Much effort has been spent in the USSR and USA for the development of MHD generators using plasma as a moving conductor. With the advancements in the field of fusion reactors, cosmology, and astrophysics, a further development of magnetic hydrodynamics becomes yet more urgent. So far, MHD generators are inferior to conventional energy converters from the economic and technical viewpoints. There are rather many modifications of MHD pumps and MHD generators, and more improved designs are likely to be devised in the future. The mathematical description of energy conversion in MHD generators comes to the simultaneous solution of Maxwell's equations defining electromagnetic processes and the Navier-Stokes equations defining the processes in liquids. The simultaneous solution of these equations is only possible for simple cases involving laminar flows.

If we assume that all particles of a liquid move at a constant speed, i.e. the liquid behaves like a solid, the problem becomes simpler and the energy conversion processes in MHD generators can be treated using the equations for conventional electric machines. Such an approach was put forward by A. I. Voldek in 1957.

The equations for electric circuits similar to Eqs. (1.34) through (1.37) or (3.3) together with the equations of magnetic hydrodynamics give a better description of energy conversion processes in MHD generators.

The effectiveness of an energy converter with a liquid or gaseous rotor depends on the magnetic field B and outflow velocity v. Therefore, the

use of superconducting magnetic systems which produce high magnetic fields offers considerable promise for improving the performance of MHD generators. The kinetic energy of the stream rises as a consequence of heating of the gas to 2 000 to 3 000 K and its acceleration as it leaves the nozzle.

The solution of the above problems in electromechanics requires the joint effort of thermal physicists and electromechanical engineers. For the advancements in this area to be more tangible, there is an urgent need for a profound learning of thermal and electromagnetic fields and the processes of conversion of energy from one form to another.

In the last years electromechanics has started using magnetizing liquids (ferromagnetic liquids). These are colloidal liquids whose critical characteristics depend on the stability and size of particles. Magnetizing liquids can perform the function of seals. Permanent magnets produce a field in the region separating a rotating member from a stationary one. Ferromagnetic particles line up in the field direction and thus make the seal tight. This approach enables improving the seals without substantial design variations. Magnetizing liquids find other, though limited, applications. The reason is that they are considerably inferior to electrical-sheet steels in magnetic properties.

Some hold a viewpoint that MHD generators directly convert thermal energy to electric energy. What is meant here is that MHD power generation dispenses with a steam turbine which is a common feature in the classical cycle of converting heat to electric energy.

Energy converters are manifold both in designs and principles of action. Electric machines are available which convert heat to electric or mechanical energy. These converters operate on the principle of changes in the permeability of ferromagnets near the Curie point. A change in the inductances with temperature causes a change in the parameters of the winding. Magnetic-thermal energy converters resemble parametric converters, since in both types energy conversion results from changes in the parameters of the coefficients that enter into electromechanical equations. Magnetic-thermal energy converters were suggested by N. Tesla and T. Edison as early as the end of the 19th century, but they did not gain recognition for the technical and economic reasons. The search for new sources of electric energy has aroused more interest in these ECs in the last years.

9.4. Other Types of Energy Converters

Heat removal from an electric machine is as an important problem as the improvement of its energy characteristics. In ventilated machines, the fan rotates together with the rotor and blows the cooling air over or through the machine. In low-speed control motors,

however, such a cooling system is ineffective. Designing a machine with two rotors can remedy the situation (Fig. 9.7). An internal rotor *1* with a large moment of inertia serves to drive the fan and an external rotor *2* in the form of a hollow cup acts as the rotor proper of the control motor. The machine has a common stator *3*. By virtue

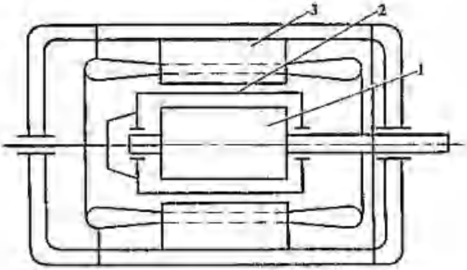


Fig. 9.7. A two-rotor energy converter

of the fact that the rotor *1* has a low resistance, its speed hardly changes with voltage, while the speed of the hollow rotor depends on the voltage.

The set of equations for a two-rotor machine includes two equations of motion, with M_e definable by the products of currents in the stator and rotor, and six voltage equations. The set of voltage equations comprises two equations for the stator windings on the α and β axes and four equations for the windings on the two rotors, arranged along the α and β axes respectively. The system of eight equations describes the processes of energy conversion in this type of machine.

For the analysis of an energy converter having n rotors and a common stator, we need to form two voltage equations for the stator, $2n$ voltage equations for the rotors, and n equations of motion. In all, the system will contain $3n + 2$ equations. The equations for an n-rotor machine can apply to an energy converter with a liquid rotor under certain assumptions.

Displacing the rotor with respect to the stator, so that in the limit the rotor almost comes in contact with the stator, gives a new energy converter called a motor with a rolling rotor (Fig. 9.8*a*). In this motor the rotor rolls by way of one-sided magnetic attraction. While in the energy converter with a uniform gap the torque is equal to the product of stator and rotor currents flowing in different phases, in the rolling-rotor motor the thrust arises from the currents due

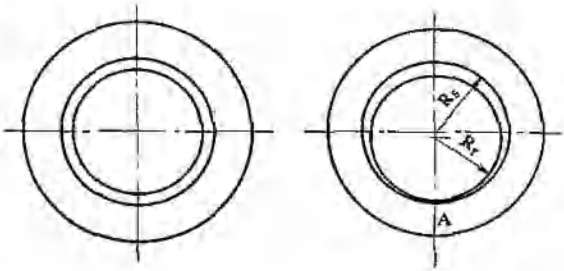
to one phase:

$$M_e = M_\alpha i_\alpha^s i_\alpha^r - M_\beta i_\beta^s i_\beta^r \qquad (9.14)$$

Since M_α and M_β differ substantially from each other in value, the thrust moment appears. The point of tangency A (Fig. 9.8b) revolves at the field velocity along the inner surface of the stator. The rotor revolves at a speed

$$n_2 = n_1 (R_s - R_r)/R_r = 60f (R_s - R_r)/R_r \qquad (9.15)$$

Since a rolling-rotor motor is made two-polar, the rotor speed depends on the supply line frequency and the difference between the



(a) (b)

Fig. 9.8. Displacement of the rotor with respect to the stator
(a) conventional energy converter; (b) rolling-rotor motor; R_s and R_r — stator and rotor radii respectively

radii of the stator and rotor. The motor combines, as it were, a mechanical speed reducer with an electric machine. This is a low-speed motor having a large torque. The disadvantages of such a motor include enhanced vibrations and a short life of bearings.

It should be noted that in a conventional machine the products of currents in one phase of the stator and the rotor do not determine a driving torque, though these currents need be taken into account in the study of vibrations. In a rolling-rotor motor, the current products of the above type do govern the driving torque.

To this type of energy converter also belongs a motor with a deformable rotor made from a flexible ferromagnetic material. As the field rotates, the rotor's deformation waves travel in synchronism with the magnetic field, so the rotor rolls over the stator interior at an angular velocity describable by the same relations as for the motor of Fig. 9.8.

If the rotor diameter is substantially smaller than the stator diameter (Fig. 9.9), the stator field that rotates at ω_s and acts on the rotor produces forces F_1 and F_2 differing in magnitude because the field near the stator is much stronger. When $F_1 \gg F_2$, the rotor turns over and begins to rotate in a direction opposite to that of the stator field (see Fig. 9.9). This effect is easy to produce by placing a metal ball inside the stator.

The interactions are more complex if we place into the stator bore two, three, and n rotors (Fig. 9.10). In constructing the mathematical

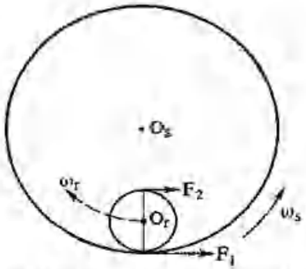


Fig. 9.9. A rolling-rotor machine at $F_1 \gg F_2$
O_s and O_r — stator and rotor centers respectively

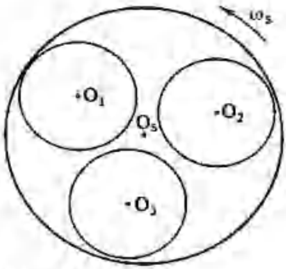


Fig. 9.10. A three-rotor machine
O_1, O_2, and O_3 — rotor centers and O_s — stator bore center

model of such an electromechanical system, consideration should be given to the effect of one rotor on the other and the interaction of each rotor field with the stator field. Practical applications of multirotor energy converters are yet unknown.

The above-described modifications of energy converters are made possible by changing the design of a rotor. As mentioned earlier, energy conversion in parametric devices results from the changes in the parameters of coefficients entering into voltage equations.

In Eqs. (1.34) and (2.3), the impedance matrix contains the terms of the form

$$(d/dt)\, Li, \quad (d/dt)\, Mi$$

Both currents and inductances usually vary in a harmonic manner. In transforming the equations, one strives to make up the equations with constants L and M and varying currents i.

It is possible to ensure energy conversion when i is constant and L and M undergo variations. The equations then have the terms of the form

$$i\,(d/dt)\, L, \quad i\,(d/dt)\, M$$

These are a few ways of designing an energy converter in which L and M may vary harmonically. A preferable design is the one shown in Fig. 9.11, where the air gap varies in a harmonic manner as the rotor is turning. This is an inductor generator which includes a star-shaped rotor *1* in which the number of slots is equal to half the number of slots in the stator. As the machine keeps running, the field set up by a dc winding *4* pulsates, thereby inducing the emf in stator windings *3*. In order that the pulsations may not heavily

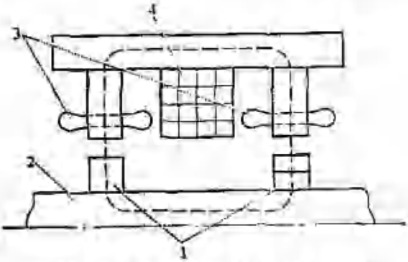


Fig. 9.11. An inductor generator

affect the torque and cause no vibrations, the machine is made complete with two rotors fitted on to the shaft *2* and disposed 90 electrical degrees from each other. Such a machine has no windings on the rotor, which is its advantage.

Parametric energy converters can also be divided into synchronous and asynchronous types. They can operate at the first harmonic or at upper harmonics in motoring, generating, braking, and transformer modes. Energy conversion occurs in the air gap. The windings that produce the field in the air gap can be either on the stator or on the rotor. The electromechanical equations offer a means of finding the ways for design of new ECs as well as of analyzing ECs that are complex in the principle of action from the viewpoint of classical theory.

Parametric ECs available today hold an important place in the list of electric machines. They include inductor generators for autonomous power systems, hf generators, low-speed motors, and other types.

Among various designs of ECs, worthy of mention are electric machines of the combined type. The designs that feature the most efficient combination of electric and magnetic circuits include single-armature converters (synchronous motor-dc generator units), three-phase transformers (units comprising three single-phase transformers),

motors performing the functions of amplifiers such as magnetic amplifiers and control motors, and a number of other designs.

The way of combining various electromechanical elements to form a desired aggregate is one of the main trends in electromechanics. Although combined energy converters were under study from the beginning of industrial application of electric machines and unique discoveries have been made recently in this branch of industry, much still remains to be done to produce new original designs.

From the viewpoint of mathematical theory, further development of ECs may take the course for the use of a greater number of voltage equations $[u] = [Z][I]$ *and equations of motion that would be more complex and of other forms.* This does not rule out the addition of new equations to the system of electromechanical equations that would be compatible with the voltage equations and equations of motion. There is a possibility of creation of energy converters for transforming thermal, light, microwave, and other forms of energy into electromechanical energy.

Chapter 10

Electric-Field and Electromagnetic-Field Energy Converters

10.1. Principles of Dual-Inverse Electrodynamics

Electromechanical energy converters act as concentrators of electromagnetic energy and heat. The expression for electromagnetic field energy density is of the form

$$W = (\varepsilon\varepsilon_0 E^2 + \mu\mu_0 H^2)/2 \tag{10.1}$$

where E and H are the electric field strength and magnetic field strength respectively; ε and μ are the relative permittivity and relative permeability respectively; ε_0 and μ_0 are the permittivity and permeability of vacuum; and $\varepsilon\varepsilon_0 E^2$ and $\mu\mu_0 H^2$ are the products that determine the energy density of the electric field and the magnetic field respectively.

Modern electromechanics mainly deals with magnetic-field energy converters, i.e. converters which concentrate energy in magnetic fields. The class of magnetic-field ECs has received most attention, and the great achievements in this area have contributed much to the progress in the science of the 20th century.

There are also electric-field energy converters, i.e. electrostatic converters with electric-field energy storage. They appeared earlier than magnetic-field ECs, but did not receive recognition as commercial power sources. Much still remains to be done to bring the theory of this class of converters to the appropriate level. It seems reasonable to extend the results in the area of magnetic-field ECs into the area of electric-field ECs and then, given the mathematical description of the processes of energy conversion in electric-field electromechanical systems, to design the converters of the desired performance.

The equations describing the processes of energy conversion in magnetic-field and electric-field electromechanical systems remain the same after the interchange of dual pairs in the following table

Electric charge Q_e	$\longleftrightarrow$	Magnetic charge Q_m
Electric flux Φ_e	$\longleftrightarrow$	Magnetic flux Φ_m
Voltage u	$\longleftrightarrow$	Current i
Electromotive force e	$\longleftrightarrow$	Magnetomotive force h

The advantage of the concept of magnetic charge is that we can establish the dual of the equations for an electromagnetic field. The mathematical description of the phenomena in electric fields thus yields the behavior in magnetic fields. Based on the theory of dual-inverse systems, we write down the following expressions

$$e = -(dQ_m/dt) = -(d\Psi_m/dt) = -(d/dt)\int_S \overline{B}\,d\overline{S}$$

$$= -\int_S (\partial\overline{B}/\partial t)\,d\overline{S} + \int_L (\overline{v}\times\overline{B})\,d\overline{l} \tag{10.2}$$

$$h = -(dQ_e/dt) = -(d\Psi_e/dt) = -(d/dt)\int_S \overline{D}\,d\overline{S}$$

$$= -\int_S (\partial\overline{D}/\partial t)\,d\overline{S} + \int_L (\overline{D}\times\overline{v})\,d\overline{l} \tag{10.3}$$

If $\overline{v}\perp\overline{B}\perp d\overline{l}$ and $\overline{v}\perp\overline{D}\perp d\overline{l}$, from (10.2) and (10.3) we get

$$e = Blv \tag{10.4}$$

$$h = Dlv \tag{10.5}$$

where l is the length of a conductor in a magnetic-field EC that is equal to the width of an electrode in an electric-field EC.

The phenomenon of electromagnetic induction is put to use in magnetic-field ECs, and that of electrostatic induction in electric-field ECs. Similarly, using two-phase, three-phase, and m-phase systems of electrodes, we can produce rotating electric fields.

The total energy of an electromagnetic field is given by

$$W = W_e + W_m = 0.5 \int_{v} (\overline{E}\overline{D} + \overline{H}\overline{B})\, dv \qquad (10.6)$$

Here $W_e = 0.5 \int_{v} \overline{E}\overline{D} dv$ is the energy of an electric field; and $W_m = 0.5 \int_{v} \overline{H}\overline{B} dv$ is the energy of a magnetic field.

The processes of energy conversion in magnetic-field electromechanical systems result from the interaction of magnetic charges (magnetic poles) and buildup of an electric field whose sources are electric charges. Energy conversion in electric-field systems stems from the interaction of electric charges and buildup of a magnetic field whose sources are magnetic charges.

The equations that can advantageously form the basis of the theory of electric-field ECs are the system of equations which are dual-inverse with respect to Maxwell's equations for moving media. The theory of this class of ECs may rely on the equations analogous to those for magnetic-field ECs.

10.2. The Equations for Electric-Field Energy Converters

Proceeding from the theory of dual-inverse electrodynamics, we formulate the following dual-inverse equations for the generalized electric-field energy converter using Eqs. (1.34) and (1.35):

$$\begin{vmatrix} i_\alpha^s \\ i_\alpha^r \\ i_\beta^r \\ i_\beta^s \end{vmatrix} = \begin{vmatrix} g_\alpha^s + (d/dt)\, C_\alpha^s & (d/dt)\, C & 0 & 0 \\ (d/dt)\, C & g_\alpha^r + (d/dt)\, C_\alpha^r & C_\beta^r \omega_r & C\omega_r \\ -C\omega_r & -C_\alpha^r \omega_r & g_\beta^r + (d/dt)\, C_\beta^r & (d/dt)\, C \\ 0 & 0 & (d/dt)\, C & g_\beta^s + (d/dt)\, C_\beta^s \end{vmatrix} \times \begin{vmatrix} u_\alpha^s \\ u_\alpha^r \\ u_\beta^r \\ u_\beta^s \end{vmatrix} \qquad (10.7)$$

$$M_e = C\,(u_\beta^s u_\alpha^r - u_\alpha^s u_\beta^r) \qquad (10.8)$$

Equations (10.7) and (10.8) follow from (1.34) and (1.35) after the interchange of u and i, inductances $L_{\alpha,\beta}^{s,r}$ and total capacitances $C_{\alpha,\beta}^{s,r}$, mutual inductances M and interelectrode capacitances C, and resistances $r_{\alpha,\beta}^{s,r}$ and conductances $g_{\alpha,\beta}^{s,r}$.

The total capacitance includes the capacitance C between the stator and rotor electrodes and self-capacitance c^s_α:

$$C^s_\alpha = C + c^s_\alpha \tag{10.9}$$

The dual of the generalized magnetic-field energy converter is the generalized electric-field EC with electrodes at potentials $u^{s,r}_{\alpha,\beta}$ instead of windings $w^{s,r}_{\alpha,\beta}$.

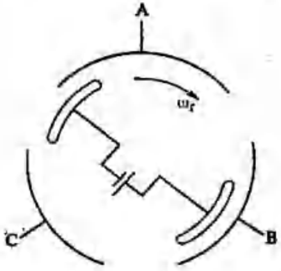


Fig. 10.1. A synchronous electric-field machine

Electric-field ECs, like magnetic-field ECs, include synchronous, asynchronous and, commutator machines, and also transformers.

In a synchronous electric-field machine the velocity $\omega_r = \omega_s$, and the rotor takes current from a dc circuit (Fig. 10.1). Electrodes A, B, and C (which replace windings) produce a rotating electric field, and interelectrode capacitance and self-capacitance present in the machine vary in a harmonic manner with field rotation.

The direct- and quadrature-axis equation for a synchronous machines has the form

$$\begin{vmatrix} i^s_d \\ i^r_d \\ i^r_q \\ i^s_q \end{vmatrix} = \begin{vmatrix} g^s_d + (d/dt)\,C^s_d & (d/dt)\,C & 0 & 0 \\ (d/dt)\,C & g^r_d + (d/dt)\,C^r_d & C^r_q\omega_r & C\omega_r \\ -C\omega_r & -C^r_d\omega_r & g^r_q + (d/dt)\,C^r_q & (d/dt)\,C \\ 0 & 0 & (d/dt)\,C & g^s_q + (d/dt)\,C^s_q \end{vmatrix} \times \begin{vmatrix} u^s_d \\ u^r_d \\ u^r_q \\ u^s_q \end{vmatrix} \tag{10.10}$$

$$M_e = C\,[u^s_q u^r_d - u^s_d u^r_q] \tag{10.11}$$

Here u^s_d, u^s_q, u^r_d, u^r_q are the d-q stator and rotor voltages in the two-phase machine; i^s_d, i^s_q, i^r_d, i^r_q are the currents in the stator and rotor electrodes; g^s_d, g^s_q, g^r_d, g^r_q are the conductances of the stator and rotor electrodes; $C^{s,r}_{d,q}$ represents total capacitances equal to

$$C^{s,r}_{d,q} = C + c^{s,r}_{d,q}$$

where $c_{d,q}^{s,r}$ represents the self-capacitances on the stator and rotor electrodes; and C stands for capacitances between the stator and rotor electrodes.

An asynchronous electric-field machine comes from the synchronous machine if in the latter we replace the rotor by a dielectric disk or use a rotor that has the same number of phases as the stator and apply to the rotor a voltage at a slip frequency.

A commutator machine can also be built from the synchronous machine by inserting a commutator into its ac circuit.

In an electric-field transformer it is an electric field that links the electrodes. The equations of this transformer have the form

$$\left|\begin{matrix} i_1 \\ -i_2 \end{matrix}\right| = \left|\begin{matrix} g_1+(d/dt)\,C_1 & (d/dt)\,C \\ (d/dt)\,C & g_2+(d/dt)\,C_2 \end{matrix}\right| \times \left|\begin{matrix} u_1 \\ u_2 \end{matrix}\right| \qquad (10.12)$$

Here subscripts 1 and 2 stand for the primary and secondary respectively. The current transformation ratio depends on the number of electrodes connected in parallel or series.

Mathematical models are available for the description of energy conversion processes in electric-field machines. However, commercial high-power converters of this class do not practically exist. This is because investigators have tried to copy magnetic-field converters in evolving electric-field types, though the latter occupy a special place among other converters and complement the former rather than replace them.

The most original and successful design of an electric-field energy converter is the convective-type Van de Graaff generator, also called the electrostatic accelerator (Fig. 10.2), in which a moving rubber belt transports charges being separated out by a corona discharge from one terminal and deposits them at the other, thereby producing a large potential difference. Even in its appearance this generator differs from conventional ECs. It is a 6-kW, 15-mln V, 1 000-mA setup 15 to 20 m high, placed in a casing and filled with a gas at high pressure. The generators of this type find use in test units.

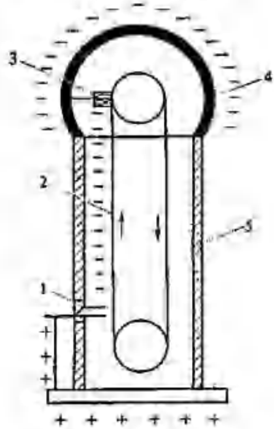


Fig. 10.2. Van de Graaff generator
1 — charging system with corona discharge; 2 — moving belt; 3 — electrode picking up charges from belt; 4 — high-voltage electrode (sphere); 5 — insulator

10.3. Parametric Electric-Field Energy Converters

In magnetic-field ECs the magnetic field energy is kept stored in the air gap thanks to the steel magnetic core. Because of the low breakdown voltage of air, near 30 kV cm^{-1}, the volume force and specific power of electric-field ECs is a factor of about 10^4 below those of magnetic-field ECs. Attempts were made to concentrate the energy of an electric field in liquid dielectrics. Academician A.F. Ioffe used kerosene for the purpose ($\varepsilon = 2$). On filling the gap with a compressed gas, the electric field strength can reach 600 kV cm^{-1}. Solid

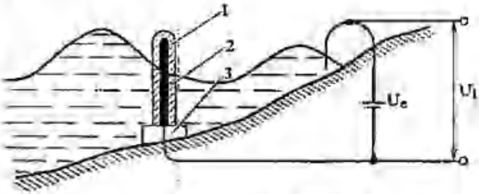


Fig. 10.3. An electric-field generator utilizing the energy of sea waves

dielectrics and ferroelectrics offer still greater possibilities. The crystals of barium titanate and potassium dihydrophosphate have a permittivity between 9×10^3 and 9×10^5. While magnetic-field ECs store the magnetic field energy in the air gap, electric-field ECs must concentrate the electric-field energy in liquid or solid dielectrics. Since energy converters must have elements moving with respect to one another, electric-field EC designs provide for mechanical gaps or envisage the use of liquid materials to serve the purpose of a rotor.

Parametric electric-field energy converters operate on the principle of a periodic change in capacitance at constant excitation voltage U_e. An example of such an energy converter is a generator utilizing the energy of surf (Fig. 10.3). Many attempts to convert the energy of chaotic motion of sea waves to electric energy by use of mechanical arrangements and conventional electric machines have not led to the acceptable engineering solutions.

The electric-field generator of Fig. 10.3 consists of a metal rod *1* coated with a layer of high-permittivity dielectric *2* and made fast on a base *3*. The rod serves as one of the capacitor plates and the sea surface acts as the other plate. A wave changes the device capacitance as it runs in at the rod and away from it; the capacitor charge varies at constant U_e, thereby inducing an alternating current $i = dq/dt$ at U_l that flows in the load circuit. Connecting such rods

in parallel and rectifying the current can give a substantial power at the output. Of course, such a generator differs from the conventional type, but it also belongs to electromechanical energy converters.

Of interest are electric-field machines with a liquid or gaseous rotor. There is a possibility of creating such machines that would have high outflow velocities (above the sound velocity) and strong electric fields.

It does not seem right to think that electric-field ECs may replace some conventional electric machines only where the latter do not produce the desired technical effect. Electric-field ECs are likely to find use in original applications in the near future.

10.4. Piezoelectric Energy Converters

A mechanical stress applied in a certain direction to the crystals of quartz, barium titanate, Rochelle salt, and others produces the electric charges of opposite sign of the crystal faces. This phenomenon is known as *the piezoelectric effect*. In quartz, this effect shows up along the electric axes of the piezocrystal and normal to its principal optic axis. A change in the direction of stress placed on the crystal causes the charges of reverse sign.

A reverse piezoelectric effect occurs under the influence of an electric field, which changes the linear dimensions of a crystal. Changing the electric field direction reverses the sign of mechanical stress.

The studies on piezoelectric materials reveal the linear relations between the deformation and charges at certain mechanical stresses. At strong stresses the relation becomes nonlinear, so that the crystal displays a dielectric hysteresis which resembles the *B-H* hysteresis curve of a steel core.

Piezoelectric energy converters are suitable for use as pulse generators in ignition systems. Attempts are made to employ these converters as hf pulse motors. Although the strains in crystals are negligible, there is a possibility of translating them into the desired linear displacements.

Piezoelectric ECs have two electrical and two mechanical terminals. Electromechanical resonance is of particular importance for these devices. As in other electric-field ECs, in piezoelectric devices the energy conversion takes place in a crystal. Extending the findings in electromechanics into the area of piezoelectric ECs, we can assume that the T equivalent circuit which includes capacitances and conductances is applicable for the representation of the intrinsic resistance of these converters.

The processes occurring in piezoelectric converters follow the basic laws of electromechanics and are amenable to the description

in terms of the theory of electromechanical energy conversion. Apart from the piezoelectric effect, a magnetostrictive effect occurs in certain ferromagnetic metals, which appears as a change in the volume and form of a ferromagnet when placed in a magnetic field. It is possible to create magnetostrictive vibrators utilizing the phenomenon of magnetostriction to convert the energy of a magnetic field to mechanical energy. The reverse magnetostrictive effect also occurs, which shows itself as a change in the magnetization of a ferromagnet when subjected to compression or tension.

Both the theory and the practice of implementing piezoelectric and magnetostrictive energy converters cannot as yet boast of great achievements. These converters are of much practical interest and are likely to find wide applications in the future.

10.5. Electromagnetic-Field Energy Converters

Let us have a look at electromagnetic-field ECs which store energy in an electromagnetic field (Fig. 10.4). These converters are not yet

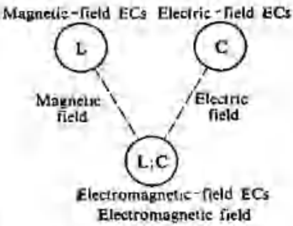


Fig. 10.4. Basic classes of electromechanical energy converters

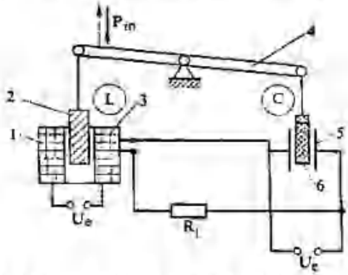


Fig. 10.5. Electromagnetic-field energy converter comprising a magnetic-field system (L) and an electric-field system (C)

commercially available, though they are widespread in nature. Biological energy converters can probably be thought to belong to this class of converters.

A machine illustrated in Fig. 10.5 can be taken as an example of the electromagnetic-field EC. The machine consists of two parts. The first part on the left of Fig. 10.5 includes a coil *1* in which a steel rod *2* reciprocates and an inductance coil *3* excited by a dc voltage U_e. This is in essence a linear magnetic-field EC coupled via an arm *4* to the other part of the machine, which consists of a capacit-

or *5* switched into a dc voltage circuit U_e and a dielectric *6* moving between the capacitor plates. The coil *3* is connected to the capacitor via a load resistance R_L. At a resonance frequency $\omega_0 = 1/\sqrt{LC}$, when $\omega_0 L = 1/\omega_0 C$, electromechanical resonance sets in. The supply line frequency and the mechanical frequency are the same, and the energy converter at electromechanical resonance exhibits the best characteristics.

This energy converter functionally and structurally combines the magnetic-field machine with the electric-field machine. The reactive power may not flow from the outside. The electromagnetic-field converter, just like any other electric machine, is convertible. The machine can operate in the generating mode, taking in the mechanical energy P_{in}, and can also act as a motor converting the electric energy absorbed in its electric circuit into mechanical energy.

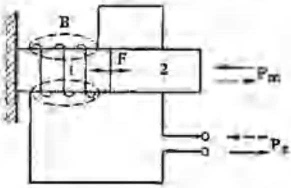


Fig. 10.6. Electromagnetic-field energy converter

In Fig. 10.6 is shown the schematic diagram of another electromagnetic converter which utilizes the changes in the linear dimensions of a core under the influence of an electromagnetic field. The converter consists of a magnetostrictive part *1* and piezoelectric part *2*. Permendure or pure nickel can be chosen for the construction of the magnetostrictive part and the solid solution of lead-zirconium titanate, which shows the best piezoelectric properties, for the construction of the part *2*.

The converter of Fig. 10.6 utilizes the magnetostrictive effect of changes in the shape and volume of a ferromagnet during its magnetization. Jointing mechanically a magnetostrictive and a piezoelectric material together, we can adjust both parts to resonance. In turn, the electric circuit which includes a winding for producing the field B and an internal resistance of the piezoelectric material comes to resonance with respect to the mechanical vibrations of the cores. In these conditions the process of energy conversion takes place. The energy converter of this type is reversible: it can transform energy from mechanical to electrical form, and vice versa. Although this converter differs in construction from conventional electric machines, the analysis of its operation is possible in the framework of the theory of electromechanical energy conversion.

Attempts are being made to produce composite magnetoelectrics from piezoelectric and magnetostrictive materials. In composite magnetoelectric materials, the magnetoelectric effect occurs through the mechanical interaction of a piezomagnetic and a piezoelectric

phase. The mechanical deformation establishes a linear link between the magnetic and the electric field.

In real magnetoelectric materials, which are mixtures of barium metatitanate and cobalt ferrite, the magnetoelectric effect accounts for merely a few percent of the energy input.

The further advances in electromechanics largely depend on the progress in the area of electrical-engineering materials. Thus, the creation of new materials which could behave like good electrical-sheet steels in strong magnetic fields and like dielectrics in strong electric fields would open the ways for producing new types of energy converters.

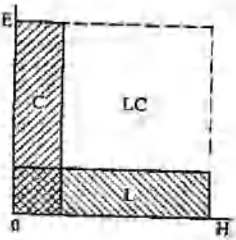


Fig. 10.7. The domains of different types of energy converters

Reciprocating-type electromagnetic machines can be made to function as rotating machines by coupling the magnetic-field structure to the electric-field structure through a common shaft. The converters of such a design would be able to offer a greater speed of response and operate without reactive power sources. In some design versions these converters would probably be more efficient and economically more advantageous than magnetic-field ECs.

Of particular significance are electromagnetic energy converters with a liquid rotor that afford the motor-generator operation by continuously regenerating the energy, energy converters with intrinsic feedback and computer-controlled converters. It would be important to combine the magnetic-field machine and the electric-field machine into a single unit in such a manner as is done in the classical designs of magnetic-field energy converters which have common structural parts for the flows of active and reactive powers.

Figure 10.7 shows (in E-H coordinates) the feasible region of electric-field machines (large values of E and small values of H), the feasible region of magnetic-field machines (large values of H and small values of E) and the feasible region of electromagnetic-field machines (large values of E and H). Composite-type ECs can exhibit still higher values of E and H.

To attain a high performance of electric machines, use is made of copper for magnetic-field ECs and gaseous and solid dielectrics of high permittivity for electric-field ECs. The above objective is difficult to accomplish if the working medium used is inorganic matter. The requirements on a working medium in electromagnetic ECs can be less stringent, so the list of active materials for electric machinery of this class can be fairly long.

The mathematical description of energy conversion in electromagnetic-field machines involves the use of the set of equations for a magnetic-field and an electric-field machine. This set of equations may have the form

$$\begin{vmatrix} u_L \\ i_C \end{vmatrix} = \begin{vmatrix} Z_L & 0 \\ 0 & Z_C \end{vmatrix} \times \begin{vmatrix} i_L \\ u_C \end{vmatrix} \tag{10.13}$$

$$M_{eL} = M\,(I^t I^r), \quad M_{eC} = C\,(u^t u^r) \tag{10.14}$$

Here u_L and i_L are the voltage and current submatrices for the magnetic-field machine, which are similar to (1.34) or (3.3) through (3.12); i_C and u_C are the current and voltage submatrices for the electric-field machine, which are similar to those for the magnetic-field machine; Z_L is the impedance matrix of the magnetic-field machine, such as in (1.34) or (3.3); Z_C is the impedance matrix of the electric-field machine, such as for the magnetic-field machine; M_{eL} is the torque of the magnetic-field machine; and M_{eC} is the torque of the electric-field machine.

Torque equations (10.14) can have any of the forms, from those of (1.35) to those of (3.12).

The set of equations (10.13) and (10.14) together with the equations of motion describes the behavior of electromagnetic ECs in transient and steady-state conditions.

It can readily be seen that, given the general theory of all the classes of electric machines, it will be more convenient to begin the analysis of energy conversion with the solution of equations for an electromagnetic-field machine and then, as a particular case, proceed with the study by solving the equations for magnetic-field and electric-field machines. The representation of electromagnetic-field ECs as a more general case of electric machines offers the possibility of a more comprehensive use of the theory of magnetic-field machines. First, the magnetic-field machine is a concentrator of energy; second, electromechanical resonance shows itself more vividly following the analysis of electromagnetic-field machines; and, third, energy converters, particularly electric-field ECs, separate out the charges, which is also the case for magnetic-field ECs since $i = dQ/dt$.

Electromechanics as a branch of science requires further development. There are the whole classes of electric machines the study of which is only at the starting stage. Research into the processes in electric-field and electromagnetic-field machines offers great promise for evolving new types of energy converters.

Chapter 11

Application of Experimental Design to Electric Machinery Analysis

11.1. General Information on the Theory of Experimental Design

The final end pursued by the investigations in the field of electric machinery is to solve the problems of synthesis of energy converters and thus design optimal machines. The technique of experimental design (ED) which can drastically cut down the number of experiments and the scope of calculations on analog and digital computers has recently gained still wider recognition. In the procedure of optimization of an electric machine, it becomes necessary to handle various problems, for example, to attain the desired performance of the machine and effect a maximum saving in materials, embody the design that would have better dynamic characteristics and improve energy characteristics, etc.

Given the mathematical model, the computer-aided solution of the problem comes to finding the optimal operating factors and making the right choice of the set of parameters by use of one variational method or another. As is known, a change in one parameter, others being kept constant, excludes the possibility of determining the desired relations and takes a great deal of machine time. The ED technique enables the correct choice of the course of the experiment, decreases the number of trials by a factor of 8 to 10, and opens the ways for optimization of energy converters.

The ED technique has a particular significance in the analysis of ECs on computing facilities. In the last years this approach has found use in most of the research works. We presume that the ED technique is generally known, so the text below will only give the systematic account of the experience gained in applying this technique in electromechanics.

The use of the ED technique in the analysis procedure permits the engineer to choose the strategy of performing tests according to the preliminarily drawn optimization scheme in order to obtain the relations between the parameters of an electric machine and its operating factors in a simple mathematical form, namely, as a polynomial

$$y=b_0+\sum_{i=1}^{n} b_i x_i+\sum_{i<j}^{n} b_{ij}x_i x_j+\sum_{i=1}^{n} b_{ii}x_i^2+\dots \qquad (11.1)$$

where b_0, b_i, b_{ij}, and b_{ii} are polynomial coefficients; x_i and x_j are variable parameters, or factors; y is the machine operating factor under study; and n is the number of variables (factors).

Thus, the ED technique offers the advantages of (1) high effectiveness since it calls for a smaller number of tests to obtain the desired information; (2) simultaneous study of the effect of a few variable parameters of the machine on its operating factors, and (3) the possibility of carrying out the tests so that the variance $\sigma^2\{b_i\}$ of polynomial coefficients in the case of random errors is at a minimum.

The ED theory relies on the fact that the results of any experiments in the n-dimensional factor space can be represented by linearized equations of the form

$$\begin{aligned} y_i &= x_{0i}b_0 + x_{1i}b_1 + x_{2i}b_2 + \ldots + x_{ki}b_k + \ldots \\ y_{i+1} &= x_{0(i+1)}\,b_0 + x_{1(i+1)}b_1 + x_{2\,(i+1)}\,b_2 + \ldots \\ &\quad \ldots + x_{k(i+1)}b_k + \ldots \end{aligned} \tag{11.2}$$

The results of N tests have the following matrix form

$$\overline{Y} = \overline{X}\overline{B} \tag{11.3}$$

where $\overline{Y}$ is the column vector of observations; $\overline{X}$ is the information matrix; and $\overline{B}$ is the column vector of coefficients. The equations for $\overline{B}$, $\overline{Y}$, and $\overline{X}$ are of the form

$$\overline{B} = \begin{vmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_N \end{vmatrix} \quad (11.4) \qquad \overline{Y} = \begin{vmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{vmatrix} \quad (11.5) \qquad \overline{X} = \begin{vmatrix} x_{01}x_{11} & \cdots & x_{k1} \\ x_{02}x_{12} & \cdots & x_{k2} \\ \cdot \quad \cdot & & \cdot \\ \cdot \quad \cdot & & \cdot \\ \cdot \quad \cdot & & \cdot \\ x_{0N}x_{1N} & \cdots & x_{kN} \end{vmatrix} \quad (11.6)$$

Deriving the matrix $\overline{B}$, i.e. defining the coefficients of the polynomial, involves the transposition and inversion of the information matrix $\overline{X}$. The final expression for $\overline{B}$ has the form

$$\overline{B} = \overline{C}^{-1}\overline{X}_t\overline{Y} \tag{11.7}$$

where $\overline{C}^{-1}$ is the inverse matrix with respect to $\overline{C} = \overline{X}_t\overline{X}$; and $\overline{X}_t$ is the transposed matrix.

The term 'design of experiments' essentially has to do with a special construction of the information matrix $\overline{X}$. The ways of how the information matrix is set up specify the types of design (orthogonal design, rotatable design, etc.). The structure of the information matrix determines the design formulas for estimating the polynomial coefficients and the variance of these coefficients. By specially

forming the matrix $\bar{X}$, one can impart, for example, an important property to the first-order design, estimate the polynomial coefficients with a variance that is a factor of N below the variance observed in conducting one test:

$$\sigma^2 \{b_i\} = \sigma^2 \{y\}/N \tag{11.8}$$

In practice the information matrix defines the values of variable parameters $x_1, x_2, \ldots, x_n$ when performing the tests. If we represent a two-dimensional factor space as a coordinate plane (Fig. 11.1), the 2^2 design matrix of the complete factorial experiment (CFE) has the form

Test No.	Variable factor	
	x_1	x_2
1	−1	−1
2	+1	−1
3	−1	−1
4	+1	+1

From the above matrix it follows that the experiment is set up only at points whose coordinates represent all combinations of the upper and lower values of variable factors, namely, at points *1*, *2*, *3*, and *4*. The second-order design matrix that has the property of rotatability requires conducting the experiment at additional points *5*, *6*, *7*, *8* and *9*. Depending on the information matrix, i.e. the points at which the experiment is to be run, the researcher obtains the corresponding polynomial relations between the variable factors and machine operating factors. These relations have important properties, namely, they enable a simple calculation of the polynomial coefficients and their variance. One attaches much importance to the calculation of polynomial coefficients because the method of least squares applied in experimental design gives cumbersome expressions for these coefficients. Thus in the simplest case of one factor, the formulas

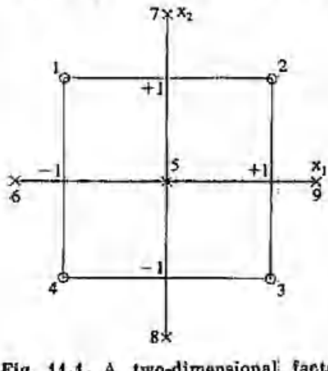


Fig. 11.1. A two-dimensional factor space

for b_0 and b_1 have the form

$$b_0 = \frac{\sum_{u=1}^{N} y_u \sum_{u=1}^{N} x_u^2 - \sum_{u=1}^{N} y_u x_u \sum_{u=1}^{N} x_u}{N \sum_{u=1}^{N} x_u^2 - \left(\sum_{u=1}^{N} x_u\right)^2} \tag{11.9}$$

$$b_1 = \frac{N \sum_{u=1}^{N} y_u x_u - \sum_{u=1}^{N} y_u \sum_{u=1}^{N} x_u}{N \sum_{u=1}^{N} x_u^2 - \left(\sum_{u=1}^{N} x_u\right)^2} \tag{11.10}$$

If the information matrix is orthogonal, the polynomial coefficients are found from simple formulas of the first-order design

$$b_0 = b_i = \sum_{u=1}^{N} x_{iu} y_u / N \tag{11.11}$$

and of the second-order design

$$b_i = \sum_{u=1}^{N} x_{ui} y_u / \sum_{u=1}^{N} x_{ui}^2 \tag{11.11a}$$

The variance of the calculated polynomial coefficients determines the accuracy of the polynomial as a whole if we use this polynomial to calculate the objective function (the machine operating factors) within the factor space.

The CFE information matrix for, say, the first-order design displays the optimal property of simplicity of the design formulas for polynomial coefficients and also the property of the identical and minimum variance of the coefficients. In the second-order design, the first and second properties are contradictory. If the matrix is orthogonal, the variance estimates of polynomial coefficients are not identical:

$$\sigma^2 \{b_i\} = \sigma^2 \{y\} / m \sum_{u=1}^{N} x_{ui}^2 \tag{11.12}$$

Hence, the prediction accuracy for the objective function y varies with the direction of motion in the factor space.

If the information matrix $\overline{X}$ exhibits the property of rotatability, the solution accuracy does not depend on the direction of motion,

and the design formulas have the form

$$b_0 = \frac{A}{N}\Big[2\lambda_y^2 (n+2)\sum_{u=1}^{N} x_{u0}y_u - 2\lambda_y C \sum_{i=1}^{N}\sum_{u=1}^{N} x_{ui}^2 y_u\Big] \quad (11.13)$$

$$b_{ii} = \frac{A}{N}\Big\{C^2[(n+2)\lambda_y - n]\sum_{u=1}^{N} x_{ui}^2 y_u + C(1-\lambda_y) \times \sum_{i=1}^{n}\sum_{u=1}^{N} x_{ui}^2 y_u - 2\lambda_y C \sum_{u=1}^{N} x_{u0}y_u\Big\} \quad (11.14)$$

$$b_i = \frac{C}{N}\sum_{u=1}^{N} x_{ui}y_u \quad (11.15)$$

$$b_{ij} = \frac{C^2}{N\lambda_y}\sum_{u=1}^{N} x_{ui}x_{uj}y_u \quad (11.16)$$

$$C = N/\sum_{u=1}^{N} x_{ui}^2 \quad (11.17)$$

$$A = \frac{1}{2\lambda_y[(n+2)\lambda_y - n]} \quad (11.18)$$

$$\lambda_y = \frac{nN\sum_{\omega=1}^{k} N_\omega p_\omega^y}{(n+2)\Big(\sum_{\omega=1}^{k} N_\omega p_\omega^2\Big)^2} \quad (11.19)$$

where N_ω is the number of points on a sphere of radius p_ω; and k is the number of spheres ($k = 3$).

The goodness of fit of the polynomial to the object under study defines the degree to which the objective function $\overline{y}$ obtained from the experiment corresponds to the objective function $\hat{y}$ calculated with the polynomial. The quantity characterizing the discrepancy between the calculated and experimental values is the inadequacy variance defined by the formula

$$S_{ad}^2 = \frac{1}{N-d}\sum_{\eta=1}^{N} (\overline{y}_v - \hat{y}_v)^2 \quad (11.20)$$

where d is the number of significant terms in the approximating polynomial.

The test of the hypothesis of adequacy is made by use of Fisher's variance ratio (F-test):

$$F = S_{ad}^2/S^2\{y\} \tag{11.21}$$

If the calculated value of the ratio is lower than the critical ratio F_{cr} taken from the corresponding table (see Table AIV.5) at the given significance level q_s, the description is considered to fit the object under study.

The design matrices applied to the analysis of electric machines introduce the definite features in the technique of experimental design. The description of these features is given below.

11.2. The Technique of Experimental Design Applied in Electromechanics

Consider the main classes of problems handled by the ED technique. The first class, being most closely related to the classical scheme of experimental design, includes the problems of testing electric machines, their elements, or electromechanical systems. The second class covers the problems of analysis of physical and mathematical models and analogs which are too specific and complex to be used directly for the solution to the problems of synthesis of electric machines. The third class includes approximation-type problems for the cases where it is possible to replace the complex mathematical model of energy conversion in electric machines by a simple polynomial noted for an explicit link between the variable parameters and the machine operating factors. The ED technique applied to the solution of each class of problems shows distinguishing features which we can illustrate by the examples that follow.

Consider the possibilities of the ED technique by examining a simple, but fairly instructive, example. For the accelerated reliability tests to be run (both evaluation and control tests), the accelerated test ratio k_a must be known, i.e. the ratio between the test time under normal conditions and that under accelerated test conditions. The ratio k_a can be determined from an experiment on a test object. As a rule there is a need to know a functional relation between k_a and accelerated test factors rather than the unique value of k_a. Such an approach offers the flexibility and universality of test procedures, enables the use of the evolved technique to carry out tests not only at a certain plant and on a particular test bed but at various plants differing in technical and production basis.

The objective of testing a machine for reliability is to check the machine serviceability over the specified time period under given operating conditions. To speed up the test procedure, the machine is run under more stringent conditions of operation. The test time is cut down depending on the test stringency being chosen. For machine

with a mean service life of 10 000 hours, the determination of the set of accelerated test condition factors which enable cutting the test time by a factor of 15 to 20 is an urgent problem from the economy point of view. The solution to the problem depends on the estimation of the functional relations between the machine operating factors and accelerated test condition factors. These relations can be defined experimentally using the ED technique.

Figure 11.2 illustrates the schematic diagram of conducting accelerated tests on an energy converter for nine accelerated test fac-

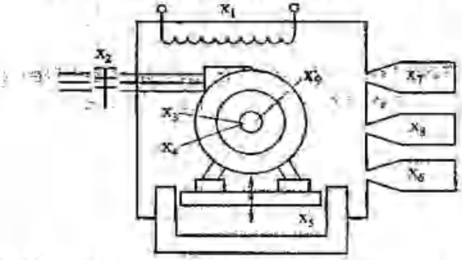


Fig. 11.2. The schematic diagram for conducting accelerated tests on an energy converter

tors. To these factors belong the ambient temperature x_1; repetition rate of starting, x_2; angular velocity x_3; load on the shaft, x_4; vibrations x_5; aggressivity of a medium, x_6; humidity x_7; dust content x_8; and grade of a lubricant, x_9. Knowing the machine operation time t_{op} recorded during the tests, we can establish the relation $t_{op} = f(x_1, \ldots, x_9)$ and thereby solve the stated problem using the ED technique.

What complicates the procedure of performing such tests is the nonuniformity of electric machines of even the same batch. The causes are the differences in electrical-sheet steel properties, variations in phase resistances and air-gap dimensions, differences in the performance of bearings, and other causes associated with manufacturing errors. These causes necessitate carrying tests on a few machines simultaneously and determining the mean operation time $\bar{t}_{op}$ and variance $S^2\{t_{op}\}$ from the test results. A higher value of the operation time variance calls for performing more repetition tests on more electric machines. To determine the dependence $t_{op} = f(x_1)$ requires staging a set of experiments at several values of x_1. The variance of the coefficient b_1 that accounts for the effect of temperature changes during the machine operation time is found from the formula $\sigma^2\{b_1\} = \sigma^2\{t_{op}\}/2$. This coefficient calculated by the ED technique evidently has a lower variance $\sigma^2\{b_1\} = \sigma^2\{t_{op}\}/4$.

As the number of accelerated test factors grows, the advantages of ED become still greater, as is clear from Table 11.1.

Table 11.1

The Number of Tests to be Run to Estimate Polynomial Coefficients Using the Classical Procedure and the ED Technique

Number of factors	Number of tests	
	classical procedure	ED technique
2	6	4
3	16	8
4	40	18
5	96	32

One of the important reasons of using the ED technique is that a smaller number of tests enables a more accurate calculation of the coefficients that account for the influence of test factors. The ED technique has a decided advantage in that the set of experiments run according to the design matrix does not tend to accumulate the error. Rather, the influence of the error decreases in determining the polynomial coefficients. For example, the formula for defining the polynomial coefficient variance $\sigma^2\{b_i\}$ in the CFE or FFE (fractional factorial experiment) has the form

$$\sigma^2\{b_i\} = \sigma^2\{y\}/N \tag{11.22}$$

where N is the number of experimental points in the factor space.

The ED technique has one more essential point in its favor. The thing is that where the test involves a large number of accelerated test factors, the analysis of the performance of the machine under test becomes a difficult problem. It is quite possible that a combination of various factors rather than each factor separately affects the machine operation time. The joint effect of a few accelerated test factors is possible to estimate without a complete knowledge of the physical nature of the processes by use of the ED technique. Consideration here is given to the limits on the number of machines put to test and the total test time, a large number of test factors, and also a high cost of the experiment.

The choice of the design matrix and test schedule and also the calculation of polynomial coefficients are made according to the conventional ED procedure. Let us only note two elements in the organization of the studies on electric machines: one relates to the choice of the limits of variation of the factors, and the other to the experimental technique.

The procedure of designing the experiment begins with the estimation of the variation range of accelerated test factors. This is one of the important stages when making the test on electric machines. On the one hand, it is advantageous that the variation range should be as wide as possible, for the test factors will then produce a greater effect and the test can be run faster. On the other hand, too wide a variation range involves a risk of qualitatively changing the physical essence of the object under study. For example, in testing a machine over a wide interval of variation of the ambient temperature (factor x_1), the grease may leak out of a bearing subassembly causing it to operate in the abnormal conditions, i.e. the conditions of dry friction. A small interval of variation of the factor is also undesirable because of a longer test time.

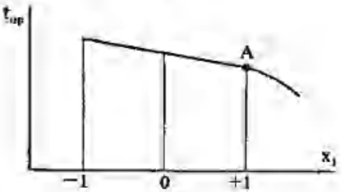


Fig. 11.3. Machine-operation time versus temperature

To make an estimate of the number of machines to be tested, Table 11.2 gives the data on determining the variation ranges after the first, second, and the third trial. Note that these data refer only to the first-order design; the second-order design will call for a much greater number of tests.

Table 11.2

Variation Ranges for Various Factorial Designs

Trial No.	Design				
	2^3CFE	2^4CFE	2^5CFE	2^{5-1}FFE	2^{6-1}FFE
1	8	16	32	16	32
2	16	32	64	32	64
3	32	64	128	64	128

The practical recommendations of how to choose the variation limits reduce to a thorough analysis of a priori information on the test of machines (analogs) depending on each of the factors set up separately (Fig. 11.3). So long as the one-dimensional relation $t_{op} = f(x_1)$ is linear or approaches the linear form, the entire range can be chosen as the interval of factor variation. If the range extends beyond the limit A, the polynomial model may become inadequate.

That is why in performing tests on costly objects such as electric machines, it is obligatory to collect a maximum amount of a priori information which must be given proper scrutiny to make a decision on the range of variables.

Assume the stated problem requires to estimate $k_a = f(x_i)$, where x_i represents accelerated test factors. Consider an example of the solution to this problem using a modified version of the ED technique. For the illustrative purpose, we will handle the problem for four test factors (a larger number of factors requires the use of standard procedures set forth in the ED theory). Let the test factors be the test chamber temperature x_1, machine vibration x_2 (of course more intensive than the normal), angular velocity x_3 and time factor t which is certainly present in the experiments of this type. The relations $k_a = f(x_1, x_2, x_3, t)$ can be found by use of the formalized ED procedures which involve the following: gathering a priori information; determining the curvature of the factor space through the construction of one-dimensional sections of $k_a = f(x_1)$, $k_a = f(x_2)$, and $k_a = f(x_3)$; choosing the lower and upper limits of variation of the factors; carrying out the experiment on appropriate test beds; giving the mathematical treatment of the results and interpreting them.

Such a way of the direct solution to the stated problem by the formalized ED procedures means that we have to realize the matrix of a power of at least 2^4 (Table 11.3). The ED technique enables us to perform the tests successfully and determine the relations $k_a = f(x_1, x_2, x_3, t)$ if the constraints on the scope of the experiment (i.e. the number of machines to be put to test) are reasonable and an ample amount of a priori information is available, namely, the results of preliminary experiments and properly chosen variation ranges, particularly the variation range of the time factor.

Experience shows that it is more advantageous to use not the standard ED methods but a somewhat refined technique based on the same rules of the ED theory. Account must be taken of the fact that an increase in the scope of the experiment requires a corresponding increase in the number of test beds and in the test time.

In solving the reliability problems and estimating the functional relations of the type $k_a = f(x_i)$, a little-known branch of electric machine engineering has to be kept in mind. What is meant here is the physics of the processes of aging and wear of electric machine elements such as commutators, bearings, slip rings, and a winding under the complex action of elevated temperatures, intensified vibrations, and increased speeds. The data on these processes, let alone the data in analytical form, is usually not available for a particular machine or for a batch of the machines of the same series. In view of this fact, setting up the variation limits on the time factor becomes problematic. An underestimated upper limit does not allow for the

Table 11.3

The 2^4 Design Matrix

Machine No.	Test No.	Test factor				Response
		x_1 (t)	x_2	x_3	x_4	y
1	1	−	−	−	−	y_1
	2	+	−	−	−	y_2
2	3	−	+	−	−	y_3
	4	+	+	−	−	y_4
3	5	−	−	+	−	y_5
	6	+	−	+	−	y_6
4	7	−	+	+	−	y_7
	8	+	+	+	−	y_8
5	9	−	−	−	+	y_9
	10	+	−	−	+	y_{10}
6	11	−	+	−	+	y_{11}
	12	+	+	−	+	y_{12}
7	13	−	−	+	+	y_{13}
	14	+	−	+	+	y_{14}
8	15	−	+	+	+	y_{15}
	16	+	+	+	+	y_{16}
9	17	0	0	0	0	
10	18	0	0	0	0	

complete utilization of the experiment. The obtained values of the accelerated test coefficients prove lower than the possible ones, which, in the final analysis, lengthens out the accelerated reliability tests. An overestimated upper limit of the time factor may render the experiment unsuccessful because at least one unrealizable row of the matrix means a failure of the entire experiment or, at best, the transition to an upper level of fractioning, i.e. the transition from the 2^4 complete factorial to the 2^{4-1} fractional factorial, from the 2^{4-1} to the 2^{4-2} factorial, etc.

The sequence of the solution to this problem is as follows. Prior to conducting the experiment, the levels of the time factor are left normal. The lower, zero, and upper levels are found only for the accelerated test factors such as temperature, vibration, and speed. In doing this, it proves possible to implement the 2^4 design matrix for the complete factorial experiment. The first factor is the factor of time. The time factor permits running tests not on 16 but on 8 machines in the 2^4 CFE. One and the same machine is put to test both at the lower and at the upper level of the time factor.

Each of the tests is run until all the elements under investigation fail to operate. This means that if, for example, a bearing subassembly fails, the time of failure is recorded, the subassembly is replaced and the experiment is continued, but the new counterpart is taken out of consideration. If, further, the sparking at the commutator exceeds the permissible level and it is difficult to remedy the defect, the commutator is left to stand and the machine to run since the operator has received the information on the failure of the commutator and excluded it from consideration.

The measurements of the optimization parameter (controllable parameters such as noise, runout time, sparking, insulation resistance) are taken continuously, whenever feasible, rather than at individual points corresponding to the vertices of the hypercube under study. This can be the case for, say, the commutator unit and slip rings. The above measurements can also be made discretely for, say, bearings and insulation over insignificant time intervals, from 24 to 48 hours.

This sequence of realization of the design matrix shows important features which make the solution of the stated problem possible. The point is that the mathematical model of the process, i.e. the polynomial dependences of the optimization parameter on the test factors, including the time factor, proves optimal since the record of the levels and the intervals of variation of the time factor is kept after obtaining the experimental data and performing the mathematical treatment. The next step involves the estimation of the accelerated test factors from the obtained polynomial dependences.

Another advantage derived from the absence of fixed levels and intervals of variation in time is the following. If the obtained mathematical model is inadequate, i.e. the mathematical description does not correspond to the real process, we can pass to the nonlinear transformation of coordinates, replacing for the purpose the independent variables by new ones, $\xi_i = x_i^{\alpha_i}$ or $\xi_i = \ln x_i$, and thus converting to the logarithmic or exponential time scale. This feature does much toward the success of the experiment.

If the transformation of coordinates does not give the desired effect (the model remains inadequate), the entire time interval can be split up into a few portions so as to obtain the adequate model.

The four-factor CFE matrix (Table 11.4) realized by the above method and required to estimate the accelerated test ratios is simultaneously the three-factor CFE matrix employed to determine the time to failure (the fourth factor, i.e. time, undergoes transformation to become the optimization parameter).

The design so set up provides the relations between the failure time and test factors, this being a rather important result of the experiment. Also, the scope of the experiment is cut down practic-

Table 11.4

The CFE Design Matrix for Estimation of Failure Time

Machine No.	Test No.	Test factor			Response t
		x_1	x_2	x_3	
1	1	—	—	—	t_1
2	2	+	—	—	t_2
3	3	—	+	—	t_3
4	4	+	+	—	t_4
5	5	—	—	+	t_5
6	6	+	—	+	t_6
7	7	—	+	+	t_7
8	8	+	+	+	t_8
9	9	0	0	0	t_9
10	10	0	0	0	t_{10}

ally by one half. The derived relations generally give different accelerated test ratios for each of the elements such as bearings, commutators, slip rings, and windings. The simultaneous solution of equations offers the possibility of obtaining a common accelerated test ratio for the entire item.

Thus the described method can determine the accelerated test ratios for various elements of the machine under test and also the failure time as a function of the test factors in severe test conditions, i.e. in the conditions of both the limited amount of a priori information and the limited scope of the experiment. This method can be extended to cover the designs for various numbers of accelerated test factors.

The use of experimental design to evaluate accelerated test ratios in testing electric machinery represents one of the complex examples illustrative of the potentialities of the ED technique.

Consider the application of this technique in studying mathematical and physical models, for example, an induction machine model developed from the mathematical theory of electric machinery and set up on an analog computer or an induction-machine magnetic core model formed on electrical conductive paper. The induction machine model in the form of a computer analog permits a fast investigation of the most diverse modes of operation of the machine (its dynamic and static action, changes in the parameters with time, etc.).

However, the attempts at searching for the optimal parameters of a machine prove impracticable because of the unstable analog

and a long time required to rearrange the gain factors of the analog. This is also true of the physical model built up on conductive paper. The calculation of fields and admittances enables evaluating practically any pattern of the magnetic circuit, but the search for the optimal geometric parameters of the machine requires a multiple reference to the problem setup. In either of the two cases it is necessary to find a convenient form of presentation of the information derived

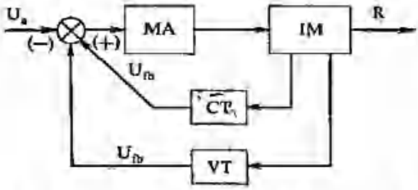


Fig. 11.4. The system comprising a magnetic amplifier (MA) and induction motor (IM)

CT — current transformer; VT — voltage transformer; U_a — accelerated test voltage; U_{fb} — feedback voltage; R — response

from the models so as to use it further for the solution of optimization problems. The polynomial notation is a convenient form of presentation.

Let us illustrate the features of ED for the second class of problems by considering an example of the analog of an induction motor-magnetic amplifier system. The block diagram of this system with current and voltage feedback mechanisms is shown in Fig. 11.4. To analyze the induction machine construction, we need to study the machine analog and obtain the functional relation

$$\Delta n = f(K_U, K_I)$$

where Δn is the motor's angular velocity difference with a change in the load torque over the specified limits: K_U and K_I are voltage and current feedback factors.

The analysis of the machine design should be made within the predetermined ranges of variation of factors x_1 and x_2: $x_1 = K_U = 1.5$ to 2.1 and $x_2 = K_I = 2.4$ to 3.6.

The ED technique is expedient for use in dealing with the given problem for the following reason. We need to obtain a convenient and concise relation between Δn and K_U, K_I in the form of a polynomial:

$$\Delta n = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 + b_{11}x_1^2 + b_{22}x_2^2 + \ldots \tag{11.23}$$

We should decrease the effect of instability of the analog on the results of experimental investigations. Cutting down the number of experimental (design) points in the problem of interest is not the main objective. The requirement for the stable limits within which the factors undergo changes brings in a number of features in the procedure of planning the experiment. Since the range of variation of the factors is fairly large and so the analogs obtained can be inadequate, we should raise the order of experimentation. At the first stage of studying the analog, it is necessary to use first-order designs of the CFE and FFE and then pass to second-order designs if the model is inadequate.

The second-order design substantially differs from the first-order design in that it does not include the experimental stages which combine simple design formulas with a high accuracy of polynomials.

In solving the problems in electromechanics, use is mainly made of two types of design—the orthogonal and rotatable types. The first type offers the advantages over the rotatable type in that it employs relatively simple design formulas and needs a smaller number of tests to be run.

In the example under consideration we use the second-order orthogonal design to define the relation $\Delta n = f(K_U, K_I)$ since the first-order design gives an inadequate polynomial. The matrix of orthogonal central composite design (OCCD) is given in Table 11.5.

Table 11.5

The OCCD Matrix

Test No.	x_0	x_1	x_2	x_1x_2	$\tilde{x}_1=x_1^2-0.666$	$\tilde{x}_2=x_2^2-0.666$	Objective function		Prediction by RCCD
							Δn	$\Delta \hat{n}$	
1	+1	−1	−1	+1	0.333	0.333	79	76.3	76
2	+1	+1	−1	−1	0.333	0.333	93	93.5	91
3	+1	−1	+1	−1	0.333	0.333	58	64.6	58
4	+1	+1	+1	+1	0.333	0.333	70	71.3	74
5	+1	−1	0	0	0.333	−0.666	62	58.5	64.5
6	+1	+1	0	0	0.333	−0.666	68	69.1	70.1
7	+1	0	−1	0	−0.666	0.333	100	85.6	98
8	+1	0	+1	0	−0.666	0.333	60	73	57
9	+1	0	0	0	−0.666	−0.666	66	67.3	69.1

The polynomial for Δn has the form

$$\Delta n = 67.3 + 5.34x_1 - 6.33x_2 - 0.5x_1x_2 - 3.5x_1^2 + 12x_2^2 \tag{11.24}$$

It should be noted that for the given range of variation of K_U and K_I, even the second-order design gives a polynomial of low accuracy. This can be seen from the results contained in the extreme right column, which are the values of Δn calculated with the polynomial. The discrepancies at some points, especially for tests Nos. 7 and 8 are so large that they go beyond the specified limits.

A different accuracy of prediction in various directions of the factor space, i.e. at various combination of K_U and K_I, is one of the grave disadvantages of the second-order orthogonal design. Such a polynomial is impracticable for use in the analysis and synthesis of the machine because the search for the optimal geometric parameters of the machine is done for various combinations of K_U and K_I and the polynomial error may result in a false extremum.

Where the accuracy of the polynomial over the specified intervals of variation of the factors is of primary importance, the investigation of analogs and physical models should rely on rotatable central composite design (RCCD). This type of design brings about an insignificant increase in the number of tests to be performed in comparison with the orthogonal type but offers a higher accuracy (Table 11.6).

Table 11.6

The Number of Tests Run in OCCD and RCCD Methods

Number of factors	OCCD	RCCD
2	9	13
3	15	20
4	25	31

For comparison between the accuracies of calculation of Δn employing the OCCD and RCCD, Table 11.5 lists the values of $\Delta \hat{n}$ obtained with the aid of rotatable design. Let us note that in the analysis of physical and mathematical models of electric machines rotatable designs are carried out with the use of modern digital computers. It should also be pointed out that there are other ways of experimental design in dealing with the class of problems under consideration if the model obtained with the aid of the first-order design turns out to be inadequate. The approaches mainly involve the replacement of variables and splitting of the variation range of factors into two subregions. In both cases the objective is to try to reduce the problem to the first-order design. However, these approaches are not always convenient and responsible for a number of difficulties in handling optimization problems. The second-order des-

ign, though being more complex, offers good results, given routine programs run on digital computers. One of the main advantages of this type of design is that it enables analytical estimation of the extremum for a certain class of problems.

The problems of the third class—approximation problems—are rather often dealt with in the analysis of dynamic and static modes of operation of electric machines. One of the specific problems of this class relates to the investigation of transients in induction machines supplied from nonsinusoidal voltage sources (see Ch. 5). The equations for an induction machine have the form of (5.1) and (5.2). It takes the M-220 computer only seven minutes to solve this system of differential equations involving nonsinusoidal voltages and four harmonics. The optimization problems call for making 300 to 500 such solutions. This procedure is not always realizable on digital computers. The way out is to replace the system of differential equations by a simpler and more concise relation, namely, a polynomial.

The basic feature of ED as regards the above class of problems is that the equation for the object of interest enables its rigorous mathematical treatment; in its explicit form, the equation is cumbersome and inconvenient. The equation is solvable with the aid of digital computers or other means. It uniquely determines the relation between the variable parameters and objective functions (this means that the object under study is stable). For this class of problems the term 'experimental design' is replaceable by the term 'computational design'. Specific to the third class of problems is the use of design matrices for performing the computation which permits replacing the complex mathematical model by a simple polynomial model. The accuracy of approximation is the main requirement here; the requirement for minimizing the number of design points is not rigid.

Consider the statement of the problem involving the estimation of an approximating polynomial for the speedup time t_{sp} of an induction motor with a 20% deviation of the values of the following quantities from the nominal values: rotor resistance x_1, stator resistance x_2, and moment of inertia, x_3. The matrix of the CFE, 2^3 design is given in Table 11.7. Under the experiment is meant here the solution of differential equations (5.1) and (5.2) on a digital computer at the fixed values of the machine parameters in accordance with the design matrix.

Determine the polynomial coefficients from corresponding design formulas. Since the repeat calculations of the machine factors at the same values of parameters give the same results, the experiment variance is $\sigma^2\{t_{sp}\} = 0$, and, hence, all coefficients of the polynomial are significant. A check of the model for adequacy against the F-test at vertices of the factor space cannot be carried out.

In solving the approximation problems, the check of polynomial adequacy for t_{sp} is made by selecting other points in the factor space.

Table 11.7

The 2^3 Design, CFE Matrix

Variable parameter	x_1 (r^r), Ω	x_2 (r^s), Ω	x_3 (J), kgf m s^2	x_1x_2	x_1x_3	x_2x_3	x_0	Objective function
Reference level	3.8	3.57	1.52×10^{-3}					
Upper level +	4.52	4.28	1.82×10^{-3}					
Lower level −	3.08	2.86	1.22×10^{-3}					
Designation	z_1	z_2	z_3	z_4	z_5	z_6	z_7	$t_{sp}\times10^{-1}$, s
Test No. 1	−	−	−	+	+	+	+	10
2	+	−	−	−	−	+	+	5
3	−	+	−	−	+	−	+	4.5
4	+	+	−	+	−	−	+	3.5
5	−	−	+	+	−	−	+	11
6	+	−	+	−	+	−	+	7
7	−	+	+	−	−	+	+	6.5
8	+	+	+	+	+	+	+	5

As is known, in performing the experiments at vertices (Fig. 11.5), the largest discrepancy between t_{sp} and $\hat{t}_{sp}$ occurs at points *1*, *2*, *3*, *4* and at central point *0*. Therefore, the check for the model adequacy should be done at these points using Cochran's test on the homogeneity of inadequacy variance estimates:

$$\sigma_{\max} = \max\,[S^2_{q\,ad}] / \sum_{q=1}^{k} S^2_{q\,ad} \tag{11.25}$$

where $S^2_{q\ ad}\{t_{sp}\} = (t_{sp\ q} - \hat{t}_{sp\ q})^2$ is the inadequacy variance for the qth check point; and k is the number of check points.

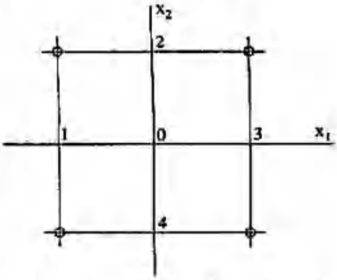


Fig. 11.5. The chart for performing checks at vertices

If the found value of $\sigma_{\max}$ is smaller than σ_{cr} taken from the table for corresponding degrees of freedom, $\nu_{1v} = 1$ and $\nu_{2v} = k$, and for the chosen significance level q (commonly equal to 5%), we accept the hypothesis of variance homogeneity. Next we make an

estimate of the generalized variance of model inadequacy:

$$s_{ad}\{t_{sp}\}=\sum_{q=1}^{k} S_{q\,ad}^{2}/k \tag{11.26}$$

If the value of $S\{t_{sp}\}$ satisfies the accuracy required of the approximation, the polynomial is adequate; if it does not, we should pass to the designs of higher order or reduce the intervals of variation of the factors. Since the limits on the number of checks in performing the computations are not rigid, it is advisable to take additional points inside the factor space using for the purpose the table of random numbers.

Applying the ED technique to the solution of approximation problems, it should be kept in mind that any approximation, while offering the simplicity of functional relations, introduces additional errors in the calculation results. Whether the ED technique is expedient for use in the solution of approximation problems is estimated for each particular case.

11.3. Transition from Experimental Design to Optimization

The solution to most optimization problems in electric machinery presents difficulties since the functional relations between the variable parameters of a machine and its operating factors have an implicit form. *The ED technique substantially facilitates the solution of optimization problems, for it enables obtaining an explicit polynomial relation between the machine's variable parameters and objective functions.* It thus becomes possible to solve optimally a number of problems by analytical methods.

If the solution of optimization problems is sought by numerical methods, polynomial and two-dimensional sections constructed with the aid of the polynomials offer a well-grounded approach to the choice of the numerical search method. The choice of the search method is an important and urgent problem.

The practice of optimum design of electric machines shows that certain search methods in current use cannot be considered universal. Random walk (random search) methods are applicable where the number of variable parameters is large, but they offer a low probability of obtaining global optima. Gradient methods and alternating direction methods lead rather fast to local optima, but they require a preliminary experimental adjustment when solving specific problems.

Relaxation methods of the Gauss-Seidel type (successive displacements methods) show algorithmic simplicity, but fail to work in the "valley" conditions and under nonlinear constraints. Local

coordinate methods of dynamic programming are free from the shortcomings of the Gauss-Seidel method. However, they take a relatively long search time at a high accuracy of the solution.

In each particular case, the choice of the method must be made after a definite analysis of the factor space region within which the solution is to be sought. Such an analysis can be performed by constructing a series of two-dimensional sections with the aid of polynomials. By applying the ED technique it is possible to obtain the relations between the motor operating factors and variable parameters such as rotor resistance R^r and stator resistance R^s. Thus, for the A-42-6 motor

$$
\begin{aligned}
\eta &= 0.851 - 0.014R^r - 0.022R^s \\
\cos\varphi &= 0.83 + 0.036R^r - 0.018R^s + 0.026R^rR^s \\
t_{st} &= 6.56 - 1.44R^r - 1.69R^s + 0.84R^rR^s \\
I_{im} &= 0.38 - 0.033R^r - 0.041R^s \\
M_{im} &= 0.83 + 0.036R^r - 0.039R^s + 0.064R^rR^s
\end{aligned}
\tag{11.27}
$$

The above polynomial relations are rather simple since the problem being solved involves the variation of only two variable parameters. It could be assumed that the search for an optimum under these conditions does not encounter difficulties. However, the analysis of variants of the solutions provides different results.

The most specific cases met with in optimization of electric machines result from a sharply reduced region of search. The zones in which the objective function satisfies the imposed constraints are small, so it is difficult to obtain such zones for the organization of stepwise procedures without constructing the sections. The solution of optimization problems of this class presents difficulties associated with the choice of the reference point needed to initiate the directed search. This is also the case for a large class of problems involved in the analysis of electric machines. What accounts for this fact is that an induction machine as the object under analysis is a fairly well studied object from both the theoretical and the practical standpoint. The machines in series production are rather close to optimal ones if the imposed constraints are taken into account. Hence, the possibilities of improving their characteristics are not large, and so the zone of permissible search for the solution is small.

The polynomial relations and optimum search zones obtained by the ED technique assist in choosing the reference point needed for the organization of the directed search. Problems are met with in practice in which the search region is discontinuous. In handling these problems by numerical methods, there is a risk of mistaking the point of a partial extremum for the optimum point. The analysis

of two-dimensional section zones helps the engineer avoid this mistake.

Appendix IV presents the assignment for the solution of design optimization problems for an induction machine with the aid of ED technique. Applying this technique, the analyst can transform the generalized machine model equations into simple polynomial relations between the operating factors of a machine and its parameters. The ED technique also allows one to single out the main and secondary factors that affect certain operating characteristics. By use of the ED technique, it becomes possible to construct higher-order mathematical models of energy converters and pass to geometric programming.

Chapter 12

Synthesis of Electric Machines

12.1. Optimization of Energy Converters. Optimization Methods

The objective of the theory of electric machinery is to find the ways for the evolution of new energy converters and optimization of conventional types. The design of a machine includes the stages of computation and engineering development. In the general case, computation represents a mathematically indefinite problem with many solutions because the number of unknowns in the problem is greater than the number of equations. The way out is therefore to preset the values of certain quantities reasoning from the experience gained in electric machine engineering. Of a few design versions, the most advantageous, optimum one, is then chosen.

A criterion of optimization can be a minimum total cost required for the production and operation of a machine, a minimum mass and cost of the machine, and other factors. An optimization criterion depends on the field of application of a particular machine, time, costs of materials and energy, etc. These factors complicate the task of seeking an optimum design.

There is a great number of methods of electric machine design. Each method relies on certain mathematical models interrelating input and output characteristics of a machine. Most methods use design equations obtained both analytically and empirically. Some methods employ equations derived from equivalent circuit models. In the last years attempts have been made to use differential equations to form a mathematical model for designing energy converters.

The use of differential equations of electromechanical energy conversion for design purposes offers a means of considering both static and dynamic characteristics of ECs. Computing facilities make it possible to establish the characteristics of steady-state operation as a particular case of transient operation. Thus it is possible to proceed with dynamics not from the equivalent circuit, i.e. not from the particular to the general, but from the general equations of energy conversion to statics.

The models being set up in this manner have brought forth the problems for simplifying the mathematical analysis of energy conversion and made it necessary to introduce the notion of specific power. It proves identical throughout the air gap in properly designed electric machines and can probably serve as a basic quantity in designing ECs.

The problems involved in the optimization of ECs generally have several solutions. *The end pursued by optimization is to seek the best solution among many potentially possible solutions.* An unambiguous problem does not need optimization.

Optimization relies on many methods, from rather complicated analytic and numerical approaches to those based on hand computation. Regarding its mathematical statement, the optimization problem reduces to the problem of nonlinear programming. Its formulation can be as follows: find the extremum (the maximum or minimum) of a definite function $F(\overline{x})$ dependent on variables $x_1, x_2, \ldots, x_n$ on condition that the two kinds of constraints are satisfied:

$$G_i \leqslant x_i \leqslant H_i, \quad i = 1, 2, \ldots, n$$

$$y_j \leqslant \varphi_j(\overline{x}) \leqslant h_j, \quad j = 1, 2, \ldots, m$$

where $\overline{x}$ is the vector whose coordinates are $x_1, x_2, \ldots, x_n$; $\varphi_j(\overline{x})$ represents some constraint functions; and G_i, H_i, y_j, h_j are the quantities defining the bounds of a permissible area.

The constraints in the design stage of energy convertors are the ones imposed on overheating, cost, dimensions, etc. The widespread methods that allow for constraints are the following.

1. The method for determining boundary values of variables, which is convenient for use in gradient and coordinate-search optimization procedures involving the constraints imposed on independent variables.

2. The penalty function method that allows for inequality constraints and relies on a new function

$$J(\overline{x}, \overline{u}, \overline{t}) = F(\overline{x}, \overline{u}, \overline{t}) + P(\overline{x}, \overline{u}, \overline{t}) \tag{12.1}$$

where $F(\overline{x}, \overline{u}, \overline{t})$ is the objective function under analysis; $J(\overline{x}, \overline{u}, \overline{t})$ is a newly derived function; and $P(\overline{x}, \overline{u}, \overline{t})$ is the penalty function.

The sign of a penalty function depends on the course of the solu-

tion: the function is negative in testing it for the maximum, and positive in testing it for the minimum. Each penalty function provides a numerical approach to the direct solution of the problem.

3. The Lagrangian multiplier method which rather efficiently allows for equality constraints. The method uses Lagrangian multipliers to introduce the constraints into the newly implemented function. The extremum of the new function can be sought by employing standard methods of mathematical programming.

Assuming we have defined the problem subject to optimization, we can choose one of the general optimization methods. These include the methods of investigation of various design versions, which involve the analysis of a few possible solutions of the same problem with the aim of finding the best; experimental methods without the study of corresponding mathematical models of the objects of interest; graphical methods based on the graphical representation of the function being optimized and dependent on one or two variables; analytical methods using the classical principles of differential and variational calculus; and numerical methods.

For the solution of engineering problems involving nonlinear relations, it is expedient to use numerical methods.

The first approaches to optimization of electric machines have mainly relied on the method of sequential tracing of spacial grid points, or nodes. In this method, the permissible area of variation of each of the factors is broken down into m meshes with a definite step and an optimum version is sought by tracing the grid points step by step. The method, being virtually simple, becomes rather cumbersome with an increase in the number of design variables and a decrease in the step length. The total number of the solutions by this method is equal to approximately the product of the steps made for all variables, i.e. $m_1 \times m_2 \ldots m_n$. The method consumes a fairly great deal of time.

The next method that has received wide application is the Gauss-Seidel relaxation (successive-displacements) method. The idea of the method lies in the sequential search for the partial extremum of the output function for each variable of the system. The search is terminated when the next point of the space turns out to be the extremum in all coordinate directions. The method is efficient for application to the objects in which the correlation between independent variables does not exist. The single traverse through all variables can then lead to the solution of the posed problem. For the objects with a large number of variables which correlate with each other, the method proves inconvenient.

The method of the steepest slope or steepest descent (Jacobi's method) is a version of the gradient method. The direction of search here is opposite to the direction of the gradient. If $f(x_{n+1}) < f(x_n)$, the minimization procedure follows: each step is taken in the same

direction until $f(x_{k+1})$ turns out to be greater than $f(x_k)$ at a certain step. After regression to the point x_k and calculation of the gradient anew, the search is continued as before. This method is more efficient than the gradient method but becomes inoperative in the presence of internal interfaces. This demerit limits the uses of the method, particularly for optimization of multiparametric objects.

The method of configurations ensures a successful search along the edge of a sharp ridge. The algorithm of search consists in the following. At the first stage the initial values of all design variables x_j, including the initial increment Δx_j, are set and the value of $f(x)$ is found. Each variable is then made to vary cyclically by the value of the adopted increment in order that all the parameters might change. If, for example, the value of $x_1^1 = x_1^0 + \Delta x_1$ does not lead to a decrease in the objective function (in solving the minimization problem), x_1^0 is varied by $-\Delta x_1$ and $f(x)$ is again tested as before. If this variation does not offer improvement, x_1^0 is kept constant. The next stage involves a variation of x_2^0 by Δx_2, then a variation of the successive variable and so long, thereby changing all design variables and completing the first cycle of the investigation. At each step made in the analysis of an independent variable the value of $f(x)$ is compared with its value at the preceding point. If the objective function decreases at a given step, its preceding value is changed for a new one and used at subsequent steps for comparison. To speed up the search for the optimum, the length of step Δx can be changed by affixing a certain multiplier $\lambda_j > 1$ to the quantity Δx_j.

What should be regarded as the main disadvantage of this method is that all tentative steps are taken parallel to coordinate axes and no information about other directions is available so it is quite possible to miss the ridge.

There are also methods based on quadratic convergence. For quadratic functions, these methods enable finding a minimum independent of the reference point for a limited number of iterations. For functions which are amenable to a high-degree of approximation near the extremum point by the quadratic dependence, these methods ensure quick convergence. But they call for the calculation of not only the first derivatives but also the second derivatives. In solving many engineering problems, however, the calculation of partial derivatives both analytically and numerically presents difficulties.

The above described numerical methods of search for the extremum can be placed into the group of deterministic methods because the direction of search with these methods is uniquely defined by the logic of the search process.

In the practice of optimum design of electric machines another group of methods finds wide use. These are stochastic (random) methods in which the direction of search is chosen quite randomly.

If the objective function shows an increase in value in any one direction, the search is carried out in this direction until the extremum is found. These methods prove rather efficient for the search of the extremum of the function whose behavior is not known at all. However, the random search does not use the information on both the previous walk and the behavior of the hyperplane, which leads to a considerable loss of time.

The methods that show most promise are the ones which successfully combine the elements of deterministic methods with those of stochastic methods of search. One of these is the complex method. This method is a modification of the simplex method invented by G. B. Dantzig and retains the basic principle of the latter. The complex method uses $N + P$ vertices ($P \neq 0$), each of which must satisfy constraints at all K stages. In the permissible area of the factor space these vertices are set up in a random manner, following which the value of the objective function is found at each vertex of the complex. The vertex at which the function $f(x)$ has the worst value is replaced by a new vertex located on the line that passes through the rejected point and the center of cluster of the remaining vertices of the complex at a distance equal to or greater than the distance from the rejected point to the center of cluster. If it happens that the new vertex has the worst value as against those of the vertices of the new complex, a new vertex is set at a half-distance from the worst point to the best vertex of the complex. If the search is successful, the complex expands and deforms in the direction of the extremum. The process of search continues until the complex is drawn together at the center of cluster within the limits of the specified accuracy. The method enables a successfull solution of multiextremum problems. In progress today is the development of new optimization methods. However, the number of problems to be solved grows faster than the number of methods required for their solution. It seems unlikely that sometime a universal method would appear and edge out all other methods. More probably, the best optimization methods will be found for the definite classes of problems.

12.2. Geometric Programming

Geometric programming is a new branch of mathematical programming, which can be used to advantage for the solution of optimization problems in electromechanics. The geometric programming technique can efficiently tackle the minimization problems, in which the optimality and constraint criteria are expressed in terms of the nonlinear functions of the definite form. *Geometric programming in combination with experimental design provides a powerful tool for the construction of new mathematical models applied in the synthesis of energy converters.*

In specific cases, geometric programming enables the solution of problems analytically for quite a definite class of functions of the form

$$g(t) = \sum_{i=1}^{n} u_i(t)$$

$$u_i(t) = C_i t_1^{a_{i1}} t_2^{a_{i2}}, \ \ldots, \ t_m^{a_{im}} \tag{12.2}$$

where $u_1, u_2, \ldots, u_n$ are the positive components of the function $g(t)$; $C_1, C_2, \ldots, C_n$ are positive constant coefficients; a_{ij} are arbitrary real numbers ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$); and $t_1, t_2, \ldots, t_m$ are positive independent variables.

Such functions are known as *positive polynomials*, or *posynomials* for short (nonlinear polynomials with positive coefficients), and form the basis of the programming technique. The basic requirement of the technique comes to the representation of the functions under analysis by the linear sums of positive components u_i.

In engineering problems the functions are often expressed in implicit form. By performing appropriate transformations, the function can be cast in the form of a posynomial to enable the solution of an ordinary problem of geometric programming.

The function of the form

$$g(t) = f(t) + [q(t)]^a h(t) \tag{12.3}$$

reduces to a posynomial

$$g(\tau) = f(t) + t_0^a h(t) \tag{12.4}$$

with the aid of an additional independent variable t_0 on condition that

$$t_0^{-1} q(t) \leqslant 1$$

In the above formulas, $f(t)$, $q(t)$, and $h(t)$ are posynomials in the independent variable: $t = (t_1, t_2, \ldots, t_m)$; $a > 0$; and $\tau = (t_0, t_1, t_2, \ldots, t_m)$ is a minimizing (transformed) vector variable.

The function of the form

$$g(\tau) = f(t) - u(t)$$

where $u(t)$ is a single-term posynomial, also belongs to the class of problems solvable by the given technique because the minimization of the function (12.4) is equivalent to the minimization of the function

$$g(\tau) = 1/t_0 \tag{12.5}$$

subject to the constraint

$$t_0/u(t) + f(t)/u(t) \leqslant 1$$

where

$$\tau = [u(t) - f(t), \ t_1, \ t_2, \ \ldots, \ t_m]$$

The subject of investigation in geometric programming is the expression

$$g(t)=\sum_i \prod_j [q_{ij}(t)]^{a_{ij}}[1-p_{ij}(t)]^{b_{ij}} \tag{12.6}$$

where $q_{ij}(t)$ and $p_{ij}(t)$ are posynomials; and a_{ij} and b_{ij} are positive constant quantities.

The sign-changing property of a posynomial leads to a variety of constraints—inequalities. Constraints are the subject of investigation in inverse geometric programming which is not a part of convex programming and therefore most of the important theorems of the original geometric programming are unacceptable for this type of programming.

The geometric programming technique relies on the classical inequality according to which the arithmetic mean does not exceed the geometric mean:

$$\delta_1 U_1+\delta_2 U_2+\ldots+\delta_n U_n \geqslant U_1^{\delta_1}\cdot\ U_2^{\delta_2}\ldots U_n^{\delta_n} \tag{12.7}$$

where $U_1, U_2, \ldots, U_n$ are arbitrary positive numbers (components of the functions); $\delta_1, \delta_2, \ldots, \delta_n$ are arbitrary positive weights satisfying the normality condition

$$\delta_1+\delta_2+\ldots+\delta_n=1 \tag{12.8}$$

For variables $u_1=\delta_1 U_1$, $u_2=\delta_2 U_2, \ldots, u_n=\delta_n U_n$, the geometric inequality assumes the form

$$u_1+u_2+\ldots+u_n \geqslant (u_1/\delta_1)^{\delta_1}(u_2/\delta_2)^{\delta_2}\ldots(u_n/\delta_n)^{\delta_n} \tag{12.9}$$

There is a one-to-one correspondence between the weights δ_i and positive components of the function. At the point of optimum the weights δ_i are the relative values of these terms, therefore the vectors $\delta\ (\delta_1, \delta_2, \ldots, \delta_n)$ and $u\ (u_1, u_2, \ldots, u_n)$ are parallel to each other.

In problems associated with constraints the positive components of the normality vector are nonnormalized components:

$$\delta_1+\delta_2+\ldots+\delta_n \neq 1 \tag{12.10}$$

Normalization occurs with the aid of the multiplier λ which is equal to the sum of nonnormalized weights:

$$\lambda=\Delta_1+\Delta_2+\ldots+\Delta_n \tag{12.11}$$

where $\Delta_1, \Delta_2, \ldots, \Delta_n$ are nonnormalized weights; λ is the multiplier relating each normalized weight to a corresponding nonnormalized weight:

$$\Delta_i=\lambda\delta_i,\ i=1, 2, \ldots, n$$

The geometric inequality with nonnormalized weights has the form

$$(u_1+u_2+\ldots+u_n)^{\lambda} \geqslant (u_1/\Delta_1)^{\Delta_1}(u_2/\Delta_2)^{\Delta_2}\ldots(u_n/\Delta_n)^{\Delta_n}\ \Lambda^{\lambda} \tag{12.12}$$

The problem of geometric programming reduces to the minimization of the objective function

$$g_0^{\lambda_0}(u) \geqslant (u_1/\Delta_1)^{\Delta_1} (u_2/\Delta_2)^{\Delta_2} \ldots (u_n/\Delta_n)^{\Delta_n} \Lambda_0^{\lambda_0} \qquad (12.13)$$

subject to constraints

$$1 \geqslant g_k^{\lambda_k}(u) \geqslant \prod_{i \in J[k]} (u_i/\Delta_i)^{\Delta_i} \prod_{k=1}^{p} \Lambda_k^{\lambda_k} \qquad (12.14)$$

$$J[k] = \{m_k, m_{k+1}, \ldots, n_k\} \qquad (12.15)$$

where $k = 1, 2, \ldots, p$ in Eq. (12.14) and $k = 0, 1, 2, \ldots, p$; $m_0 = 1$; $m_1 = n_0 + 1, \ldots, n_p = n$ in Eq. (12.15).

One of the variables δ_i corresponds to each term $g_h[u(t)]$ of the constraint functions ($k = 1, 2, \ldots, p$). The constraint functions $g_k(t)$ yield multipliers λ_k. The objective function does not entail the emergence of multipliers λ_k because $\lambda_k = 1$ for $k = 0$ according to the normality condition, i.e. because the normalization factor λ_0 is taken equal to unity. The normality condition represents the sole part of the optimization problem, in which there is a distinction between the objective function $g_0(t)$ and posynomial constraints $g_k(t)$, $k = 1, 2, \ldots, p$.

If we express the positive components of the objective function and constraint functions in terms of the independent variables $t_1, t_2, \ldots, t_m$, the relation will take the form

$$g_0(t) \geqslant \left[\prod_{i=1}^{n} (C_i/\delta_i)^{\delta_i}\right] \left(\prod_{j=1}^{m} t_j^{D_j}\right) \prod_{k=1}^{p} \Lambda_k^{\lambda_k} \qquad (12.16)$$

where C_i denotes the coefficients on positive components u_i; t_j denotes positive independent variables; and D_j represents the linear combination of exponents of independent variables:

$$D_j = \sum_{i=1}^{n} \delta_i a_{ij}, \quad j = 1, 2, \ldots, m \qquad (12.17)$$

Here, a_{ij} denotes the arbitrary real numbers used to form the exponent matrix $|a_{ij}|$.

The right side of (12.16) is termed *the pre-dual function* and designated as $V(\delta, t)$. It is a function of the positive weights and independent variables. The pre-dual function $V(\delta, t)$ takes on a minimum value when the linear combinations are equal to zero

$$\sum_{i=1}^{n} \delta_i a_{ij} = 0 \qquad (12.18)$$

In this particular case all independent variables t_j are raised to the zero power and the pre-dual function is only dependent on δ_i positive weights.

The number n of rows of the exponent matrix a_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) determines the dimension of the vector of exponents and is in strict correspondence with the number of terms of all posynomials. The number m of columns determines the dimension of the space of the exponents and is equal to the number of independent variables of the objective function.

If the number of rows is equal to or smaller than the number of columns, i.e. the dimension of the exponent vector is greater than the dimension of the column vector of the exponent matrix, the orthogonality condition cannot be met and, hence, the problem has no solution. The quantity

$$d = n - (m + 1) \tag{12.19}$$

is termed *the degree of difficulty* of a problem.

If the dimension of the exponent vector is greater than the dimension of the column vector of the exponent matrix, then, in this particular case, there is an analytical solution of the problem.

When $n = m + 1$, there exists the single direction of the solution vector that is normal to all column vectors of the exponent matrix. The orthogonality condition, which forms the vector subspace of the n-dimensional space, uniquely determines the dual vector δ without performing the normalization process. In this case the number of unknown equations is equal to the number of linear equations which satisfy the conditions of the dual problem (program) in the form of equalities. In the general case, when $n > m + 1$, all the solutions for orthogonality conditions define only the dual space whose dimension is equal to $n - m$.

Expression (12.16) then takes the form

$$v(\delta) = [\prod_{i=1}^{n} (C_i/\delta_i)^{\delta_i}] \prod_{k=1}^{p} \Lambda_k^{\lambda_k} \tag{12.20}$$

which is *a dual function*, and $\delta_1, \delta_2, \ldots, \delta_n$ are *dual variables*. Here two kinds of linear constraints appear in the form of inequalities and in the form of equalities.

The first constraint imposed on the vector δ is the condition of nonnegativity of dual variables:

$$\delta_1 \geqslant 0, \delta_2 \geqslant 0, \ldots, \delta_n \geqslant 0 \tag{12.21}$$

The constraint (12.21) implies that none of the components of δ can be negative.

According to the second kind of linear constraints, the sum of components corresponding to the primal function is equal to unity (the normality condition).

There is a third kind of constraints, namely, vector orthogonality constraints. They appear as a result of application of the inequality to which there must correspond the scalar product of n-dimensional

vectors. The orthogonality condition underlies the theories of duality and requires that the vector δ belong to the space orthogonal with respect to the column vectors of the exponent matrix. The assumption is that the column vectors of the exponent matrix are linearly independent. For geometric programming problems with the zero degree of difficulty the dual region contains one point. The solution of the dual and the primal problem reduces to the solution of the system of linear equations, in which the determinant is different from zero. This system of equations has a unique solution.

At a positive degree of difficulty, the dual region has a high degree of arbitrariness. A unique solution of the system of linear equations does not then exist.

The dual function is simpler than the primal function. A dual program offers computing advantages because the constraints are linear; the constraints of a primal program are nonlinear. That is why the first stage of the solution to geometric programming problems involves the estimation of the dual vector satisfying the conditions of orthogonality and normality.

The solution of the system of linear equations

$$\begin{aligned} \delta_1 + \delta_2 + \ldots + \delta_n &= 1 \\ a_{11}\delta_1 + a_{12}\delta_2 + \ldots + a_{1n}\delta_n &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}\delta_1 + a_{m2}\delta_2 + \ldots + a_{mn}\delta_n &= 0 \end{aligned} \tag{12.22}$$

fully defines the direction of the dual vector.

The dual vector δ' $(\delta_1', \delta_2', \ldots, \delta_n')$ offers the possibility of determining the dual function from the expression

$$v(\delta') = (C_1/\delta_1')^{\delta_1'} (C_2/\delta_2')^{\delta_2'} \ldots (C_n/\delta_n')^{\delta_n'} \Lambda_k^{\lambda_k} \tag{12.23}$$

By the duality theory, maximizing dual variables δ_i' yield minimizing independent variables of the objective function. t_i'. Therefore,

$$C_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a} = \delta_i' v(\delta'), \quad i = 1, 2, \ldots, n \tag{12.24}$$

Taking the logs of (12.24) gives the system of n linear equations in independent variables t_i' of the primal function. The solution of these equations determines the sought-for optimal value of the objective function.

In analogy to linear programming, the technique of geometric programming relies on the duality theory. But there is a difference, namely, in going from the primal to the dual program, the nonlinear function (12.6) becomes linear due to the transition from the region of independent variables to the region of exponents.

The theory of geometric programming is set forth to describe convex functions defined on a convex set. According to the basic theorem

of convex programming, any point of a local minimum of the function is also the point of a global minimum of the given function. Therefore, there is no need to obtain local extrema and compare them to make the choice of the global solution.

The basic criterion of the method of geometric programming is the objective constraint. The objective function, or optimality criterion, is subject to various constraints (in the form of equalities and inequalities) imposed on, say, the cost, dimensions, and overheat in designing electric machines.

In the text above (see Sec. 12.1) a few constraint methods have been brought out, namely, the methods of boundary values, penalty functions, and Lagrangian multipliers. In geometric programming, the procedure of solving practical problems may involve a great number of constraints which present difficulties in using this technique. The objective constraint, i.e. the functional constraint equivalent to all individual constraints, obviates these difficulties.

The objective constraint is a monomial

$$O(Ct_1^{x_1}t_2^{x_2} \ldots t_m^{x_m}) \leqslant 1 \tag{12.25}$$

where C is the coefficient of the objective constraint $(C > 0)$; $t_1, t_2, \ldots, t_m$ are independent variables of the system; and $x_1, x_2, \ldots, x_m$ are objective exponents.

Having q functional constraints, one has to define $\lambda_1, \lambda_2, \ldots, \lambda_h, \ldots, \lambda_q$ Lagrangian multipliers. Each Lagrangian multiplier corresponds to the constraint weight. The objective constraint combines all constraints, so one Lagrangian multiplier λ_0 called an *objective multiplier* corresponds to this constraint:

$$\lambda_0 = \lambda_1 \lambda_2 \ldots \lambda_q \tag{12.26}$$

Where the region of search for the objective function is a point, the search for the function extremum and corresponding coordinates comes to a well-founded single computation procedure.

The objective constraint cannot apply to all problems. It can act as a useful tool in solving problems of the zero degree of difficulty, and also in experimental design and geometric programming.

12.3. Design of Electric Machines by Geometric Programming

The design of an electric machine by geometric programming begins with calculations based on the experimental design technique. The initial stage comes to working out the program by use of the known design formulas to test the function in the region of interest and derive the first system of linear equations covering all a priori information

on the machine. Such a system of equations may take the form

$$\begin{aligned} \cos\varphi &= C_{11}x_1 + C_{12}x_2 + \ldots + C_{1m}x_m \\ \eta &= C_{21}x_1 + C_{22}x_2 + \ldots + C_{2m}x_m \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ k_I &= C_{c1}x_1 + C_{c2}x_2 + \ldots + C_{cm}x_m \end{aligned} \tag{12.27}$$

where $\cos\varphi$, efficiency η, and current ratio k_I are the output characteristics of the machine; $x_1, x_2, \ldots, x_m$ are the objective exponents of independent variables in the objective constraint; $C_{11}, C_{12}, \ldots, \ldots, C_{cm}$ are the coefficients of regression; and c is the number of constraint functions of the optimum design.

At optimum design by geometric programming, the functions are cast in the form of posynomials. Thus, the mass of the stator winding is found from the formula

$$G_{Cu} = gS_c z_1 q_{Cu}(l + l_s)\,10^{-6} \tag{12.28}$$

where G_{Cu} is the mass of the copper winding, kg; g is the density of the stator winding material, g/cm^3; S_c is the number of effective conductors in the slot; z_1 is the number of slots in the stator; q_{Cu} is the cross section of the effective conductor, mm^2; l is the stator core length, mm; and l_s is the average length of the stator-winding face portion, mm.

On substituting the value of the face portion and making simple transformations, the expression for the stator winding mass assumes the form

$$\begin{aligned} G_{Cu} = gS_c z_1 q_{Cu} l &+ gS_c q_{Cu} k_{f1} y_1 \pi D_i \\ &+ gS_c q_{Cu} k_{f1} y_1 \pi h_{z1} + 2gS_c z_1 q_{Cu} B \end{aligned} \tag{12.29}$$

where k_{f1} and B are the quantities accounting for the geometric dimensions of the winding face portion; y_1 is the stator winding pitch; D_i is the inner diameter of the stator, mm; and h_{z1} is the design height of the stator slot, mm.

The slot area is given by

$$S_s = (h_{z1} b_s k_{Cu})/q_{Cu}$$

where b_s is the slot width; and k_{Cu} is the space factor accounting for the degree of filling the slot with copper.

The expression for the winding mass finally takes the form

$$\begin{aligned} G_{Cu} = gz_1 k_{Cu} b_s h_{z1} l &+ gy_1 \pi k_{f1} k_{Cu} b_s h_{z1} D_i \\ &+ gy_1 \pi k_{f1} k_{Cu} b_s h_{z1} + 2gz_1 k_{Cu} b_s h_{z1} B \end{aligned} \tag{12.30}$$

The efficiency expression of the form

$$\eta = (1 - \Sigma P/P_{1nom})\,100\% \tag{12.31}$$

is not amenable to testing by the technique of geometric programming. But, assuming that a maximum efficiency derives from a minimum loss, we perform transformations and obtain the expression for the total loss in the posynomial form

$$\sum_{i=1}^{n} P = P_1 + P_2 + \ldots + P_n \tag{12.32}$$

where $P_1, P_2, \ldots, P_n$ are individual losses in the machine, which can be defined in terms of geometric dimensions.

In a similar way we can derive expressions for the machine cost, heating, and other factors. Depending on the statement of the problem, the unknown quantities can be the parameters of the equivalent circuit, voltage, torque, etc.

The starting torque can be expressed in the following posynomial form

$$1/M_{st} = \frac{\omega_1}{m_1 u_1^2} \frac{r_1^2}{r_2^2} + \frac{2\omega_1}{m_1 u_1^2} r_1 C_1 + \frac{\omega_1}{m_1 u_1^2} C_1^2 r_2' + \frac{\omega_1}{m_1 u_1^2} \frac{x_1^2}{r_2'} + \frac{2\omega_1}{m_1 u_1^2} \frac{C_1 x_1 x_2'}{r_2'} + \frac{\omega_1}{m_1 u_1^2} \frac{C_1^2 (x_2')^2}{r_2'} \tag{12.33}$$

The solution of Eqs. (12.27) applies to the second system of linear equations which describe the link between the elements of the exponent matrix of the design and the positive components of the dual vector

$$\begin{aligned} a_{11}\delta_1 + a_{12}\delta_2 + \ldots + a_{1n}\delta_n &= 0 \\ a_{21}\delta_1 + a_{22}\delta_2 + \ldots + a_{2n}\delta_n &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}\delta_1 + a_{m2}\delta_2 + \ldots + a_{mn}\delta_n &= 0 \end{aligned} \tag{12.34}$$

where $a_{11}, \ldots, a_{mn}$ are the exponents of independent variables in the optimum design; $\delta_1, \delta_2, \ldots, \delta_n$ are the positive components of the minimizing dual vector.

The system of equations (12.34) represents the relation between the exponent matrix of the design and the positive components of the minimizing dual vector. The last column vector of the exponent matrix results from the solution of the first system of linear equations. The independent (unknown) variables of the second system of equations are the positive components of the minimizing dual vector. The coefficients affixed to variables constitute the exponent matrix (12.34) and reflect the conditions of normality and orthogonality of the positive components of the dual vector. This system has a unique solution at the zero degree of difficulty of the problem. In this case the number of independent variables of the design is smaller than the number of terms in the problem by one.

The coefficients on the variables of the third system are equal to the exponents of the independent variables of the design:

$$\begin{aligned} a_{11}z_1 + a_{21}z_2 + \ldots + a_{m1}z_m &= b_1 \\ a_{12}z_1 + a_{22}z_2 + \ldots + a_{m2}z_m &= b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots &\ \ldots \\ a_{1n}z_1 + a_{2n}z_2 + \ldots + a_{mn}z_m &= b_m \end{aligned} \tag{12.35}$$

where $z_1, z_2, \ldots, z_m$ are the logs of the absolute values of independent variables; $b_1, b_2, \ldots, b_m$ are the logs of the product of the positive components of the minimizing dual vector and the objective function minus the logs of the coefficients on the variables of the positive components of the objective function.

The design process yields final results by taking the antilogs of the results of the solution to the third system of linear algebraic equations.

As an example, consider the problem of search for the geometric dimensions of an energy converter that ensure the specified static and dynamic output characteristics and effect a maximum saving in copper.

The objective function is given by (12.30) for determining the mass of the winding. This function depends on four variables, D_i, l, b_s, and h_{z1}. Denote the exponents by x_1, x_2, x_3, and x_4 for the variables b_s, h_{z1}, l, and D_i respectively. The expression for constraint functions will then take the form of (12.27). The solution of (12.27) enables us to form the objective constraint

$$C/(b_s^{x_1}h_{z1}^{x_2}l^{x_3}D_i^{x_4}) \leqslant 1 \tag{12.36}$$

The second equation in (12.34) will then take the form

$$\begin{aligned} 1\delta_1 + 1\delta_2 + 1\delta_3 + 1\delta_4 + 0\delta_5 &= 1 \\ 1\delta_1 + 1\delta_2 + 1\delta_3 + 1\delta_4 - x_1\delta_5 &= 0 \\ 1\delta_1 + 1\delta_2 + 1\delta_3 + 2\delta_4 - x_2\delta_5 &= 0 \\ 1\delta_1 + 0\delta_2 + 0\delta_3 + 0\delta_4 - x_3\delta_5 &= 0 \\ 0\delta_1 + 0\delta_2 + 1\delta_3 + 0\delta_4 - x_4\delta_5 &= 0 \end{aligned} \tag{12.37}$$

Solving (12.37) with consideration for (12.27) gives an optimum (minimum) value of the objective function

$$G_{\mathrm{Cu\,min}} = \left(\frac{k_{\mathrm{Cu}}z_1 g}{\delta_1}\right)^{\delta_1}\left(\frac{2k_{\mathrm{Cu}}g z_1 B}{\delta_2}\right)^{\delta_2} \times \left(\frac{\pi k_{\mathrm{Cu}}k_{f1}g y_1}{\delta_3}\right)^{\delta_3}\left(\frac{\pi k_{\mathrm{Cu}}k_{f1}g y_1}{\delta_4}\right)^{\delta_4}\left(\frac{C}{\delta_5}\right)^{\delta_5}\delta_5^{\delta_5} \tag{12.38}$$

At the optimum point the individual terms are equal to the product of the minimum value of the objective function by the respect-

ive positive components of the minimizing dual vector δ:

$$
\begin{aligned}
\delta_1 G_{\mathrm{Cu\,min}} &= k_{\mathrm{Cu}} z_1 g b_s h_{z_1} l \\
\delta_2 G_{\mathrm{Cu\,min}} &= 2k_{\mathrm{Cu}} g z_1 B b_s h_{z_1} \\
\delta_3 G_{\mathrm{Cu\,min}} &= \pi k_{\mathrm{Cu}} k_{f1} g y_1 b_s h_{z_1} D_i \\
\delta_4 G_{\mathrm{Cu\,min}} &= \pi k_{\mathrm{Cu}} k_{f1} g y_1 b_s h_{z_1}
\end{aligned}
\tag{12.39}
$$

Taking the log of each of the equations in (12.39) yields

$$
\begin{aligned}
\ln(\delta_1 G_{\mathrm{Cu\,min}}) &= \ln(k_{\mathrm{Cu}} z_1 g) + \ln b_s + \ln h_{z_1} + \ln l \\
\ln(\delta_2 G_{\mathrm{Cu\,min}}) &= \ln(2k_{\mathrm{Cu}} g z_1 B) + \ln b_s + \ln h_{z_1} \\
\ln(\delta_3 G_{\mathrm{Cu\,min}}) &= \ln(\pi k_{\mathrm{Cu}} k_{f1} g y_1) + \ln b_s + \ln h_{z_1} + \ln D_i \\
\ln(\delta_4 G_{\mathrm{Cu\,min}}) &= \ln(\pi k_{\mathrm{Cu}} k_{f1} g y_1) + \ln b_s + 2\ln h_{z_1}
\end{aligned}
\tag{12.40}
$$

Introducing the notation

$$
\begin{aligned}
b_1 &= \ln(\delta_1 G_{\mathrm{Cu\,min}}) - \ln(k_{\mathrm{Cu}} z_1 g) \\
b_2 &= \ln(\delta_2 G_{\mathrm{Cu\,min}}) - \ln(2k_{\mathrm{Cu}} g z_1 B) \\
b_3 &= \ln(\delta_3 G_{\mathrm{Cu\,min}}) - \ln(\pi k_{\mathrm{Cu}} k_{f1} g y_1) \\
b_4 &= \ln(\delta_4 G_{\mathrm{Cu\,min}}) - \ln(\pi k_{\mathrm{Cu}} k_{f1} g y_1)
\end{aligned}
\tag{12.41}
$$

$$z_1 = \ln b_s, \quad z_2 = \ln h_{z_1}, \quad z_3 = \ln l, \quad z_4 = \ln D_i$$

we get the third system of equations which have a unique solution

$$
\begin{aligned}
1z_1 + 1z_2 + 1z_3 + 0z_4 &= b_1 \\
1z_1 + 1z_2 + 0z_3 + 0z_4 &= b_2 \\
1z_1 + 1z_2 + 0z_3 + 1z_4 &= b_3 \\
1z_1 + 2z_2 + 0z_3 + 0z_4 &= b_4
\end{aligned}
\tag{12.42}
$$

Taking the antilogs of the solutions of (12.42) gives the coordinates of the minimizing point of the objective function. The final results are defined by the expressions of the form

$$b_s = e^{z_1}, \quad h_{z1} = e^{z_2}, \quad l = e^{z_3}, \quad D_i = e^{z_4} \tag{12.43}$$

The method of geometric programming can yield the relations between the output and input characteristics of the object under study in the form of the systems of linear algebraic equations.

The analysis of energy converters with the aid of the exponents of independent variables is the generalization of a higher order than is the case for conventional models. The objective exponents of independent variables in the objective constraint and the positive components of the minimizing dual vector for electric machines of the same series change over insignificant intervals. This permits extending the data on a thoroughly designed machine model to

other design versions of electric machines of the same series, thereby cutting down heavily the time required for the computations.

The design technique based on geometric programming follows from the generalization of the theory of mathematical similitude, where the design procedure relies on the elementary laws of linear algebra.

The methods of geometric programming together with experimental design methods hold much promise for the solution of optimization problems. These methods do not certainly rule out the applications of other optimization methods.

Chapter 13

Automated Design of Electric Machines

13.1. General Points on the Evolution of the Systems of Automated Design

In the USSR the use of computers for design of electric machines was begun in the late 1950s. In the mid-1970s a tendency evolved to integrate separate design approaches and form a single methodology to promote optimum design of electric machines using the concept of a generalized energy converter. However, the general design procedure at the time largely remained manual despite the fact that computers held a definite place among other design means.

In the modern period of application of computing facilities to the processes of design of electric machines, the trend is to rearrange fully the design techniques on the basis of automated design systems so as to transfer the design problem in the form of a mathematical model on to a computer to generate solutions. Thus, principally new design approaches have come into being.

Obviously, high-quality design work must be done in the shortest time possible, otherwise the ideas put into the project and the engineering solutions will be obsolescent even before the machines come into service. A design insufficiently worked out at the early stages of its development entails a lengthy period of "upgrading" the prototypes or even remodeling them at the production stage, thereby adding to the expenses and protracting the design implementation.

A principle obstacle to the improvement of the quality of designs and reduction in the time of their development is the inconsistency between complex modern machinery and the old methods and means

of designing. At the age of technical progress, an increased complexity of designed objects is inevitable, and one certainly cannot raise the quality of design work and accelerate the design process by just increasing the number of design offices. The problem is amenable to the solution only with the aid of mathematical methods and computing facilities installed at project and design services, manufacturing organizations, and at various plants.

The most effective design guideline is to convert from automation of individual designs to integrated automation evolving for the purpose automated design systems (ADSs). The ADS is an operational engineering system associated with design office subunits and intended to fulfill the assignments by the available automatic means which form a specific complex. The complex ensures methodical, programming, material, information, and organizational support.

The subunits of a design office do coordinated jobs using hardware and relying on organizational support. The basic function of the ADS is to carry out computer-aided design at all stages or some stages of the design procedure using mathematical and other models.

In the systems of automated design, the operator interacts with a computer to introduce appropriate changes into the description of the design and represent it in suitable languages. In some automatic procedures, the computer performs these operations on a closed-shop basis.

Structurally, the ADS consists of several subsystems. A subsystem is an ADS part having its own specific features and capable of yielding complete design solutions and other useful information.

There are object-oriented (object) subsystems and object-independent (invariant) subsystems. The object subsystem develops the design of a certain object (a class of objects) at a definite design stage. It can work on a design of machine parts, components, production processes, etc. The invariant subsystem performs control functions and processes information independent of the features of the object being designed. It can control the ADS; conversational procedures; numerical analysis; optimization; input, processing, and output of graphical information; and also information retrieval procedures. An ADS subsystem consists of components intended to perform a common objective function.

An ADS component is an element of the means of support that performs a definite function in its ADS subsystem. All the subsystems structurally form an integral system by virtue of the links between the ADS components of various subsystems.

The components of methodical support are the documentation which offers the following (either fully or gives references to primary sources): the theory, methods, approaches, mathematical mo-

dels, algorithms, algorithmic languages for the description of objects, terminology, specifications, standards, and other information needed for the methodology of design work in the subsystems. The components of programming support are a collection of documents comprising various programs, programs compiled on machine data carriers, and service forms and records for proper functioning of corresponding subsystems.

Programming support includes the all-system type and the application type of software. The components of all-system software are operating systems, symbolic compliers, etc. The components of application software are programs and packs of application programs intended for design solution generation.

The components of material support are computing and office mechanization facilities, data transfer means, measuring devices, and other types of hardware.

The components of information support that form the data base of the ADS are the files of documents giving the description of standard design procedures, standard design solutions, standard elements, items of sets, materials, and also other information, including the files and data units with the record of the above documents on the machine medium.

The components of organizational support are methodical and other manuals, regulations and instructions ensuring the interaction between office subunits.

The analysis of the design of an object is made by way of simulation of the object on a computer. The synthesis of the design and its optimization are put into effect by retrieving information from the computer memory in the conversational mode which involves the interaction of the designer with the computer. The design work thus acquires the features of what is known as "computer-aided design". Here the computer acts as a means of integration of the partial models and techniques (decision-making procedures) starting from the general data and programming-technical base of the ADS.

In the ADS, the design process requires the intricately coordinated work of the technical, programming, and information facilities. The computer interacts with the external world through the means of input, output, storage, and transfer of information in aplhabetic-digital and graphic forms.

Prior to working on the project of an automated design system, the steps are taken to analyze the existing design status, substantiate the project, determine the priority of automation of individual design stages, estimate the expedient level of automation, rationalize the flows of information, analyze and improve the methods of solutions to the problems.

An ADS well substantiated economically can ensure the choice of the optimum problem solution, desired accuracy of solutions,

shorter time required to solve problems, rational acquisition and processing of data; most efficient utilization of operational potentialities of computing facilities and other means.

The level of automation for the project is chosen on the basis of economy with consideration for the information capacity, labor input required for the solution, computer storage capacity, succession of solution runs, and multivariance of the solution to the problems.

The basic factors used to estimate the effectiveness of design in the framework of the ADS are the labor productivity, quality indexes of the solution to problems, and due dates on the development of design plans and specifications.

The analysis of the basic stages in the process of manual design of induction machines permits establishing the optimum level and priority of automation of various design stages in evolving electric machines with the aid of the ADS.

The Soviet industry largely produces automated design systems for induction machines (ADS IM). The system of this type offers automatic means of optimizing the designs, performing graphic operations, and making up drawings in the development of general-purpose low-voltage induction machines. Projects are put forward for the development of a system for designing and doing graphic work on man-machine basis.

The ADS IM contains improved mathematical models and graphic sets of induction motors, including standard parts and subassemblies. The system envisages the design of machines which are to include standard subassemblies and parts such as bearings, bolts, nuts, and keys made as stipulated in pertinent standards. The modified versions of a machine design can be built up by varying the dimensions of standard units and parts in a linear fashion (shafts, stator cores, etc.) and in a nonlinear fashion (face portions of windings, stator and rotor laminations, etc.). The features embodied in the system operated on the man-computer basis permit the designer to evolve new units and parts of the machine.

A motor design is made up of elements obtained from the calculation results or data read out from the computer storage. At each stage of the design procedure, the ADS IM yields the graphic solution of the design shown on a display, presented as a drawing, or in any other form.

An ADS employed for each type of electric machine has its own features. In one case, the machine can preserve its certain base structure, so a system such as ADS IM can be set to fill in only certain gaps in the design. In another case, the entire construction of, say, a machine of autonomous power must be rebuilt completely to change its element base. Here, the ADS is called upon to make a choice of the optimum design out of a variety of the design versions,

effect optimum unification and standardization of subassemblies and parts of the machine. This circumstance considerably complicates the problem of design work. The reason is that while conventional problems generally have a clear-cut mathematical statement and are solvable by formal methods, the problem of search designing shows a nonformal character and, hence, is not amenable to the solution by effective numerical methods. Nevertheless, even in the absence of nonformal mathematical methods, the pragmatic basis for the solution of design problems can be man-machine conversational procedures.

The dialog is made possible by the method of search for alternatives. As he follows the course of the solution to the problem, the designer can take a number of possible decisions at the given stage of the solution. The designer makes a choice as he adjusts the system with the graphic terminal for the step-by-step solution. The effectiveness and flexibility of the system depend on whether or not the sequence of actions suggested to the designer can afford a sufficiently large number of possible solutions at each step. In this connection, information support of the system acquires much importance. One finds it advantageous to accomplish information aids as a set of programmable models which describe standard design elements and simple geometric figures and also as semantic models reflecting the hierarchy and structure of elements, parts, and units of the machine being designed. Such a representation of graphic and semantic information has its roots in the preliminary analysis of the geometric forms of the possible designs of a machine and its elements, and also in the systematic analysis of the machine design layout and hierarchy of the isolated elements (in the form of a graph of possible solutions).

Apart from the programs of the geometric forms of elements and parts, the graphic information aids of the system also include the programs of the projections, cross sections, representative dimensions and dimensional tolerances, requirements for winding surface cleanliness, standard text information, etc.

The ADS of electric machines (ADS EM) has a general program of drawing. It envisages work on various problems and collects and stores information. The problems of storages, search, and processing of data acquire primary importance in modern design systems and have a substantial effect on the structure and the principles of action of the system as a whole.

The files of data and programs directly intended to provide for centralized storage and search of information and also to establish links with application programs which process this information form *a bank of data*. In systems using data banks, application programs receive data for processing not from external data carriers but from control systems of the data base.

One of the important problems of ADS EM is to create the data bank of electromechanical solutions. For this it is necessary that design and development work carried on in the field of energy converters should comply with the requirements of the ADS. The procedure in the development of a new energy converter should follow the adopted guidelines envisaging the storage of final results in the data bank for further use.

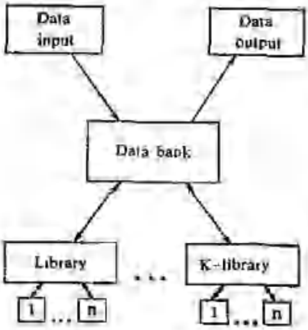


Fig. 13.1. The general schematic of a data bank for energy converters

The bank can store information in libraries organized in an hierarchic order according to the types of EC or for each leading research institute (Fig. 13.1). The library can contain data on design of EC elements and standard solutions.

The evolution of an ADS EM is a lengthy process and calls for further development in the field of electromechanics and more coordinated work of design departments at plants and research institutes.

13.2. Software of Automated Design Systems

Software of an ADS of electric machines can be of several types, such as general, special, service, and dialog control types. *General software*, being invariant with respect to the object under study, comprises application program packs for the solution to general mathematical problems of various classes, such as the problems of mathematical programming, problems for the solution of linear and nonlinear algebraic equations, the systems of differential equations, problems of mathematical statistics, etc.

Service software and *dialog control software* must provide for effective interaction between a computer and a user. *Special software* is a problem-oriented part of programming support, the development of which needs particular care, for it is the quality of the evolved mathematical models of ECs that determines, in the final analysis, the effectiveness of the ADS.

The structure of ADS EM software must meet the following basic requirements: (1) high modularity, i.e. a high degree of autonomy of subsystems; (2) open-loop ability to enable the extention of the system or its correction within the structure without changing other blocks; (3) functional completeness of the set of design operations

realizable in the ADS so that the system can exert the desired influence on all design stages, starting with the request for the proposal and ending with the forms and records on the production of an energy converter.

Consider the software of an ADS EM. *An ADS EM is capable of doing both computational and developmental work, from the moment of receiving the order to the moment of issuing the production documentation and testing experimental models of the machine (without manufacturing the production prototypes).*

The most difficult and important task is the task of evolving the systems for automated design of machines destined for work in

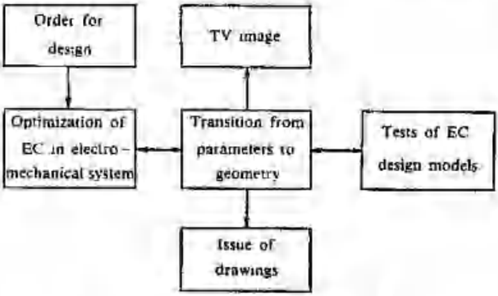


Fig. 13.2. An ADS of electric machines

an electromechanical system with control elements and feedback mechanisms. What adds to the difficulty of the task, apart from the fact that the design work must be done in the shortest possible period, is that the design must envisage the small-batch or piece-work production of the above machines for operation at varying frequencies and voltages and under diverse environmental conditions.

As mentioned earlier, the ADS EM consists of a few subsystems, each being capable of solving individual problems on the man-computer basis (Fig. 13.2).

The first subsystem effects the optimization of the parameters of a machine in the electromechanical system. Given the mathematical description of energy conversion processes and the equations of the system elements, it is possible to pass to the representation of the machine and its supply network as a system of multiports and implement the synthesis of the multiports.

In electric drives a motor is generally fed from a generator or static (semiconductor) control elements which can be connected to the stator or rotor. The supply system can be split up into an infinite

power line and a feed multiport. This multiport is in turn broken apart into a passive multiport and an emf which also represent an infinite power line.

The generalized routine enables calculating the characteristics of a machine with its stator or rotor winding supplied from the line of nonsinusoidal asymmetric voltage through a linear multiport of arbitrary form having an internal energy source. The electric machine can be of the two-phase asymmetric type or of the m-phase symmetric type with a dc or ac supply source. Following the expansion into a harmonic series, we determine the effective value of a harmonic or its initial phase. Next we calculate the characteristics for each harmonic and evaluate the impedances of the machine and generalized multiport.

The experimental design technique permits us to simplify the description of the system, single out significant and insignificant factors, and represent the appropriate relations as polynomials. Using routine programs to estimate an optimum, we can preliminarily determine the parameters of the machine and the control multiport.

The second subsystem ensures the conversion from the parameters to the geometry of the machine. The problem of transition to the optimum geometry at the given optimum parameters of the electric machine can be stated as the problem of nonlinear programming.

The use of routine optimization programs enables finding the conditional extremum of the objective function in the shortest time possible. The transition from the parameters to the geometry of the machine can be speeded up using the method of geometric programming. After determining the geometric dimensions of the machine, the next stage follows which involves the estimation of parameters (coefficients on the variables in the initial equations) and the check of the parameters for the optimum in the system. If the discrepancy between the optimum parameters in the first and the second subsystem is unacceptable, new computations are performed.

The third subsystem yields structural and production drawings. It can substantially cut down lead times.

The ADS can incorporate a subsystem for forming an image of the machine under development on a TV screen given the machine geometry data stored in the computer. Such a subsystem together with the automated control system can speed up the machine design procedure.

Modern computing facilities allow for the creation of a subsystem for testing the experimental models of an electric machine prior to manufacturing its production prototype. After translating the data on the machine geometry into the command language, we can use a program-controlled machine tool to produce a model of a machine subassembly, for example, a full-size or geometrically similar rotor.

At the modern state of the art, it is possible to collect enough useful data for constructing a general-type rotor model for most energy converters. Thus, for induction machines, a solid rotor makes for the model. Tests run on the rotor model under conditions similar to real conditions enable us to predict the mass, energy characteristics, reliability, and other factors of the machine with the aid of a digital computer and to pass to the stage of evolving a production prototype.

One of the important problems of computer-aided design of electric machines is the problem of evolving a system of optimum design computation that yields the best machine-design version which satisfies the restrictions imposed by specifications and requirements.

In its general statement, the problem of synthesis of electric machines reduces to the optimization (minimization or maximization) of a certain functional:

$$(\overline{x}^s) \to \min \Rightarrow s^*, \ \overline{x}^{s*}$$
$$s \in S, \quad x^s \in D \tag{13.1}$$

with D subject to the constraint

$$\varphi_i^s(\overline{x}^s) = \varphi_i^s(x_1^s, x_2^s, \ldots, x_n^s) \geqslant 0$$
$$i = 1, 2, \ldots, m, \quad s = \{1, 2, \ldots, s, \ldots, k\} \tag{13.2}$$

where s is a set of structures (design versions) of electric machines; $\overline{x}^{s*}$ is the vector of optimum parameters of the optimum design. The problem defined by (13.1) and (13.2) covers both structural and parametric optimization. The conditions (13.2) express the restrictions imposed on the machine design by the specifications and describe the permissible region D. The region for defining the functions $F^s(\overline{x}^s)$ and $\varphi_i^s(\overline{x}^s)$ is a certain region D_1 subject to the constraint

$$\psi_j^s(\overline{x}^s) \geqslant 0, \quad j = 1, 2, \ldots, l \tag{13.3}$$

If $s = \{1\}$, i.e. when there is a need to optimize the given type of machine, the optimum design procedure comes to the solution of the problem

$$F_1(\overline{x}) \to \min \Rightarrow \overline{x}^* \tag{13.4}$$
$$\overline{x} \in D$$

with D subject to the inequality constraints

$$\varphi_i(\overline{x}) \geqslant 0, \quad i = 1, 2, \ldots, m \tag{13.5}$$

Optimum designing reduces to the solution of a nonlinear programming problem of the general type that shows the smoothness of the objective function, multiparametric and multiextreme features, and "valleys" in the hyperspace of permissible solutions. In

most extremum-seeking problems, a "valley" associated with a poor conditionality of the matrix of second partial derivatives shows up as a strong elongation (in the topological representation) of the lines of the objective function level. However, in the problems under discussion, the valley situation more often appears as a result of a small angle formed by the objective-function level line with the boundary of the permissible region.

The procedures of rounding off conductor diameters according to the tabulated data considerably complicate the strategy of search for the optimum. Optimization methods differ from each other by the programming procedures involved, count time, rate of convergence, etc. The above factors are dependent on both the effectiveness of the methods employed and on the complexity of the optimality criterion and geometry of the permissible area.

There are two approaches to the problem of optimization in a bounded area, which differ in principle since one approach considers constraints in implicit form and the other in explicit form. The implicit methods such as the Lagrangian multiplier method and penalty function method presuppose the implementation of the generalized objective function which coincides with the initial optimality criterion within the area, but grows rather fast beyond the area or even near to its boundary. The explicit methods (possible direction methods) make it necessary to move into the permissible area and take steps inside of it.

By the character of data collection, optimization methods can be divided into the methods of local and nonlocal search. Local search methods call for the analysis of the results of each experiment (computation results for the mathematical model) and the use of the information so obtained for the next experiment. The feature peculiar to these methods is that each successive experiment uses the information on the behavior of the mathematical model of the device under study that covers only a small region of the values of the parameters in the preceding experiment. Local search takes a relatively small amount of machine time but yields at best only one local extremum. The most important stage of local search is the choice of the direction of search for an optimum. This choice can be made from the estimate of the gradient defined with the aid of finite differences, or from the statistical data (random search), or from the estimates of partial derivatives. In some cases the directions can be chosen preliminarily, for example, along the coordinate axes or along orthogonal paths.

In solving problems for many extrema and problems involving valley conditions, the autonomous functioning of algorithms of local search proves ineffective. This situation has spurred the development of nonlocal search methods which in fact require a definite organization of a succession of local search stages. Thus the nonlocal

algorithm for the solution to multiextremal problems envisages the choice of initial points within the given field and processing of the results of local search made from these points. The algorithm affords the analysis of the given field and estimation of local extrema. In use are also special nonlocal search algorithms for the solution of problems involving valley-containing fields.

Optimum design of electric machines requires a pack of application programs and programs realizing local search algorithms. These are the algorithms for optimization of the criterion in the permissible search field, a direction of motion into the field from an arbitrary initial point, and preliminary search of the given field to determine the location of extrema and valleys and to solve the problems for the field with valleys.

To accomplish computer-aided graphic constructions associated with the design, it is necessary to work out a semigraphic model (being also a mathematical model) which would use the data on the main dimensions of stator and rotor cores to enable the graphic representation of the basic design of the machine.

Automated structural design of an electric machine includes two basic stages: the development and execution of the principal view of the machine in longitudinal and cross sections; the development and execution of views of assembly units and parts in a proper way for further use of the drawings at design departments and engineering plants.

The two stages of design work primarily use the results of computer-aided electromagnetic and heat-removal analyses based on a definite criterion for the automatic search of the optimum design of the machine as regards its dimensions and parameters. The geometric dimensions of the active part of the machine (i.e. stator and rotor cores with slots and windings) form the basis for the graphic representation at both stages.

At the first stage of design, the machine structure must possess certain general features specific to the machines of the given type. Further work on the general design must be done on the basis of a dialog of the designer with the computer, in which procedure the designer must perform all additional calculations necessary for the structural design of the shaft, bearings, and fasteners, for the vibration-acoustic analysis and the choice of standard parts, etc. The general drawing of the machine can be thought of as a sum of individual elements each of which should practically represent an assembly unit or part of the construction.

The designer must assess the computations and either accept the results for further use or introduce changes in the drawings while running the computer in the conversational mode. The need for introducing corrections can appear after the calculation of a shaft and bearings, the check on the depth of shields and the length of

the frame for correspondence with the overhang of the winding's face portions, the choice of fasteners, etc., and also in the course of bringing the design to the final form in compliance with the requirements of the assignment.

After completion of the general drawing of the machine, the second stage of design can follow, which involves developing the assembly units and parts of the construction and determining their mass, dimensions, and tolerances required at the production stage. This done, the autoplotter can finally make up the working drawings in compliance with the requirements of standardization. The subsystem of drawing facilities provides design and production drawings and substantially cuts down the lead times.

The development of automated design systems poses a number of complex problems. Of much significance is the establishment of international program libraries which would pull in scientific potential of engineers for the solution of the most important problems of electromechanics.

13.3. Hardware of Automated Design Systems

The central processing unit (processor) forms the basis for ADS hardware. This is a high-capacity computer with (or without) a satellite computer.

The first Soviet-made systems used computers of the M-220 and other types and various satellite computers. More advanced systems of automated design appeared as the Soviet Union together with the countries entering into the Council for Economic Mutual Assistance created and put into production electronic computers of the third generation with a wide range of storage and peripheral units. At present ADSs are built around the computers of the EC system (EC is the abbreviation for the Russian words meaning unified system) and EC satellite computers or small-capacity computers of the international system.

EC computers represent the family of program-compatible electronic machines performing from a few thousands to a few millions of operations in a second and having a unified range of peripheral units. Hardware of computers used in specific systems may vary in structure over a wide range.

The ADS requires well-developed means of data input, output, and search, means of copying graphic and text documentation, and means of on-line interaction with a computer. The list of EC computer peripheral units produced in series today extends to over 200 titles. By the purpose they have to serve, the peripheral units of EC computers are broken down into the following groups: external storages; graphic information input-output devices; conversational processing devices; and input preparation equipment.

In accordance with the features of the automated design process, output devices for graphic information can generally be divided into on-line mapping (display) means and means of plotting final graphs and forms. This is because the decision on the choice of the design version is commonly taken in the course of the iterative procedure involving consideration of a few designs before arriving at the final decision. In this connection the devices of on-line mapping (display) of graphic information must meet the requirements of a high rate of mapping and the plotting devices must meet the requirements of a high accuracy and high quality of graphs and drawings.

At present there are various methods of output of graphic information. These are the methods of making up images on paper and photographic paper, displaying images on a CRT screen, changing the color of paper by the reaction of electrolysis, etc. The systems of automated design widely use electromechanical, electronic, and scanning devices for output of graphic information.

Electromechanical automatic drawing machines are similar to numerically controlled millers in design and principle of action. Drawing on paper (or tracing paper) is brought about by an executive unit which consists of a plotting board or drum, electric drive, and a tracing unit. In automatic machines (graph plotters) of the plotting board type, the tracing unit moves in two mutually perpendicular directions x and y while the chart carrier remains stationary. The principle of a drum-type plotter differs from that of a board-type plotter. In the former the step motor-driven tracing unit moves only along the x axis and the driving drum shifts the paper sheet along the y axis. With the paper being reeled out from the roll, the tracing element draws a path as it shifts in the x direction. The tracing unit has pen holders to fasten ball-pen or pen-and-ink recorders; the number of recorders can vary from one to four. Each recorder traces lines or draws symbols of a definite thickness or a definite color. Automated design systems widely employ the above two types of automatic drawing machines.

In use are also automatic drawing machines of the unified system. The board-type machine of this class with a plotting board measuring 1 200 by 1 150 mm traces at a rate of 50 mm/s. The machine has a data converter which performs linear and curvilinear interpolation and affords automatic tracing of up to 253 symbols and three types of lines, namely, solid, dash, and dot-and-dash lines. There are two versions of drum-type machines with paper rolls of 420 $\times$ 80 000 mm and 878 $\times$ 20 000 mm in size, which have a maximum rate of tracing of 200 mm/s and 150 mm/s respectively.

Electromechanical drawing machines offer a number of advantages; namely, they ensure a high accuracy and quality of lines and symbols, can make up drawings of whatever size, trace lines of any types and colors, are adaptable for doing other jobs such as engrav-

ing and marking-off, and can act as autonomous devices. However, they show a low rate of tracing, do not allow for correcting an error in the process of drawing, have relatively large overall dimensions, and also present other drawbacks.

Electrochemical and electrothermal drawing machines relate to raster-type devices. A comb of electrodes forms a raster and serves as a tracing unit to produce an image on electrochemical paper impregnated with a special electrolyte. On one of its sides the paper comes in contact with comb electrodes, and on its other side with a metal electrode shaped like a cylinder. The voltage applied to individual electrodes induces the electrolysis reaction that changes the paper coloration. Changing the voltage on the comb electrodes gives different lines on the paper continuously unreeled from the roll.

In comparison with drawing machines of the electromechanical type, raster-type automatic machines have a higher rate of drawing. But they demonstrate difficulties in obtaining lines of various thicknesses, require microfilming and special moistened paper, fail to function in the autonomous mode, etc.

Electronic devices using a CRT as an executive unit are rather promising graphic display setups. Control of an image on the screen is brought about through a repetitive display of the image at a definite regeneration frequency. The image on the screen becomes stable and flickers vanish at a frequency of 40 Hz. Electronic display devices with a cathode-ray storage tube (CRST) enjoy use today. The number of addressing points on the screen reaches 4 096 × 4 096, with the working field measuring about 50 cm along the diagonal. This makes it possible to produce quality images on the screen and then photograph the drawings up to the 24th sheet size.

The main advantages of CRST-based devices are a high speed of image formation on the screen, possibility of producing color and half-tone images, on-line correction of errors by erasing the lines and displaying repetitively the images, low cost and small overall dimensions. The limitations are a relatively low quality of lines and symbols, low resolution, necessity of microfilming, etc.

Automation of the process of input of graphic information is a rather urgent problem. In the ADS, graphic information input devices (GIDs) supplement the main set of equipment of the computer. They can be automatic and semiautomatic. Automatic devices are of the scanning and tracking types. In scanning devices, the scanner beam sweeps over the drawing field line-by-line. Tracking devices track the lines of the drawing and predict their possible extention where a few lines intersect. Automatic devices can set only rather simple graphic data into the computer and require drawings of enhanced quality.

The storage capacity of a computer should be appreciably large to store the code obtained in automatic readout. However, reliable

algorithms for high-speed recognition of geometric patterns are not yet available. For this reason, automatic graphic data input devices have not found wide applications in ADSs.

Semiautomatic GIDs operate on the following general principle. While analyzing the drawing, the operator fixes the actuator at a definite point of the drawing and then brings the device into operation to calculate the point coordinates and represent the data in the numerical code. Thus the device automatically calculates the coordinates of points chosen by the operator. ADSs use to advantage semiautomatic data input devices having a working field 1 000 mm by 1 000 mm in size, which measure coordinates to better than 0.25 mm. Semiautomatic GIDs using CRTs with regeneration, light pens, coordinate balls, and other actuators hold rather considerable promise for use in ADSs.

Along with the facilities performing the functions of data input or data output, devices have recently found use which can serve both functions. Setups are available which combine a semiautomatic graphic data input device with an electromechanical automatic drawing machine. The approach to integrating data input and output devices into single CRT-based input-output units has led to the creation of a device for the on-line graphic access to the computer storage, which is known as a graphic display. The systems of automated design now employ graphic displays of various types developed in several countries (USSR, USA, France, Japan, etc.).

In evolving an automated design system, the set of facilities is chosen in each particular case with consideration for the problems to be solved; the body and streams of information; the time of generation, processing, and transfer of information; the form and kind of input and output data carriers; compatibility of available equipment, code and program facilities; the time required for construction of the system; and the cost.

Figure 13.3 shows the block diagram of a problem-oriented automated design complex with a universal set of peripheral facilities. This complex can form the basis for the development of various systems of automated design. As seen from the figure, the system has three groups of hardware means which enable the computer to be run in the interactive mode. The first group includes the facilities having a direct link with the central processor. These are a card reader, tape reader, alphanumeric printer, graphic display, and operator console comprising a video keyboard and devices of group control and block control. Tape and disk storages form external storage facilities.

The second group of hardware forms a satellite system having a link with the central processor via a front-end computer of the international system type with a small storage capacity. The external storage units here are disk storage and tape cartridge storage units.

Data input facilities are card and tape readers, and a semiautomatic data input device of the plotting board type. Other means of material support include an alphanumeric printer, graph plotter, graphic monitor, microfilming setup, and device for on-line execution of records.

The third group of hardware forms a terminal system connected to the central processor via a communication line. This system whose

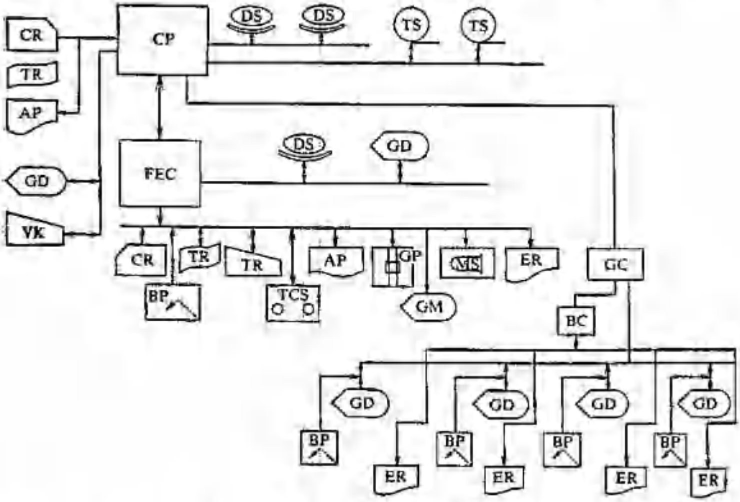


Fig. 13.3. The block diagram of ADS

VK — video keyboard; *GD* — graphic display; *AP* — alphanumeric printer; *TR* — tape reader; *CR* — card reader; *CP* — central processor; *DS* — disk storage; *TS* — tape storage; *FEC* — front-end computer; *BP* — board plotter; *TCS* — tape cartridge storage; *GP* — graphic plotter; *GM* — graphic monitor; *MS* — microphotography setup; *ER* — on-line execution of records; *GC* — group control; *BC* — block control

siting can be in a design office or a subdivision ensures parallel work of a few automated setups in the network of the functionally oriented complex.

The automated setup for an operator (designer) generally comprises a small-capacity computer, graphic data input-output and display devices, external storage units, and means of communication with the central processor. Soviet industry has started to turn out automated setups for design of radioelectronic equipment, solution to problems involved in tooling-up for production, and design of machinery. The Soviet-made standard automated setup consists of a pro-

cessor, primary storage unit, symbolic data input and display device, punched-tape input-output device, external storage unit, graphic terminal (graphic display), analog-to-number converter (image coder), tape reader, and board plotter. The software of the setup permits the simultaneous operation of all devices, including the input-output device and graphic editor.

The hardware engineering is making progress at a tremendous pace and more advanced devices take over constantly. But the principal architecture of the ADS and its basic hardware change little.

13.4. Conclusion

One of the important tasks of electromechanics is to evolve electric machines of extremely high powers, machines of novel unified series, and also special machines for various applications. The growing demand for energy has raised the problems of seeking new energy sources and developing new energy converters apart from the problem of improving the energy characteristics of conventional electric machines. In this connection it is highly important to find ways of creating efficient generators that would convert the solar energy into electric form and harness the thermonuclear reactor for generation of high-voltage energy.

The problem confronting engineers today can only be coped with by advancing the theory of electric machines still further, using for the purpose computing facilities and primarily digital computers. In recent years engineers have managed to investigate a number of problems earlier considered unsolvable. These are the problems for the solution of equations with nonlinear parameters and equations of transients in energy converters operating on nonsinusoidal and asymmetric supply voltages. Besides, it has become possible to solve problems relating to transients in multiwinding machines, energy conversion processes in machines with many degrees of freedom, buildup of the electromagnetic torque in one revolution, and interaction of an energy converter with elements connected to the stator and rotor.

The problems for the analysis of energy conversion in electric machines remain the focus of attention at the present time too.

Analog and digital computers enable the analyst to pass from differential equations describing transients to complex equations and investigate steady-state processes as a specific case of the solution of complicated differential equations. The analyst can proceed from the general to the particular rather than from the circuit models and equations of statics to dynamics as was the case before the advent of computing machines. There is an urgent need for conducting investigations with the aim to generalize the application of analog

and digital computers to the solution of the definite types of problems.

An increase in the number of complex systems of differential equations poses an important problem of simplifying the mathematical models and evaluating the accuracy of solutions. Simplified mathematical models obtained by the experimental design technique are finding ever increasing application. The development of polynomial models applied to the solution of the problems of synthesis of electric machines will facilitate the evolution of more improved methods of geometric programming.

It should be recognized that the advancements in the synthesis of energy converters are far from being as high as they are in the field of analysis, so much still remains to be done to raise the practice of synthesis to a higher level. The search for more improved methods of optimization will certainly continue and probably yield several optimization methods for the definite classes of problems.

Automated design systems represent the highest achievement in the field of synthesis of energy converters. Of paramount importance here is the establishment of data banks and program libraries. To accomplish the end requires the pooling of efforts not only within a particular branch of industry but also in the framework of the International Electrotechnical Commission. The creation of an ADS of electric machines that could do jobs starting from the request for the proposal and ending with the shipment of machines is an economically warranted task, though it is one of the difficult tasks and requires considerable efforts.

The creation of the general theory of electric machines is essential for the unified mathematical description of energy conversion processes in magnetic-field, electric-field, and electromagnetic-field energy converters. As is known from the history of electromechanics, many scientists made attempts to work out the general theory of energy converters. There are at present equations describing energy conversion processes in electric-field and electromagnetic-field machines. However, many difficulties have yet to be overcome to produce commercial versions of these types of machine. More research in the field of the general theory of electric machines and extensive work on the creation of new electrical engineering materials will obviously facilitate the development of new classes of electric machines.

The equations for a generalized electromechanical energy converter permit formulating a mathematical model practically for any problem in modern electromechanics. The notion of the generalized energy converter will undergo changes with time. The general model will obviously be needed to write equations for electric-field and electromagnetic-field energy converters with many degrees of freedom, equations for describing the conversion of energy in electric

machines in other forms of energy, and equations for the solution of unique problems in electric machine engineering. But the principal approach to the simultaneous analysis of fields and currents taking part in energy conversion processes will largely remain the same.

The notion of the electric machine as an electromechanical energy converter of any design version will have to extend concurrent with the search for new physical phenomena that would enable the creation of an energy converter with unique properties.

Theoretical investigations into magnetic, electric, thermal, and mechanical fields and their complicated interaction in an energy converter will promote further the development of the theory of electric machines and will thus offer innovations in the field of electric machine engineering. The statement that an electric machine converts energy from electric to mechanical form or from mechanical to electric form with the attendant transformation of energy into heat calls for extensive investigation on thermal-physical problems. There are machines in which it is difficult to give preference either to electromechanical phenomena or thermal-physical phenomena. Cryogenic electric machines, MGD energy converters, and energy storage devices may serve as an example. No doubt further advancements in the theory of electromechanical energy conversion will greatly promote electric machine engineering.

Appendices

Appendix 1. The Equations of the Basic Types of Electric Machine. Block Diagrams for Solution of the Equations on Computers

The equations (in currents) of an induction motor have the form

$$i_\alpha^s = \frac{1}{\dot{p}}\left(\frac{1}{L^s}u_\alpha^s - \frac{R^s}{L^s}i_\alpha^s\right) - \frac{M}{L^s}i_\alpha^r$$

$$i_\beta^s = \frac{1}{\dot{p}}\left(\frac{1}{L^s}u_\beta^s - \frac{R^s}{L^s}i_\beta^s\right) - \frac{M}{L^s}i_\beta^r$$

$$i_\alpha^r = \frac{1}{\dot{p}}\left[-\frac{R^r}{L^r}i_\beta^r - \omega_r\left(i_\beta^r + \frac{M}{L^r}i_\beta^s\right)\right] - \frac{M}{L^r}i_\alpha^s$$

$$i_\beta^r = \frac{1}{\dot{p}}\left[-\frac{R^r}{L^r}i_\beta^r + \omega_r\left(i_\alpha^r + \frac{M}{L^r}i_\alpha^s\right)\right] - \frac{M}{L^r}i_\beta^s$$

$$M_e = \frac{3}{2}pM\left(i_\alpha^r i_\beta^s - i_\beta^r i_\alpha^s\right)$$

$$\dot{p}\omega_r = \frac{1}{J}p\left(M_e - M_r\right)$$

where $\dot{p} = d/dt$; and p is the number of pole pairs. The block diagram for the solution of these equations is shown in Fig. A1.

The equations (in flux linkages) of an induction motor are

$$\frac{d\Psi_\alpha^s}{dt} = U_m\cos\omega t - \frac{R^sL^r}{L^sL^r - M^2}\Psi_\alpha^s + \frac{R^sM}{L^sL^r - M^2}\Psi_\alpha^r$$

$$\frac{d\Psi_\beta^s}{dt} = U_m\sin\omega t - \frac{R^sL^r}{L^sL^r - M^2}\Psi_\beta^s + \frac{R^sM}{L^sL^r - M^2}\Psi_\beta^r$$

$$\frac{d\Psi_\alpha^r}{dt} = -\frac{R^rL^s}{L^sL^r - M^2}\Psi_\alpha^r + \frac{R^rM}{L^sL^r - M^2}\Psi_\alpha^s - \omega_r\Psi_\beta^r$$

$$\frac{d\Psi_\beta^r}{dt} = -\frac{R^rL^s}{L^sL^r - M^2}\Psi_\beta^r + \frac{R^rM}{L^sL^r - M^2}\Psi_\beta^s + \omega_r\Psi_\alpha^r$$

$$M_e = \frac{mp}{2}\frac{M}{L^sL^r - M^2}\left(\Psi_\beta^s\Psi_\alpha^r - \Psi_\alpha^s\Psi_\beta^r\right), \qquad \frac{d\omega_r}{dt} = \frac{p}{J}\left(M_e - M_r\right)$$

The block diagram for the solution of these equations appears in Fig. A2. The current equations are given by

$$i_\alpha^s = \frac{L^r}{L^r L^s - M^2}\Psi_\alpha^s - \frac{M}{L^s L^r - M^2}\Psi_\alpha^r, \quad i_\beta^s = \frac{L^r}{L^s L^r - M^2}\Psi_\beta^s - \frac{M}{L^s L^r - M^2}\Psi_\beta^s$$

$$i_\alpha^r = \frac{L^s}{L^s L^r - M^2}\Psi_\alpha^r - \frac{M}{L^s L^r - M^2}\Psi_\alpha^s, \quad i_\beta^r = \frac{L^s}{L^s L^r - M^2}\Psi_\beta^r - \frac{M}{L^s L^r - M^2}\Psi_\beta^s$$

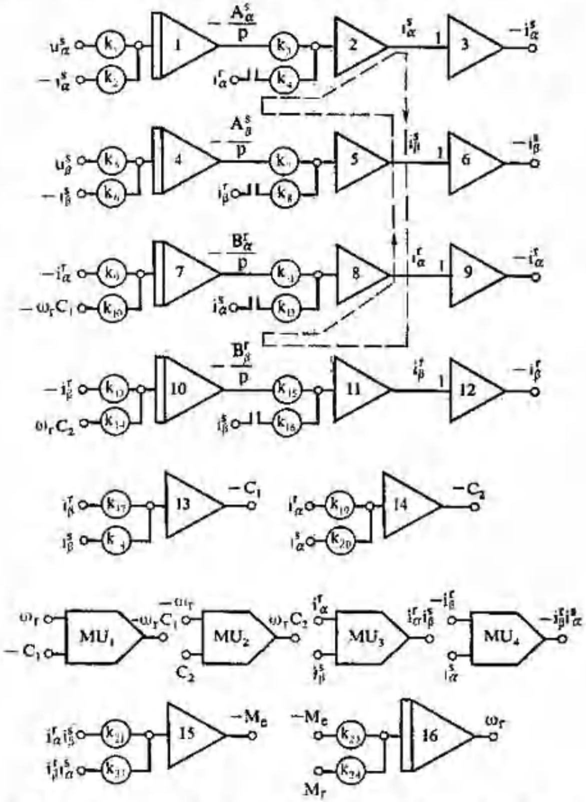


Fig. A1. The block diagram for the solution of equations of an induction motor (in currents)

1 to *16* — amplifiers; *MU* — multiplier units

The block diagram for the solution of equations (in flux linkages and currents) of an induction motor is shown in Fig. A3. The equa-

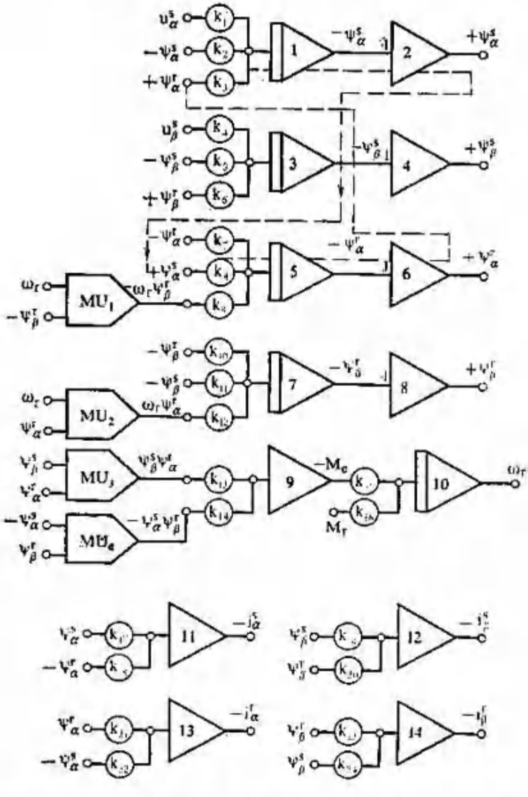


Fig. A2. The block diagram for the solution of equations of an induction motor (in flux linkages)

1-14 — amplifiers; *MU* — multiplier units

tions have the form

$$\frac{d\Psi_\alpha^s}{dt} = U_m \cos \omega t - R^s i_\alpha^s, \quad \frac{d\Psi_\beta^s}{dt} = U_m \sin \omega t - R^s i_\beta^s$$

$$\frac{d\Psi_\alpha^r}{dt} = -\omega_r \Psi_\beta^r - R^r i_\alpha^r, \quad \frac{d\Psi_\beta^r}{dt} = \omega_r \Psi_\alpha^r - R^r i_\beta^r$$

$$i_\alpha^s = \frac{L^r}{L^s L^r - L_m^2} \Psi_\alpha^s - \frac{L_m}{L^s L^r - L_m^2} \Psi_\alpha^r$$

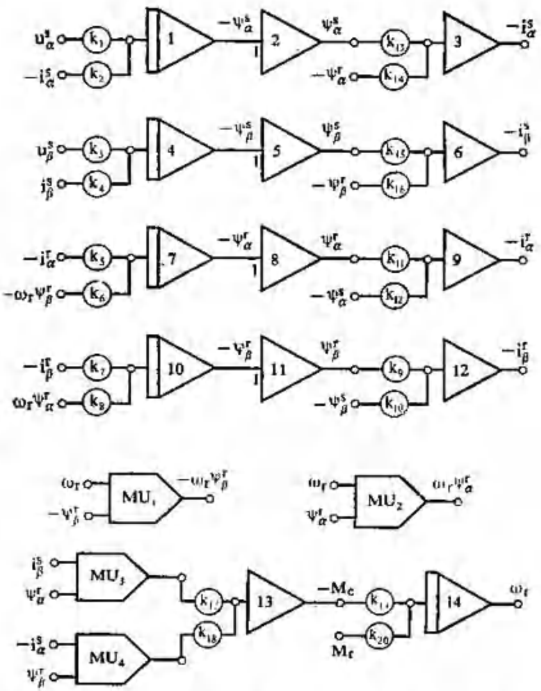


Fig. A3. The block diagram for the solution of equations of an induction motor (in flux linkages and currents)
1-14 – amplifiers; *MU* – multiplier units

$$i^s_\beta = \frac{L^r}{L^s L^r - L^2_m}\Psi^s_\beta - \frac{L_m}{L^s L^r - L^2_m}\Psi^r_\beta$$
$$i^r_\alpha = \frac{L^s}{L^s L^r - L^2_m}\Psi^r_\alpha - \frac{L_m}{L^s L^r - L^2_m}\Psi^s_\alpha$$
$$i^r_\beta = \frac{L^s}{L^s L^r - L^2_m}\Psi^r_\beta - \frac{L_m}{L^s L^r - L^2_m}\Psi^s_\beta$$

The block diagram for the solution of equations of a dc motor is given in Fig. A4. The equations are defined as

$$\frac{dI_a}{dt} = -\frac{C_e}{L_a} n\Phi_{res} - \frac{R_a}{L_a} I'_a + \frac{1}{L_a} u, \qquad \frac{d\Phi_{res}}{dt} = -\frac{r_e}{2p\sigma w_e} I'_e + \frac{1}{2p\sigma w_e} u$$
$$\Phi_{res} = \Phi_m - \Phi_{res.\,a}, \qquad \frac{dn}{dt} = \frac{375}{J_m}(M_m - M_r), \qquad M = C_m \Phi_{res} I'_a$$

where I'_a and I'_e are the currents in the armature circuit and excitation circuit respectively; σ is the leakage coefficient of main poles; C_e and C_m are design constants of the motor; L_a and R_a are the inductance and resistance of the armature circuit respectively;

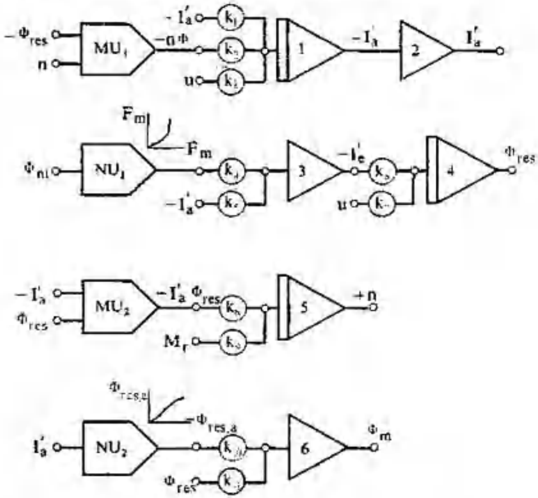


Fig. A4. The block diagram for the solution of equations of a dc motor
1-6 — amplifiers; *NU* — nonlinearity units; *MU* — multiplier units

M_m and M_r are the motor torque and load torque (moment of resistance) respectively; $\Phi_{res} = \Phi_m - \Phi_{res,a}$ is the resultant magnetic flux due to excitation windings; n is the rotational speed; and J_m is the moment of inertia of the motor.

The block diagram for the solution of equations of a dc generator is illustrated in Fig. A5. The equations are

$$\frac{di_a}{dt} = -\frac{1}{L_a}u_l - \frac{R_a}{L_a}i_a - \frac{1}{L_a}2\Delta u_b + \frac{1}{L_a}e_{res} + \frac{M_b}{L_a}\frac{di_e}{dt}$$

$$\frac{di_e}{dt} = \frac{1}{R_e}u_e - \frac{L_e}{dt}\frac{di_a}{dt} - \frac{M_d}{R_e}\frac{di_a}{dt}$$

$$\frac{d\omega_r}{dt} = \frac{1}{J_m + J_g}(M_m - M_g), \quad M_m = i_a C_e \Phi_{res}$$

where u_l is the instantaneous value of voltage across the load; e_{res} is the instantaneous (resultant) value of emf of generator rotation; M_d and M_b are respectively the coefficients for the direct and back-

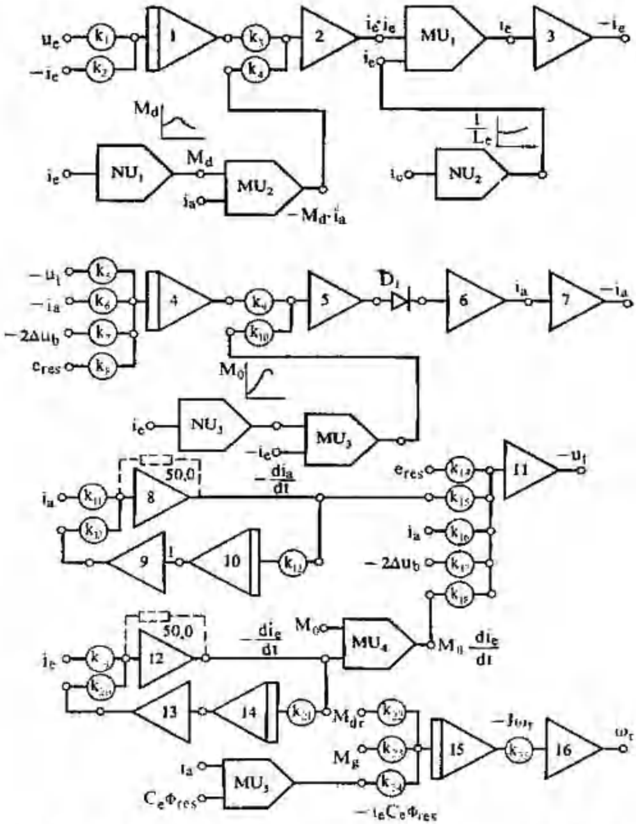


Fig. A5. The block diagram for the solution of equations of a dc generator
1-16 — amplifiers; *NU* — nonlinearity units; *MU* — multiplier units

ward mutual inductances between the excitation circuit and armature circuit; $2\Delta u_b$ is the voltage drop across the brush contact; i_a and i_e are the instantaneous values in the armature and excitation

circuits respectively; M_m and M_g are the torques on the shafts of the motor and of the generator; and J_m and J_g are the moments of inertia of the motor and generator respectively.

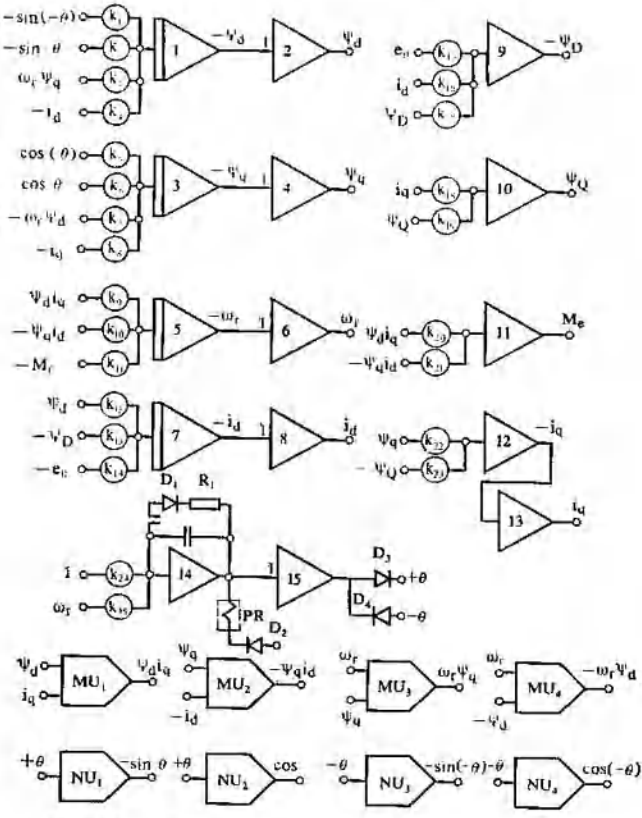


Fig. A6. The block diagram for the solution of equations of a synchronous permanent-magnet motor

1-15 — amplifiers; *PR* — polarized relay; *MU* — multiplier units; *NU* — nonlinearity units

The block diagram for the solution of equations of a synchronous permanent-magnet motor is shown in Fig. A6. The equations are of

the form

$$\frac{d\Psi_d}{dt} = -u_m \sin\theta + \omega_r \Psi_q - i_d R_s, \qquad \frac{d\Psi_q}{dt} = u_m \cos\theta - \omega_r \Psi_d - i_q R_s$$

$$\frac{d\Psi_D}{dt} = \frac{R_D x_{ad}}{x_{DD}} i_d - \frac{R_D}{x_{DD}} \Psi_D + \frac{R_D}{x_{DD}} e_0, \qquad \frac{d\Psi_Q}{dt} = \frac{R_Q x_{aq}}{x_{QQ}} i_q - \frac{R_Q}{x_{QQ}} \Psi_Q$$

$$i_d = \frac{x_{DD}}{x_d x_{DD} - x_{ad}^2} \Psi_d - \frac{x_{ad}}{x_d x_{DD} - x_{ad}^2} \Psi_D - \frac{x_{DD} - x_{ad}}{x_d x_{DD} - x_{ad}^2} e_0$$

$$i_q = \frac{x_{QQ}}{x_q x_{QQ} - x_{aq}^2} \Psi_q - \frac{x_{aq}}{x_q x_{QQ} - x_{aq}^2} \Psi_Q$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} (\Psi_d i_q - \Psi_q i_d) - M_r, \qquad \frac{d\theta}{dt} = 1 - \omega_r$$

Appendix II. Data on Induction Motors of Series 4A with a Height of Axes of Rotation from 80 to 250 mm. Basic Quantities, Scales of Variables, and Gains of Amplifiers

Table AII.1

Data on Motors

Factor	Unit of measure	Motor type										
		4AA-50A4	4AA-56A4	4A-71A4	4A-80A4	4A-112M4	4A-132M4	4A-180M4	4A-200L4	4A-225M4	4A-250M4	4AH-250M4
$2p$		4	4	4	4	4	4	4	4	4	4	4
m		3	3	3	3	3	3	3	3	3	3	3
f	Hz	50	50	50	50	50	50	50	50	50	50	50
P_2	W	60	120	550	1100	5500	11 000	30 000	45 000	55 000	90 000	110 000
$U_{n.ph}$	V	220	220	220	220	220	220	220	220	220	220	220
$I_{n.ph}$	A	0.27	0.43	1.58	2.66	11.1	21.53	54.97	81.36	99.31	158.5	195.5
R_1	Ω	152.9	97.72	16.39	9.53	1.32	0.462	0.160	0.091	0.067	0.032	0.03
R_2	Ω	192	72.48	15.08	5.619	0.922	0.312	0.078	0.045	0.032	0.019	0.0172
x_1	Ω	160	42.68	12.27	5.464	1.439	0.831	0.362	0.249	0.210	0.130	0.117
x_2	Ω	134.7	76.9	24.33	9.2	2.35	1.262	0.513	0.386	0.305	0.21	0.19
x_μ	Ω	837	600	195.9	140.45	51.5	27.5	15.34	10.1	9.01	6.75	4.83
L_m	H	2.66	1.91	0.624	0.447	0.164	0.0876	0.0489	0.0322	0.0287	0.0215	0.0154
L_1	H	3.175	2.046	0.663	0.484	0.169	0.0903	0.05	0.033	0.0294	0.022	0.0158
L_2	H	3.095	2.155	0.7015	0.476	0.1715	0.0916	0.051	0.0334	0.0297	0.022	0.0158
k_M		2.37	2.2	2.47	2.51	2.5	3.32	2.92	2.78	2.79	2.86	2.68
k_I		2.81	3.28	4.06	4.95	6.47	6.83	7.01	7.01	7.1	7.11	6.5
s_n		0.078	0.076	0.075	0.056	0.035	0.027	0.019	0.015	0.014	0.013	0.015
GD^2	kgf m²	0.0008	0.0012	0.043	0.0103	0.0824	0.1852	0.898	1.7369	2.483	4.567	3.871
J	kgf m²	0.000189	0.000275	0.0011	0.0026	0.0206	0.0463	0.2245	0.434	0.621	1.142	0.968
$\cos\varphi$		0.687	0.693	0.737	0.822	0.882	0.876	0.910	0.914	0.908	0.921	0.908

Table AII.2

Basic Quantities

Quantity	Unit of measure	Motor type					
		4AA-50A4	4AA-56A4	4AA-63A4	4A-71A4	4A-80M4	4A-112M4
$U_b = U_{n.ph}\sqrt{2}$	V	310	310	310	310	310	310
$i_b = I_{n.ph}\sqrt{2}$	A	0.381	0.606	1.099	2.23	3.76	15.7
$z_b = U_b/I_b$	Ω	815.8	500	300	139	82.45	19.7
$P_b = (3/2)\,U_b i_b$	W	177.2	281.9	511.4	1036.9	1748.4	7300.5
$\omega_b = \omega_0$	s^{-1}	314	314	314	314	314	314
ω_b^2	$(s^{-1})^2$	98 596	98 596	98 596	98 596	98 596	98 596
$t_b = 1/\omega_b$	(s)	0.00318	0.00318	0.00318	0.00318	0.00318	0.00318
$M_b = P_b/\omega_b$	N m	0.564	0.898	0.163	3.3	5.57	23.25
$J_b = M_b/\omega_b^2$	kg m²	5.68×10^{-6}	9.3×10^{-6}	16.5×10^{-6}	33.5×10^{-6}	56.5×10^{-6}	0.24×10^{-3}
$\Psi_b = U_b/\omega_b$	Wb	0.987	0.987	0.987	0.987	0.987	0.987
$\Psi_b^2 = U_b^2/\omega_b^2$	Wb²	0.974	0.974	0.974	0.974	0.974	0.974
Ψ_b/M_b		1.73	1.06	0.597	0.295	0.175	0.042
$\dot{j} = J/J_b$		17.54	32.94	32.0	32.1	45.6	85.8
Ψ_b/i_b		2.6	1.63	0.898	0.44	0.26	0.063

Table AII.2 (*continued*)

Quantity	Unit of measure	Motor type						
		4A-132M4	4A-180M4	4A-200L4	4A-225M4	4A-250S4	4A-250M4	4AH-250M4
$U_b = U_{n.ph}\sqrt{2}$	V	310	310	310	310	310	310	310
$i_b = I_{n.ph}\sqrt{2}$	A	30.44	77.73	115	140.4	187.9	224.1	277.85
$z_b = U_b/I_b$	Ω	10.2	3.99	2.7	2.2	1.7	1.38	1.12
$P_b = (3/2)\,U_b i_b$	W	14154.5	36 144	53 476	65 286	87396.7	104 206	129 200
$\omega_b = \omega_0$	s^{-1}	314	314	314	314	314	314	314
ω_b^2	$(s^{-1})^2$	98 596	98 596	98 596	98 596	98 596	98 596	98 596
$t_b = 1/\omega_b$	(s)	0.00318	0.00318	0.00318	0.00318	0.00318	0.00318	0.00318
$M_b = P_b/\omega_b$	N m	45.1	115	170.3	207.9	278.3	331.8	411
$J_b = M_b/\omega_b^2$	kg m^2	0.46×10^{-3}	1.17×10^{-3}	1.73×10^{-3}	2.11×10^{-3}	2.8×10^{-3}	3.36×10^{-3}	4.17×10^{-3}
$\Psi_b = U_b/\omega_b$	Wb	0.987	0.987	0.987	0.987	0.987	0.987	0.987
$\Psi_b^2 = U_b^2/\omega_b^2$	Wb^2	0.974	0.974	0.974	0.974	0.974	0.974	0.974
Ψ_b/M_b		0.022	0.0085	0.0057	0.0047	0.0035	0.003	0.002
$\dot{J} = J/J_b$		100.6	192	250.9	294	351.96	339.8	232.1
Ψ_b/i_b		0.032	0.013	0.0086	0.007	0.0053	0.0044	0.0035

Table AII.3

Coefficients of Variables

Coefficient	Motor type												
	4AA-50A4	4AA-56A4	4AA-63A4	4AA-71A4	4A-80A4	4A-112M4	4A-132M4	4A-180M4	4A-200L4	4A-225M4	4A-250S4	4A-250M4	4AM-250M4
$\dot{a}_2=\dot{a}_5$	0.55	0.88	0.67	0.484	0.675	0.333	0.21	0.17	0.15	0.125	0.102	0.0887	0.0957
$\dot{a}_3=\dot{a}_6$	0.477	0.78	0.597	0.43	0.649	0.32	0.20	0.17	0.144	0.12	0.1	0.086	0.09
$\dot{a}_7=\dot{a}_9$	0.714	0.621	0.53	0.421	0.388	0.19	0.14	0.088	0.073	0.059	0.059	0.052	0.054
$\dot{a}_8=\dot{a}_{10}$	0.599	0.579	0.49	0.396	0.383	0.186	0.138	0.022	0.071	0.058	0.0576	0.051	0.0531
$\dot{a}_{11}$	5.09	8.47	8.1	7.38	11.24	10.14	9.72	8.9	8.47	8.48	8.64	8.06	6.15
$\dot{a}_{12}$	0.144	0.0607	0.063	0.0623	0.044	0.024	0.02	0.01	0.0079	0.007	0.0057	0.006	0.086
$\dot{a}_{13}$	2.96	4.6	4.5	4.08	5.79	5.3	4.9	4.58	4.41	4.37	4.46	4.07	3.78
$\dot{a}_{14}$	2.54	4.09	4.014	3.63	5.56	5.06	4.7	4.54	4.25	4.2	4.32	3.94	3.68
$\dot{a}_{15}$	3.024	4.38	4.27	3.85	5.64	5.2	4.85	4.65	4.36	4.31	4.41	4.01	3.58

Note. $\dot{a}_1=\dot{a}_4=1$.

Table AII.4

Scales of Variables

Scale	Motor type												
	4AA-50A4	4AA-56A4	4AA-63A4	4AA-71A4	4A-80M4	4A-112M4	4A-132M4	4A-180M4	4A-200L4	4A-225M4	4A-250S4	4A-250M4	4AH-250M4
M_{ω}	100	100	100	100	100	100	100	100	100	100	100	100	100
M_{Ψ}	50	50	50	50	50	50	50	50	50	50	50	50	50
M_{M_e}	35	30	20	25	20	20	20	20	28	20	25	20	40
M_t	1	1	1	1	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159
M_i	36.6	29.2	28.1	25	20	16.2	14.6	14.2	14.2	14.2	13.4	14.1	15.4

Table AII.5

Gains of Amplifiers of the Computer Analog

$\dot{k}_1=\dot{k}_4$	0.5	0.5	0.5	0.5	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14
$\dot{k}_2=\dot{k}_5$	0.55	0.88	0.67	0.484	4.23	2.09	1.31	1.06	0.942	0.78	0.64	0.81	0.6
$\dot{k}_3=\dot{k}_6$	0.477	0.78	0.6	0.43	4.07	2	1.56	1.06	0.904	0.75	0.625	0.59	0.56
$\dot{k}_7=\dot{k}_{10}$	0.714	0.621	0.53	0.421	2.44	1.19	0.88	0.55	0.458	0.37	0.369	0.358	0.33
$\dot{k}_8=\dot{k}_{11}$	0.599	0.579	0.494	0.396	2.4	1.18	0.86	0.51	0.42	0.36	0.36	0.35	0.33
$\dot{k}_9=\dot{k}_{12}$	1	1	1	1	6.28	6.28	6.28	6.28	6.28	6.28	6.28	6.28	6.28
$\dot{k}_{13}=\dot{k}_{14}$	7.126	9.8	6.4	7.38	28.46	8.1	7.76	7.12	9.49	6.78	8.64	6.68	9.85
$\dot{k}_{15}=\dot{k}_{16}$	0.57	0.202	0.314	0.249	0.437	0.754	0.628	0.314	0.84	0.220	0.143	0.185	0.135
$\dot{k}_{17}=\dot{k}_{19}$	2.1	2.69	2.53	2.05	2.3	1.7	1.43	1.3	0.807	1.24	1.496	1.176	1.164
$\dot{k}_{18}=\dot{k}_{20}=\dot{k}_{22}=\dot{k}_{24}$	1.81	2.388	2.26	1.81	2.22	1.54	1.37	1.29	1.21	1.19	1.158	1.14	1.106
$\dot{k}_{21}=\dot{k}_{23}$	2.15	2.56	2.39	1.925	2.26	1.68	1.42	1.32	1.24	1.22	1.18	1.16	1.133

Appendix III. Block Diagrams of the Models of Nonsinusoidal Voltage Generators

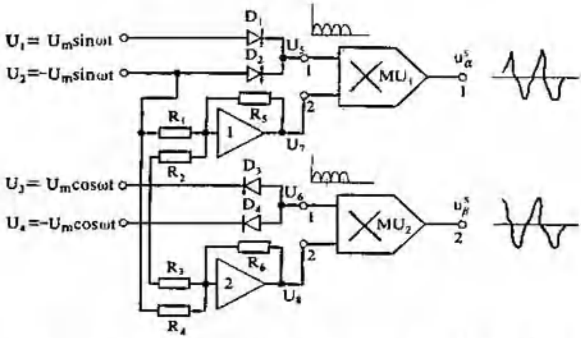


Fig. A7. The block diagram of the model of a nonsinusoidal voltage generator
1-2 — amplifiers; *MU* — multiplier units

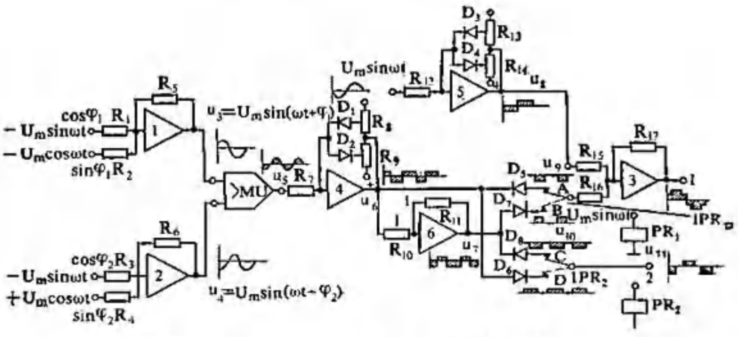
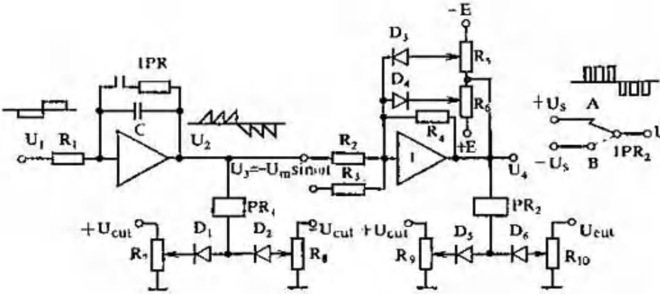


Fig. A8. The block diagram of a rectangular pulse generator
1-6 — amplifiers; *MU* — multiplier unit; *PR* — polarized relays



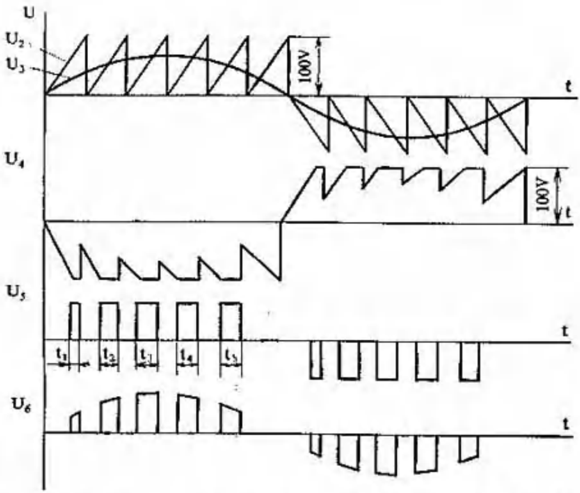


Fig. A9. The block diagram of an inverter model with pulse-duration modulation and waveforms at generator output

U_{cut} – cut-off voltage; *PR* – polarized relay

Appendix IV. An Example of the Electric Machine Design Assignment

Questions to be Treated

1. Make up the equations for an induction squirrel-cage motor in stationary coordinates α and β and reduce them to the form convenient for their solution on an analog computer. Generate the solutions to the equations in terms of currents and flux linkages.
2. Construct the block diagram for the solution of the system of equations and calculate the gain for each input of the amplifier in the chosen analog of the motor.

Design of the Experiment and the Solution to the Optimization Problem

1. Choose the variable parameters of the motor and form the experimental design matrix.
2. Simulate the experiment on an analog computer and define the polynomial relations between the characteristics and parameters of the motor.
3. Determine the optimum parameters of the induction motor under the specified conditions of optimization.
4. Evaluate the accuracy of search for the optimum parameters.

Directions for the Solution to the Problem

1. For the derivation of induction machine equations in terms of currents and flux linkages, see Chapter 2. These equations and the block diagram for their solution are given in Appendix I.

To illustrate the way of how to reduce the equations to the form convenient for their solution on an analog computer, we consider an example of the A42-6 three-phase squirrel-cage motor using its equations expressed in terms of currents. The A42-6 has the following nominal characteristics: $P_{2n} = 1.7$ kW, $U_n = 220/380$ V, $I_n = 7.5/4.5$ A, and $n_n = 930$ rpm.

The motor parameters at a working temperature of 75°C are as follows:

$$r_\alpha^r = r_\beta^r = 3.8\Omega \qquad r_\alpha^s = r_\beta^s = 3.57\Omega$$

$$L_\alpha^s = L_\beta^s = 0.279 \text{ H} \qquad L_\alpha^r = L_\beta^r = 0.289 \text{ H}$$

$$M = 0.263 \text{ H} \qquad J = 152 \times 10^{-5} \text{ kg m}$$

$$p = 3 \qquad m = 3$$

The equation for U_α^s has the form

$$U_\alpha^s = i_\alpha^s \left[r_\alpha^s + (d/dt)\, L_\alpha^s\right] + i_\alpha^r\, (d/dt)\, M$$

or

$$U_\alpha^s = i_\alpha^s + i_\alpha^s (d/dt) L_\alpha^s + i_\alpha^r (d/dt) M$$

whence

$$i_\alpha^s (d/dt) L_\alpha^s = U_\alpha^s - r_\alpha^s i_\alpha^s - (d/dt) M i_\alpha^r$$

Introduce the designation $d/dt = \dot{p}$. Then,

$$i_\alpha^s = U_\alpha^s / L_\alpha^s \dot{p} - r_\alpha^s i_\alpha^s / L_\alpha^s \dot{p} - M i_\alpha^r / L_\alpha^s$$

Substitute here the motor parameters

$$i_\alpha^s = 220\sqrt{2} \cos \omega t / 0.279 \dot{p} - 3.57 i_\alpha^s / 0.279 \dot{p} - (0.263/0.279)\, i_\alpha^r$$

or

$$i_\alpha^s = 116 \cos \omega t / \dot{p} - 12.8 i_\alpha^s / \dot{p} - 0.935 i_\alpha^r$$

This form of the equation is more preferable for its solution on an analog computer since the analog set up on the computer proves more stable.

The equations for the rotor voltage are brought to the desired form in a similar manner with consideration for the fact that $U_\alpha^r = U_\beta^r = 0$ because the rotor is of the cage type:

$$0 = \dot{p} M i_\alpha^s + [r_\alpha^r + (d/dt) L_\alpha^r]\, i_\alpha^r + L_\beta^r \omega_r i_\beta^r + M \omega_r i_\beta^s$$

$$\dot{p} L_\alpha^r i_\alpha^r = -\dot{p} M i_\alpha^s - i_\alpha^r r_\alpha^r - \omega_r (L_\beta^r i_\beta^r + M i_\beta^s)$$

$$i_\alpha^r = -\frac{M}{L_\alpha^r} i_\alpha^s - \frac{r_\alpha^r i_{\alpha}^{r}}{L_\alpha^r \dot{p}} - \frac{\omega_r (L_\beta^r i_\beta^r + m i_\beta^s)}{L_\alpha^r \dot{p}}$$

After substitution of the numerical values, the final expression becomes

$$i_\alpha^r = -13.14 i_\alpha^r / \dot{p} - 0.91 i_\alpha^s - \omega_r (0.91 i_\beta^s + i_\beta^r)/\dot{p}$$

The same approach works for reducing the remaining equations to the form convenient for setting up the computer analog. The system of equations for the A42-6 motor has the following final form

$$i_\alpha^s = 1\,116 \cos \omega t / \dot{p} - 12.8 i_\alpha^s / \dot{p} - 0.935 i_\alpha^r$$

$$i_\beta^s = 1\,116 \sin \omega t / \dot{p} - 12.8 i_\beta^s / \dot{p} - 0.935 i_\beta^r$$

$$i_\alpha^r = -13.14 i_\alpha^r / \dot{p} - 0.91 i_\alpha^s - \omega_r (0.91 i_\beta^s + i_\beta^r)/\dot{p}$$

$$i_\beta^r = -13.14 i_\beta^r / \dot{p} - 0.91 i_\beta^s + \omega_r (0.91 i_\alpha^s + i_\alpha^r)/\dot{p}$$

$$M_e = 0.121\,6\, (i_\alpha^r i_\beta^s - i_\alpha^s i_\beta^r)$$

$$d\omega_r/dt = 1\,975\, (M_e - M_r)$$

To clear up point 1 of the optimization problem, we should take the motor parameters from Table AIV. 1 in accordance with the number of the assignment variant and reduce each initial equation expressed in currents or flux linkages to the desired form.

The block diagram for the solution of the system of equations of the induction motor is built up in stages. The first stage involves

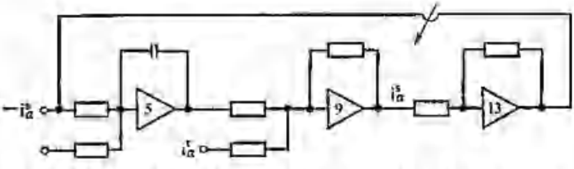


Fig. A10. The block diagram for determining stator currents

the choice of appropriate elements and calculation of the gain for each equation and the second stage involves the connection and switching of the entire analog network.

As an example, consider the construction of a computing network and the calculation of gain factors for one of the equations:

$$i^s_\alpha = 1\,116 \cos \omega t/\dot{p} - 12.8 i^s_\alpha/\dot{p} - 0.935 i^r_\alpha$$

The computer analog for this equation is shown in Fig. A10. It contains an integrator with two inputs to integrate the first and the second term of the equation and also a summer to add together the integration results and the values of the subsequent term, $-0.935\, i^r_\alpha$. To derive from the output the value of $-i^s_\alpha$ which is then sent to the input of an amplifier *5* and to other computing elements, the network incorporates an inverter built around an amplifier *13*. After constructing the block diagram, we need to calculate the amplifier gains equivalent to the constants in the equations. The calculation for the summing and the integrating units is made by use of the following formulas:

$$k_\Sigma = M_{out}a/M_{in}, \quad k_\int = M_{out}a/M_{in}M_t$$

where M_{in}, M_{out}, and M_t are the scales of the input quantity, output quantity, and time respectively. Since $M_i = 4$ V/A, $M_U = 100$ volts per unit, and $M_t = 50$, we have $k_{5-1} = 4 \times 12.8/4 \times 50 = 0.256$, $k_{5-2} = 4 \times 1\,116/100 \times 5 = 0.893$, $k_{9-1} = 4/4 = 1$, and $k_{9-2} = 4.0 \times 935/4 = 0.935$.

In a similar way the calculation is performed for other units and the gain factors are found. The entire block diagram of the computer setup for the solution of the system of equations is shown in Fig. A11.

In the experiment being designed, the variable parameters (factors) of the motor are chosen from Table AIV.2 (in accordance with the assignment variant number) and the variation ranges of the factors are preset. A typical design procedure comes to designing the CFE of the 2^3 type (see Table AIV.3). In each assignment variant being

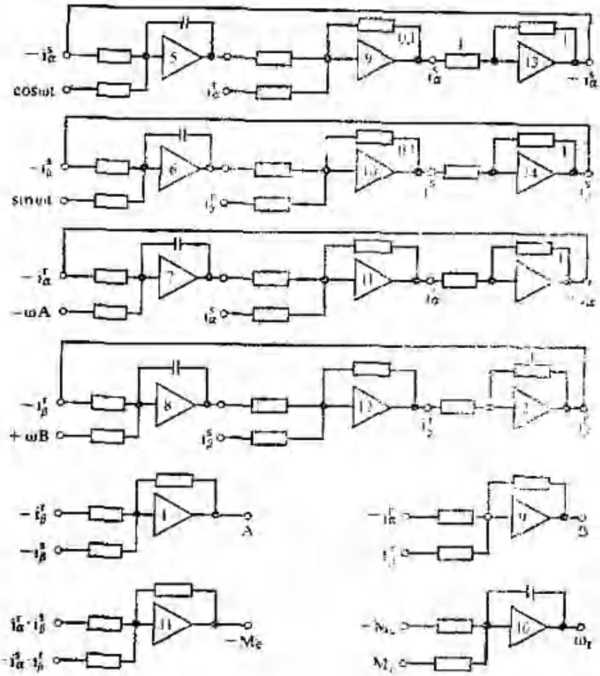


Fig. A11. The block diagram for the solution of equations of an induction motor

tackled, the motor parameters presented in Table AIV.1 are taken as basic values. The upper and lower limits of the variable parameter are defined in accordance with the specified intervals and the data are put down in the table (matrix). The experiment being designed calls for the variation of a few variable factors at different levels. The 2^3 CFE involves eight combinations in the variation of variable factors, therefore for each row of the design matrix we need

to carry out the appropriate recalculation of the gains of the analog. For example, if the variable factor is taken to be r^s, then its value determines the coefficient affixed to $i_\alpha^s/\dot{p}$, which is equal to 12.8 at a basic value of r^s equal to 3.57Ω. In proportion to this value we calculate the gain $k_{5-1} = 0.256$. In setting up the experiment, the value of r^s is made to vary at two levels: the upper level, +1 or 4.28Ω; the lower level, −1 or 2.86Ω. In accordance with these values, the gain k_{5-1} for the network of Fig. A11 need be recalculated.

For the level with a 'minus' sign

$$k_{5-1} = M_{ou} r^s / M_{in} M_t = 4 \times 11.65/4 \times 50 = 0.233$$

and for the level with a 'plus' sign

$$k_{5-1} = 4 \times 14.65/4 \times 50 = 0.293$$

In clarifying point 1 of the problem, we should see to it, considering the specified variable parameters, what coefficients in the system of equations are variable, then recalculate the corresponding gain factors and summarize the results in the table. As an example, Table AIV.3 gives the results of the calculation of gain factors in modeling the A42-6 motor by solving its equations expressed in flux linkages. A similar table should be made up for the assignment variant involving the solution of the motor equations expressed in terms of currents.

It is now necessary to analyze the obtained design matrix and determine the expedient sequence of its implementation on any analog computer so as to recalculate a smaller number of the gain factors in going from one trial to the other.

The trial in experimental design means the calculation on an analog computer of the characteristics of the transient and steady-state processes in the induction motor at the fixed values of its parameters. The fixed values of the motor parameters are chosen for each assignment variant from Table AIV.2. Since in the computer analog the amplifiers show zero drift, the experiment must be rerun at least three times and the results entered in the table.

2. The experimental design enables us to obtain a simple relation between the variable parameters of the machine and its characteristics. This relation has the form of a polynomial

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i<j}^{n} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} x_i^2 + \dots$$

where b_0, b_i, b_{ij}, b_{ii} are polynomial coefficients; x_i, x_j are the variable parameters of the induction motor; y is the operating factor or factors (objective functions) of the motor in its static and dynamic operation; and n is the number of variable factors (parameters).

The polynomial coefficients in the CFE are found by the formula

$$b_{ij} = b_i = \sum_{g=1}^{n} z_{vi} y_v / N, \qquad (i = 0, 1, \ldots, n)$$

After determining the polynomial coefficients b_i, b_{ij}, we should check their significance (test the null hypothesis).

A check of the hypothesis is done by the Student t-test formulated in this case as

$$t_i = |b_i| / S\{b_i\}, \quad S\{b_i\} = \sqrt{S^2\{b_i\}}$$

where $S^2\{b_i\}$ is the error variance in estimating the coefficient b_i. In the CFE this variance for all values of b_i is given by

$$S^2\{b_i\} = S^2\{y\}/Nm$$

where N is the number of points of the factor space in which the designed experiment is set up; $S^2\{y\}\,[1/(m-1)] \sum_{l=1}^{m} (\bar{y}_{vl} - y_y)^2$ is the experiment variance obtained in conducting m trials for one of the points of the factor space under study; and $\bar{y}_v = \sum_{l=1}^{m} y_{vl}/m$.

If the found value of the t-test exceeds the value taken from Table AIV.4 for the number of degrees of freedom, $\nu_s = N(m-1)$, at the specified significant level g_s (%), the hypothesis is rejected and the coefficient b_i is assumed significant: otherwise, the hypothesis is accepted and the coefficient b_i is considered insignificant, i.e. equal to zero.

The check of the hypothesis for the adequate representation of the experiment results by the found polynomial is done from the estimate of the discrepancy between the output value of $\hat{y}_v$ and the experimental values of y_v at all points of the factor space.

The dispersion of the experiment results on the approximating polynomial is describable by the inadequacy variance σ^2_{ad} whose estimate S^2_{ad} is found from the formula

$$S^2_{ad} = [1/(N-d)] \sum_{g=1}^{N} (y_v - \hat{y}_v)^3$$

where d is the number of the significant terms in the approximating polynomial.

The inadequacy variance is dependent on the number of the degrees of freedom

$$\nu_{ad} = N - d$$

The check on adequacy consists in estimating the deviation of the inadequacy variance σ^2_{ad} from the reproducibility variance $\sigma^2\{y\}$.

If σ^2_{ad} does not exceed the experiment variance, the obtained mathematical model adequately represents the results of the experiment; if $\sigma^2_{ad} > \sigma^2\{y\}$, the description is considered inadequate to the object under analysis.

The check of the hypothesis for adequacy is carried out using Fisher's variance ratio test. The F-test permits checking the null hypothesis for the equality of two generalized variances S^2_{ad} and $S^2\{y\}$ in the case where sampling variances $S^2_{ad} > S^2\{y\} = S^2$.

The F-test is defined as the variance ratio

$$F = S^2_{ad}/S^2$$

If the calculated value of the F-test is smaller than its critical value found from Table AIV.5 for the corresponding degrees of freedom, $\nu_{1ad} = \nu_{ad} = N - d$ and $\nu_{2ad} = \nu_S = N(m-1)$, at the specified significant level q_{ad} (%), the null hypothesis is accepted. Otherwise the hypothesis is rejected and the description is considered inadequate to the object under study. If the hypothesis of adequacy is rejected, a smaller step of variation is taken and the experiment is run anew.

3. Optimum parameters of the motor are found using the polynomials defined in the text of point 2 of the problem. The optimization criterion and constraint functions are chosen for each assignment variant from Table AIV.6. The polynomials for η and $\cos\varphi$ in all assignment variants are the same and equal to

$$\eta = 0.815 - 0.014r^r - 0.022r^s$$

$$\cos\varphi = 0.83 + 0.036r^r - 0.018r^s + 0.026r^r r^s$$

In the typical design procedure, the optimum parameters are determined by the semigraphical method in which two parameters are varied within the limits set up in the assignment. Consider the way of estimating the optimum parameters. Assume that the designed experiment conducted for a certain type of induction motor has given the polynomial relations for the impact current, impact torque, and starting (speedup) time as functions of two parameters. These relations are of the form

$$I_{im} = 4.5 - 0.3r^r - 0.4r^s + 0.1r^s r^r$$

$$M_{im} = 3.5 + 0.5r^r - 0.5r^s$$

$$t_{st} = [50 - 12r^r + 2r^s + 2r^r r^s]\,(10^{-1})$$

where r^s and r^r are the stator and rotor winding resistances, respectively, expressed in relative units.

What we need to determine are the values of r^r and r^s at which the starting time is a minimum, with the constraints being imposed on I_{im} and M_{im}. In its mathematical form, the optimization prob-

lem can be written as

$$t_{st} = 50 - 12r^r + 2r^s + 2r^r r^s \to \min$$

at

$$I_{im} = 4.5 - 0.3r^r - 0.4r^s + 0.1r^r r^s = 4.5$$

$$M_{im} = 3.5 + 0.5r^r - 0.5r^s \leqslant 4$$

if $-1 \leqslant r^r \leqslant +1$ and $-1 \leqslant r^s \leqslant +1$ (the variation ranges are expressed in relative units in analogy with the design matrix).

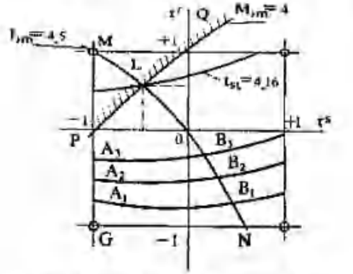


Fig. A12. The two-dimensional factor space for variables r^r and r^s

The two-dimensional factor space for the variables r^r and r^s is shown in Fig. A12. The vertices of the square are the points for setting up the factorial experiment. The space confined within the square is the initial field of the permissible solutions with the constraints disregarded.

The first constraint

$$I_{im} = 4.5 - 0.3\ r^r - 0.4\ r^s + 0.1r^r r^s = 4.5$$

represents an intercept of the hyperbola branch MLN which passes through a central point and one of the vertices. This section can be plotted by bringing the equation for I_{im} into canonical form or by substituting certain fixed values of r^r and r^s.

The constraint

$$M_{im} = 3.5 + 0.5r^r - 0.5r^s \leqslant 4$$

represents an intercept of the line PLQ passing through the points with coordinates $(-1;\ 0)$ and $(0;\ +1)$.

The constraints so imposed limit the field of permissible solutions to a polygon $PLNG$.

The relation

$$t_{st} = [50 - 12r^r + 2r^s + 2r^r r^s]\ (10^{-1})$$

defines the family of curves A_1B_1, A_2B_2, A_3B_3 . . ., etc. On each of these curves the value of t_{st} is constant. In moving from, say, A_1B_1 to A_2B_2, the value of time t_{st} decays. The point L is the solution to the problem since at this point the time t_{st} takes a minimum value upon satisfying the constraints set up on I_{im} and M_{im}. The coordinates of the optimum point or the values of the optimum parameters of the machine can be found directly on the grid of Fig. A12:

$$r^s_{opt} = -0.45,\ \ r^r_{opt} = 0.58,\ \ t_{st} = 4.16\ \text{s}$$

4. To estimate the accuracy of the solution, we should compare the calculated data with the simulation results for the optimization criterion and constraint functions at the optimum point. For this it is necessary to convert from the relative units used in designing the experiment to the real units. The conversion is accomplished by the formula

$$\bar{x}_i = (x_i - x_{i\,m})/x_{i\,m} - x_{i\,\min}$$

where $\bar{x}_i$ is the running value of the motor parameter in relative units; $x_{i\,m}$ is the mean basic value of the variable parameter of the motor; $x_{i\,\min}$ is the lower limit of the variable parameter; and $\tilde{x}_i$ is the running value of the motor parameter in real units.

Thus, for r^r varied over the range of up to 20% (see Table AIV.3),

$$x_i = r^r = 3.8,\ x_{i\,\min} = r^r_{\min} = 3.4$$

The optimum value of the rotor resistance r^r_{opt} in relative units is equal to 0.58. So, $0.58 = (r^r_{opt} - 3.8)/(3.8 - 3.04)$ or $r^r_{opt} = 4.24\Omega$.

In a similar way we calculate the stator resistance, $r^s_{opt} = 3.25\Omega$, and then substitute the found values of the optimum parameters into the system of equations of the induction motor with the simulation in terms of currents or flux linkages. In accordance with the optimum parameters of the motor, we recalculate the gain factors of the analog and set up the problem on an analog computer to measure the optimization criterion and constraint functions for the optimum point.

Table AIV.1

Assignment Variants

Variant No.	Motor parameter					
	$r^r_\alpha = r^r_\beta$	$r^s_\alpha = r^s_\beta$	$L^s_\alpha = L^s_\beta$	M	$L^r_\alpha = L^r_\beta$	p
1	3.8	3.57	0.279	0.263	0.289	3
2	4	3.7	0.285	0.26	0.28	2
3	3.6	3.7	0.285	0.26	0.28	1
4	4	3.5	0.285	0.26	0.28	3
5	3.6	3.5	0.285	0.26	0.28	3
6	4	3.7	0.285	0.25	0.28	2
7	3.6	3.7	0.285	0.25	0.28	3
8	4	3.5	0.285	0.25	0.28	3
9	3.6	3.5	0.285	0.25	0.28	2
10	4	3.7	0.275	0.26	0.28	2
11	3.6	3.7	0.275	0.26	0.28	3

Table AIV.1 (*continued*)

Variant No.	Motor parameter					
	$r^r_\alpha = r^r_\beta$	$r^s_\alpha = r^s_\beta$	$L^s_\alpha = L^s_\beta$	M	$L^r_\alpha = L^r_\beta$	p
12	4	3.5	0.275	0.26	0.28	2
13	3.6	3.5	0.275	0.26	0.28	3
14	4	3.7	0.275	0.25	0.28	1
15	3.6	3.7	0.275	0.25	0.28	2
16	4	3.5	0.275	0.25	0.28	3
17	3.6	3.5	0.285	0.25	0.28	1
18	4	3.7	0.285	0.26	0.27	3
19	3.6	3.7	0.285	0.26	0.27	2
20	4	3.5	0.285	0.26	0.27	3
21	3.6	3.5	0.285	0.26	0.27	2
22	4	3.7	0.285	0.25	0.27	3
23	3.6	3.7	0.285	0.25	0.27	1
24	4	3.5	0.285	0.25	0.27	3

Notes. 1. For all variants the following values are to be set: $J = 152 \times 10^{-5}$ [kg m²], $m = 3$, and $U_n = 220/380$ V.

2. Even serial numbers of the variants involve simulation in terms of currents, and odd serial numbers in terms of flux linkages

Table AIV.2

Data for the Experiment to be Designed

Variant No.	Variable parameter			Machine factor (objective function)			Variation range, %
1	r^r	r^s	J	t_{st}	I_{im}	M_{im}	20
2	r^r	r^s	M	t_{st}	I_{im}	M_{im}	20
3	r^r	J	M	I_{im}	t_{st}	—	20
4	r^s	J	M	I_{im}	t_{st}	—	20
5	r^r	r^s	J	t_{st}	I_{im}	—	20
6	r^r	r^s	M	I_{im}	M_{im}	—	20
7	r^r	J	M	I_{im}	M_{im}	—	20
8	r^s	J	M	M_{im}	—	—	20
9	r^r	r^s	J	M_{im}	—	—	20
10	r^r	J	M	I_{im}	—	—	20
11	r^r	J	M	t_{st}	—	—	20
12	r^s	J	M	t_{st}	—	—	20
13	r^r	r^s	J	t_{st}	—	—	20
14	r^r	r^s	M	M_{im}	—	—	20

Table AIV.2 (*continued*)

Variant No.	Variable parameter			Machine factor (objective function)			Variation range, %
15	r^r	J	M	t_{st}	M_{im}	—	20
16	r^r	J	M	I_{im}	—	—	30
17	r^r	r^s	M	I_{im}	—	—	30
18	r^r	r^s	J	M_{im}	I_{im}	—	30
19	r^r	M	J	t_{st}	—	—	30
20	r^s	M	J	M_{im}	t_{st}	—	30
21	r^r	r^s	M	M_{im}	I_{im}	—	30
22	r^r	r^s	J	M_{im}	—	—	30
23	r^r	r^s	J	I_{im}	t_{st}	—	30
24	r^s	M	J	t_{st}	I_{im}	—	30

Table AIV.3

The Gains of Amplifiers of the Computer Analog

Variable parameters		r^r (x_1), Ω	r^s (x_2), Ω	J $(x_3)\times10^{-5}$, kg m²					Gain of the analog						Objective function	
Reference		3.8	3.57	1.52												
Upper level	+	4.52	4.28	1.82												
Lower level	—	3.08	2.86	1.22	x_1x_2	x_1x_3	x_2x_3									
Designation	Test No.	z_1	z_2	z_3	z_4	z_5	z_6	z_0	$k_{2-5}=k_{2-6}$	$k_{3-5}=k_{3-6}$	$k_{1-7}=k_{1-8}$	$k_{2-7}=k_{2-8}$	$k_{1-12}=k_{2-12}$	$k_{1-10}=k_{2-16}$	M_{im}	t_{st}
	1	—	—	—	+	+	+	+	1.252	1.138	1.285	1.211	2.77	1.195	—	—
	2	+	—	—	—	—	+	+	1.252	1.138	2.39	2.25	2.77	1.195	—	—
	3	—	+	—	—	+	—	+	2.33	2.11	1.285	1.211	2.77	1.195	—	—
	4	+	+	—	+	—	—	+	2.33	2.11	2.39	2.35	2.77	1.195		
	5	—	—	+	+	—	—	+	1.252	1.128	1.285	1.211	1.85	0.644	—	—
	6	+	—	+	—	+	—	+	1.252	1.138	2.39	2.25	1.85	0.644	—	—
	7	—	+	+	—	—	+	+	2.33	2.11	1.285	1.211	1.85	0.644	—	—
	8	+	+	+	+	+	+	+	2.33	2.11	2.39	2.25	1.85	0.644	—	—

Table AIV.4

The Values of the Student t-Test at 95% Confidence Level

f	t	f	t	f	t
1	12.71	11	2.20	21	2.08
2	4.30	12	2.18	22	2.07
3	3.18	13	2.16	23	2.07
4	2.78	14	2.14	24	2.06
5	2.57	15	2.13	25	2.06
6	2.45	16	2.12	26	2.06
7	2 36	17	2.11	27	2.05
8	2.31	18	2.10	28	2.05
9	2.26	19	2.09	29	2.04
10	2.23	20	2.09	∞	1.96

Table AIV.5

Fisher's Variance Ratios at 95% Confidence Level

f_2	f_1								
	1	2	3	4	5	6	12	24	∞
1	164.4	199.5	215.7	224.6	230.2	234.0	244.9	249.0	254.3
2	18.5	19.2	19.2	19.3	19.3	19.3	19.4	19.4	19.5
3	10.1	9.6	9.3	9.1	9.0	8.9	8.7	8.6	8.5
4	7.7	6.9	6.6	6.4	6.3	6.2	5.9	5.8	5.6
5	6.6	5.8	5.4	5.2	5.1	5.0	4.7	4.5	4.4
6	6.0	5.1	4.8	4.5	4.4	4.3	4.0	3.8	3.7
7	5.5	4.7	4.4	4.1	4.0	3.9	3.6	3.4	3.2
8	5.3	4.5	4.1	3.8	3.7	3.6	3.3	3.1	2.9
9	5.1	4.3	3.9	3.6	3.5	3.4	3.1	2.9	2.7
10	5.0	4.1	3.7	3.5	3.3	3.2	2.9	2.7	2.5
11	4.8	4.0	3.6	3.4	3.2	3.1	2.8	2.6	2.4
12	4.8	3.9	3.5	3.3	3.1	3.0	2.7	2.5	2.3
13	4.7	3.8	3.4	3.2	3.0	2.9	2.6	2.4	2.2
14	4.6	3.7	3.3	3.1	3.0	2.9	2.5	2.3	2.1
15	4.5	3.7	3.3	3.1	2.9	2.8	2.5	2.3	2.1
16	4.5	3.6	3.2	3.0	2.9	2.7	2.4	2.2	2.0
17	4.5	3.6	3.2	3.0	2.8	2.7	2.4	2.2	2.0
18	4.4	3.6	3.2	2.9	2.8	2.7	2.3	2.1	1.9
19	4.4	3.5	3.1	2.9	2.7	2.6	2.3	2.1	1.9

Table AIV.5 (*continued*)

f_2	f_1								
	1	2	3	4	5	6	12	24	∞
20	4.4	3.5	3.1	2.9	2.7	2.6	2.3	2.1	1.8
22	4.3	3.4	3.1	2.8	2.7	2.6	2.2	2.0	1.8
24	4.3	3.4	3.0	2.8	2.6	2.5	2.2	2.0	1.7
26	4.2	3.4	3.0	2.7	2.6	2.5	2.2	2.0	1.7
28	4.3	3.4	3.0	2.8	2.6	2.5	2.2	2.0	1.7
30	4.2	3.3	2.9	2.7	2.5	2.5	2.1	1.9	1.6
40	4.1	3.2	2.9	2.6	2.5	2.3	2.0	1.8	1.5
60	4.0	3.2	2.8	2.5	2.4	2.3	1.9	1.7	1.4
120	3.9	3.1	2.7	2.5	2.3	2.2	1.8	1.6	1.3
∞	3.8	3.0	2.6	2.4	2.2	2.1	1.8	1.5	1.0

Table AIV.6

Optimization Criterion and Constraint Functions

Variant No.	Optimization criterion	Constraint functions	
1	t_{st} min	$I_{im} \leqslant 0.35$	$M_{im} \leqslant 0.8$
2	I_{im} max	$t_{st} \leqslant 6$	$M_{im} \leqslant 0.8$
3	η_{max}	$t_{st} \leqslant 6$	$I_{im} \leqslant 0.37$
4	I_{im} min	$\eta \geqslant 0.83$	$t_{st} \leqslant 5$
5	t_{st} min	$\eta \geqslant 0.83$	$I_{im} \leqslant 0.37$
6	I_{im} min	$M_{im} < 0.85$	$\eta \geqslant 0.85$
7	η_{max}	$I_{im} \leqslant 0.35$	$M_{im} \leqslant 0.8$
8	η_{max}	$M_{im} \leqslant 0.85$	$\cos\varphi \geqslant 0.8$
9	$\cos\varphi_{max}$	$M_{im} \leqslant 0.85$	$\eta \geqslant 0.85$
10	η_{max}	$\cos\varphi \geqslant 0.8$	$I_{im} \leqslant 0.37$
11	I_{im} min	$\eta \geqslant 0.83$	$\cos\varphi \geqslant 0.8$
12	t_{st} min	$\eta \geqslant 0.83$	$\cos\varphi \geqslant 0.82$
13	η_{max}	$\cos\varphi \geqslant 0.8$	$t_{st} \leqslant 5$
14	η_{max}	$M_{im} \leqslant 0.85$	$t_{st} \leqslant 5$
15	t_{st} min	$\eta \geqslant 0.83$	$M_{im} \leqslant 0.8$
16	I_{im} min	$\cos\varphi \geqslant 0.8$	$\eta \geqslant 0.85$
17	η max	$I_{im} \leqslant 0.35$	$\cos\varphi \geqslant 0.8$
18	I_{im} min	$M_{im} \leqslant 0.75$	$\eta \geqslant 0.82$
19	t_{st} min	$\eta \geqslant 0.82$	$\cos\varphi \geqslant 0.8$
20	η_{max}	$M_{im} \leqslant 0.75$	$t_{st} \leqslant 4.35$
21	η_{max}	$I_{im} \leqslant 0.35$	$M_{im} \leqslant 0.75$
22	$\cos\varphi_{max}$	$M_{im} \leqslant 0.75$	$\eta \geqslant 0.82$
23	η_{max}	$t_{st} \leqslant 4.35$	$I_{im} \leqslant 0.35$
24	I_{im} min	$\eta \geqslant 0.85$	$t_{st} \leqslant 4.35$

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